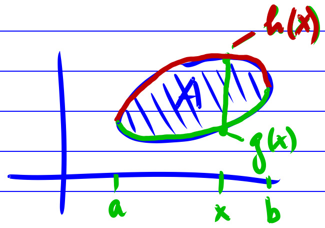
Def $$\int\limits_{[a,b]\times[c,d]} f(x,y)\,d(x,y) := \int_c^d dy \int_a^b dx\; f(x,y)$$

Flächenintegral

Wollen



$$\int_A f(x,y)\,d(x,y) = \int_a^b dx \int_{g(x)}^{h(x)} dy\; f(x,y)$$

10.6 Def $A \subset \mathbb{R}^2$ heißt x-Normalbereich $\Longleftrightarrow$

$\exists\, [a,b] \subset \mathbb{R}\ \exists\, g,h : [a,b] \to \mathbb{R}$ st.:

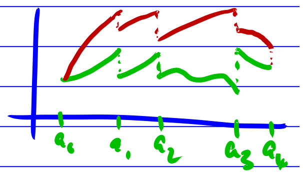
g, h st. diffbar auf (a,b) und

$$A = \{(x,y) \in \mathbb{R}^2 \mid a \le x \le b,\ g(x) \le y \le h(x)\}$$

oder stückweise so

(d.h. $a = a_0 < a_1 < \dots < a_r = b$

$g_i, h_i \in C^1(a_i, a_{i+1})$



$\exists \lim\limits_{x \nearrow a_{i+1}} g_i(x),\ h_i(x)$

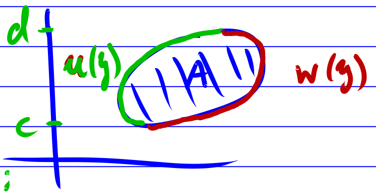
$\exists \lim\limits_{x \searrow a_i} g_i(x),\ h_i(x)$)

A heißt y-Normalbereich $\Longleftrightarrow$

$\exists\, [c,d] \subset \mathbb{R}$

$\exists\, u, w :$

$[c,d] \to \mathbb{R}$ st.:



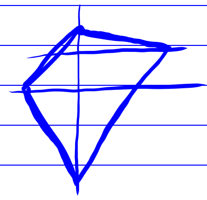
u, w auf (c,d) st. diffbar und

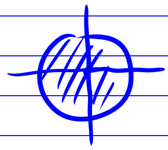
$$A = \{(x,y) \in \mathbb{R}^2 \mid c \le y \le d,\ u(y) \le x \le w(y)\}$$

A heißt Normalbereich $\Longleftrightarrow$

A ist x-Norm.ber. und y-Norm.ber.

Bsp 10.7 • $[a,b]\times[c,d]$ ist Norm. Ber.

•  • 

• 

$g(x) = -\sqrt{1-x^2}$

$u(y) = -\sqrt{1-y^2}$

Def Sei $A \subset \mathbb{R}^2$, $f: A \to \mathbb{R}$ st.

Falls A x-Norm. Ber. dann

$$\int_A f(x,y)\,d(x,y) := \overbrace{\int_a^b dx \int_{g(x)}^{h(x)} dy\; f(x,y)}^{I_x}$$

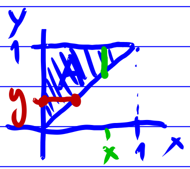
y-Norm. Ber.:

$$:= \underbrace{\int_c^d dy \int_{u(y)}^{w(y)} dx\; f(x,y)}_{I_y}$$

Satz Falls A Norm. Ber., dann
$I_x = I_y$.

(ohne Beweis)

Bsp $A = \{(x,y) \in \mathbb{R}^2 \mid 0 \le x \le y \le 1\}$

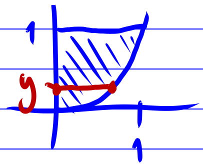


$$\int_A f(x,y)\,d(x,y)$$

$$= \int_0^1 dx \int_x^1 dy\; f(x,y)$$

$$= \int_0^1 dy \int_0^y dx\; f(x,y)$$

$$B = \{(x,y)\in\mathbb{R}^2 : 0\le x\le 1,\ x^2 \le y \le 1\}$$



$$\int_B f(x,y)\, d(x,y) = \int_0^1 dx \int_{x^2}^1 dy\ f(x,y)$$

$$= \int_0^1 dy \int_0^{\sqrt{y}} dx\ f(x,y)$$

Def 10.10 Sei A Norm. ber.,

$$F(A) := \int_A d(x,y)$$

$(f=1)$ heißt die Fläche von A.

Def Der Mittelwert einer Fkt $f: A \to \mathbb{R}$ ist

$$\bar{f} := \langle f \rangle := \frac{1}{F(A)} \int_A f(x,y)\, d(x,y)$$

Def Der Schwerpunkt von A ist $\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}$.

Ebenso in $\mathbb{R}^n$:

Normalbereiche in $\mathbb{R}^n$: $\exists g,h: D \to \mathbb{R}$, $\quad \mathbb{R}^{n-1} \supset D$ Norm. ber.

$$A = \{(x_1 \dots x_n) \in \mathbb{R}^n : g(x_1..x_{n-1}) \le x_n \le h(x_1..x_{n-1})\}$$

$$Vol(A) = \int_A d(x_1,\dots,x_n)$$

Volumenintegral $\int_A f(x_1 \dots x_n)\, d(x_1 \dots x_n) =$

$$\int dx_1 \int dx_2 \cdots \int dx_n\ f(x_1 .. x_n)$$

Notation $d(x_1,\dots,x_n) = dV = \underset{n=2}{dA} = d\underline{x} = d^n\underline{x}$

$= dx_1 dx_2 \cdots dx_n$