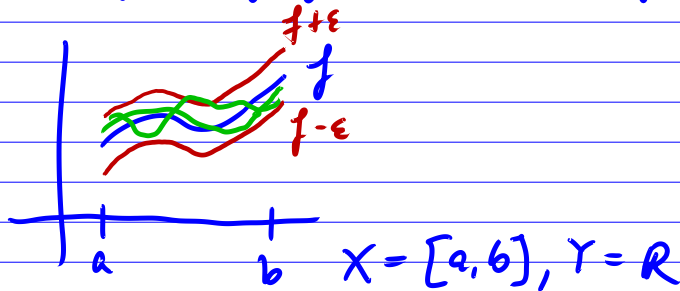


Rep.: Pktweise und glm. Konv.

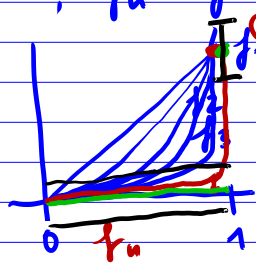
Satz Seien X, Y metr. R.e., $f_n, f: X \rightarrow Y$
 f_n stetig, $f_n \rightarrow f$ glm. Dann ist f stetig.



Möglich oder nicht? Jeweils $f_n, f: [a, b] \rightarrow \mathbb{R}$

1) f_n st. $\forall n$, $f_n \rightarrow f$ pktweise, f nicht st.

~~Nein:~~
Ja:



$$f_n(x) = x^n$$

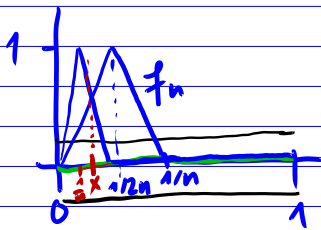
$$f(x) = \begin{cases} 0 & \text{für } x < 1 \\ 1 & \text{für } x = 1 \end{cases}$$

nicht glm.

2) f_n st. $\forall n$, $f_n \rightarrow f$ pktw. aber nicht glm.

f st.

Ja:

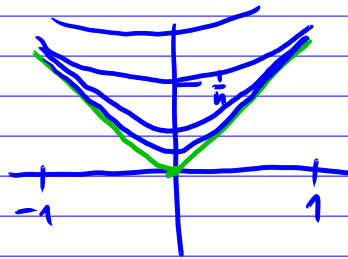


$$f_n(x) = \begin{cases} 0 & \text{für } x \geq \frac{1}{n} \\ 2nx & \text{für } 0 \leq x \leq \frac{1}{2n} \\ 2-2nx & \text{für } \frac{1}{2n} \leq x \leq \frac{1}{n} \end{cases}$$

$$f(x) = 0 \quad \forall n > \frac{1}{x} \\ \forall x > 0$$

3) f_n st. diffbar $\forall n$, $f_n \rightarrow f$ glim.
 f nicht diffbar

Ja:



$$f(x) = |x|$$

$$f_n(x) = \sqrt{x^2 + \frac{1}{n^2}}$$

$$|f_n(x) - f(x)| \leq \frac{1}{n}$$