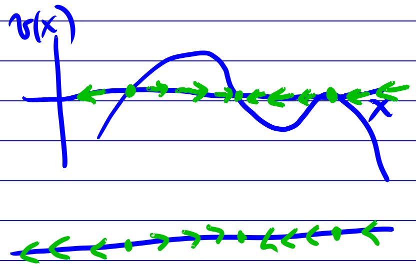
Phasendiagramm (autonom)

$\dot{x} = v(x)$ $\qquad n=1$

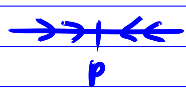


Def Für die ODE $\dot{x} = v(x)$ $\quad (n=1)$

heißt ein stationärer Pkt. p $(v(p)=0)$

<u>stabil</u>, wenn $\exists\, \varepsilon > 0$:

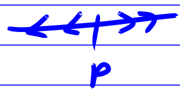
$\forall x \in (p-\varepsilon, p): v(x) > 0$



und $\forall x \in (p, p+\varepsilon): v(x) < 0$

<u>instabil</u>, wenn $\exists\, \varepsilon > 0$:

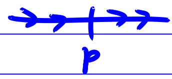
$\forall x \in (p-\varepsilon, p): \ v(x) < 0$



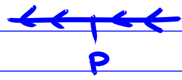
und $\forall x \in (p, p+\varepsilon): \ v(x) > 0$

<u>halbstabil</u>, wenn $\exists\, \varepsilon > 0$:

$v > 0$ auf $(p-\varepsilon, p+\varepsilon) \setminus \{p\}$



oder $\quad v < 0$ auf $(p-\varepsilon, p+\varepsilon) \setminus \{p\}$



Dauer bis p?

Wenn $v \in C^1$, dann braucht $x(t)$ ∞ lange Zeit, um p zu erreichen.

Heuristisch: (Bew. später)

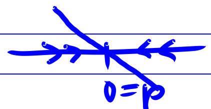
Wenn $v(p)=0$ und $v'(p)\neq 0$, dann

$v(x) \approx (x-p)\, v'(p)$

Angenommen $v(x) = c\,(x-p)$

$\mathbb{R} \ni c \neq 0$, z.B. $p=0$

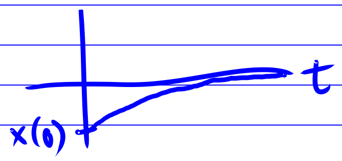
$c<0$



Sep. Var. $\Longrightarrow$ $t = \int_{x(0)}^{x(t)} \frac{dx'}{cx'} = \frac{1}{c}\Big(\underbrace{\ln|x(t)| - \ln|x(0)|}_{\ln\left|\frac{x(t)}{x(0)}\right|}\Big)$

$\Rightarrow$ $\underline{\underline{x(t) = x(0)\, e^{ct}}}$

$\dot{x} = cx$



∞ lange bis p.

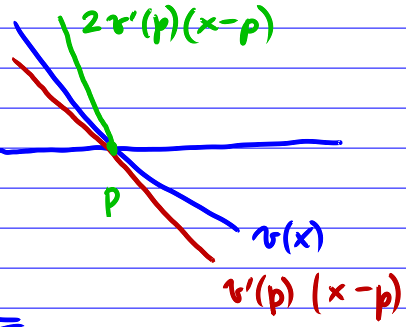
Oder direkt: Zeit T bis p

$T = \int_{x(0)}^{p} \frac{dx'}{v(x')}$. $\quad v(x) = v'(p)(x-p) + o(|x-p|)$

(z.B. $v'(p) < 0$)

Für $p-\varepsilon < x < p$

gilt $0 \le v(x) \le \underbrace{2v'(p)(x-p)}$

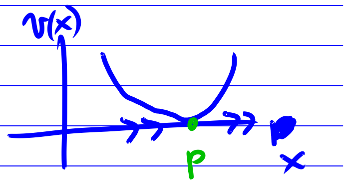


$T \geq \int_{x(0)}^{p} \frac{dx'}{2v'(p)(x'-p)} =$

$$= \frac{1}{2v'(p)} \int_{-\varepsilon}^{0} \frac{du}{u} = \infty$$

Bsp $\quad v(x) = c\,(x-p)^2$

$c>0, \quad p=0$

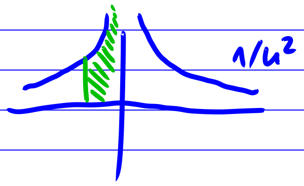


Sep. Var. $\Rightarrow \quad t = \int_{x(0)}^{x(t)} \frac{dx'}{c\,x'^2} = \frac{1}{c}\left(-\frac{1}{x(t)} + \frac{1}{x(0)}\right)$

$$\Rightarrow \quad x(t) = \frac{1}{a - ct} \qquad \left(a = \frac{1}{x(0)} < 0\right)$$

$x(t) \xrightarrow{t\to\infty} 0 \quad$ (∞ lange bis p)

Oder direkt: $\quad T = \int_{x(0)}^{p} \frac{dx'}{c(x'-p)^2} = \frac{1}{c}\int_{-\varepsilon}^{0} \frac{du}{u^2}$



$= \infty$