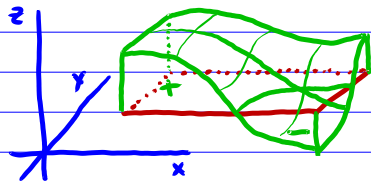
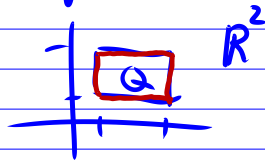


Kap. 10: Integration in \mathbb{R}^n

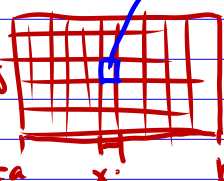
geom. Bed. des Flächenintegrals

$$Q := [a, b] \times [c, d]$$

$$f: Q \rightarrow \mathbb{R} \text{ st.}$$



$\int_Q f(x, y) dA =$ signiertes Volumen
zw. xy -Ebene u. Graph(f)

$$\approx \sum_{i=1}^n \left(\sum_{j=1}^m \int_c^d f(x, y) dy \right) \Delta x_i \Delta y_j$$
A red grid is drawn in the xy-plane. The x-axis is labeled with $x_0 = a$, x_i , and $b = x_n$. The y-axis is labeled with y_j . A small red rectangle is highlighted in the grid, labeled ΔA_{ij} .

$\Delta A_{ij} = \Delta x_i \Delta y_j$

$$\Delta x_i = x_i - x_{i-1} \text{ klein}$$

$$\Delta y_j = y_j - y_{j-1}$$

legt nahe, dass

$$\int_a^b \int_c^d f(x, y) dA = \int_a^b \left(\int_c^d f(x, y) dy \right) dx$$

$$= \int_c^d \left(\int_a^b f(x, y) dx \right) dy$$

Wir nehmen das als Def, müssen aber zeigen:

10.5 Satz von Fubini

Sei $f: [a,b] \times [c,d] \rightarrow \mathbb{R}$ st., dann

$$\int_a^b \left(\int_c^d f(x,y) dy \right) dx = \int_c^d \left(\int_a^b f(x,y) dx \right) dy$$

Vgl. $\sum_i \sum_j a_{ij} = \sum_j \sum_i a_{ij}$

$$\sum_i \sum_j \begin{array}{ccc} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{array}, \quad \sum_j \sum_i \begin{array}{ccc} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{array}$$