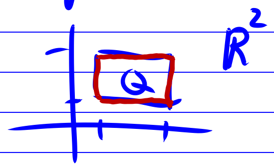
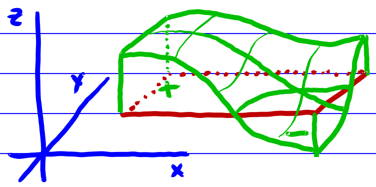
Kap. 10: Integration in $\mathbb{R}^n$

geom. Bed. des Flächenintegrals

$Q := [a,b] \times [c,d]$



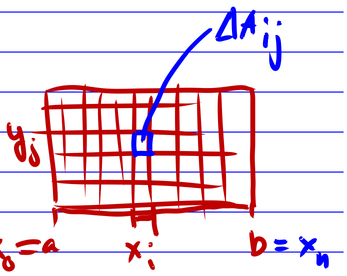
$f : Q \to \mathbb{R}$ st.



$$\int_Q f(x,y)\,dA = \text{signiertes Volumen zw. } xy\text{-Ebene u. graph}(f)$$

$$\approx \sum_{i=1}^{n} \sum_{j=1}^{m} f(x_i, y_j)\, \underbrace{\Delta x_i\, \Delta y_j}_{\Delta A_{ij}}$$

$$\sum_{i=1}^{n} \Big(\overbrace{\sum_{j=1}^{m} f(x_i,y_j)\, \Delta y_j}^{\int_c^d f(x,y)\,dy} \Big) \Delta x_i$$



$\Delta x_i = x_i - x_{i-1}$ klein

$\Delta y_j = y_j - y_{j-1}$

legt nahe, dass

$$\int_Q f(x,y)\,dA = \int_a^b \left(\int_c^d f(x,y)\,dy \right) dx$$

$$= \int_c^d \left(\int_a^b f(x,y)\,dx \right) dy$$

Wir nehmen das als Def, müssen aber zeigen:

10.5 Satz von Fubini

Sei $f: [a,b] \times [c,d] \to \mathbb{R}$ st., dann

$$\int_a^b \left(\int_c^d f(x,y)\, dy \right) dx = \int_c^d \left(\int_a^b f(x,y)\, dx \right) dy$$

Vgl. $\sum_i \sum_j a_{ij} = \sum_j \sum_i a_{ij}$

$$\sum_i \sum_j \quad \begin{matrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{matrix}, \qquad \sum_j \sum_i \quad \begin{matrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{matrix}$$