Nochmal Wegintegral:

Invarianz unter Reparametrisierung

Erinnerung $\int_\gamma \underline{v} \cdot d\underline{x}$

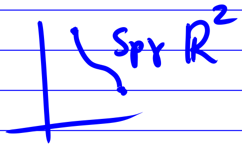
Für $\gamma \in C^1([a,b], G)$, $G \subset \mathbb{R}^n$ offen, $\underline{v} \in C^0(G, \mathbb{R}^n)$ heißt

$$\int_\gamma \underline{v} \cdot d\underline{x} := \int_a^b dt \, \langle \underline{v}(\underline{\gamma}(t)), \underline{\dot{\gamma}}(t) \rangle \in \mathbb{R}$$

das Wegintegral von v entlang γ

Spur $\quad \mathrm{Sp}\,\gamma = \gamma([a,b]) = \mathrm{Bild}\,\gamma$

$= \text{durchlaufene Menge}$



Reparametrisierung

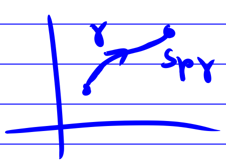
$\phi : [c,d] \to [a,b]$ Diffeo

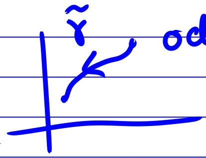
$\tilde{\gamma} := \gamma \circ \phi$, $\tilde{\gamma} : [c,d] \to G$, C^1

$\mathrm{Sp}\,\tilde{\gamma} = \mathrm{Sp}\,\gamma$.

("$\mathrm{sgn}(\phi') = +1$")

Prop 10.21 Entweder stets $\phi' > 0$





oder stets $\phi' < 0$

("$\mathrm{sgn}(\phi') = -1$")

und $\forall \underline{v} \in C^0(G, \mathbb{R}^n)$

$$\int_{\tilde{\gamma}} \underline{v} \cdot d\underline{x} = \mathrm{sgn}(\phi') \int_\gamma \underline{v} \cdot d\underline{x}$$

Bew ϕ Diffeo $\Rightarrow \phi'(t) \neq 0 \;\; \forall t.$

Substitutionsregel in $\mathbb{R}^1$

$\tilde{\gamma} = \gamma \circ \phi$

$$\int_{\tilde{\gamma}} \underline{v} \cdot d\underline{x} = \int_c^d dt \left\langle \underline{v}(\underline{\tilde{\gamma}}(t)), \underline{\dot{\tilde{\gamma}}}(t) \right\rangle \overset{\text{Kettenregel}}{=}$$

$$= \int_c^d dt \left\langle \underline{v}(\underline{\gamma}(\phi(t))), \dot{\gamma}(\phi(t))\, \phi'(t) \right\rangle$$

$s = \phi(t)$

$ds = \phi'(t)\, dt$

$$= \int_a^b ds \left\langle \underline{v}(\underline{\gamma}(s)), \underline{\dot{\gamma}}(s) \right\rangle \operatorname{sgn}(\phi')$$

$$= \operatorname{sgn}(\phi') \cdot \int_{\gamma} \underline{v} \cdot d\underline{x} \qquad \square$$