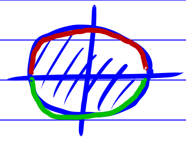
Bspe zum Flächenintegral

Bsp 10.9

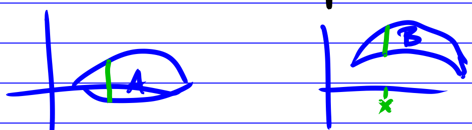
$$A := \overline{B_1(0)}$$



$$F(A) = \int_A d(x,y)$$

$$= \int_{-1}^{1} dx \int_{\underbrace{-\sqrt{1-x^2}}_{g(x)}}^{\overbrace{\sqrt{1-x^2}}^{h(x)}} dy = \int_{-1}^{1} dx \; \underbrace{2\sqrt{1-x^2}}_{(h(x)-g(x))}$$

Cavalierisches Prinzip:



Wenn $\forall x: h_A(x) - g_A(x) = h_B(x) - g_B(x)$

dann $F(A) = F(B)$.

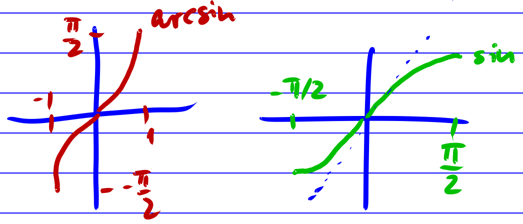
$$\frac{d}{dx}\left(x\sqrt{1-x^2} + \arcsin x\right)$$

$$= \sqrt{1-x^2} + x\,\frac{1}{2}\underbrace{(1-x^2)^{-\frac{1}{2}}}_{\frac{1}{\sqrt{1-x^2}}}(-2x) + \frac{1}{\sqrt{1-x^2}}$$

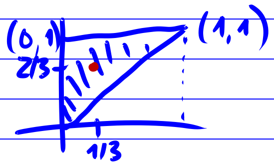
$$= \frac{1-x^2 - x^2 + 1}{\sqrt{1-x^2}} = 2\sqrt{1-x^2}$$

Also $F(A) = \left[x\sqrt{1-x^2} + \arcsin x \right]_{-1}^{1}$

$= \frac{\pi}{2} - \left(-\frac{\pi}{2}\right)$



$= \pi$.

Bsp $A =$ 

$= \{ 0 \le x \le y \le 1 \}$

Schwerpunkt = ?

Schwerpunkt $= \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$

$$\bar{x} = \frac{1}{F(A)} \int_A x \, d(x,y) .$$

$$F(A) = \int_A d(x,y) = \int_0^1 dx \int_x^1 dy$$

$$= \int_0^1 dx \,(1-x) = \left[x - \frac{1}{2}x^2 \right]_0^1$$

$$= 1 - \frac{1}{2} = \frac{1}{2} .$$

$$\bar{x} = 2\int_0^1 dx \left(\int_x^1 dy \; x \right) = 2\int_0^1 dx \; x(1-x)$$

$$= 2\left[\frac{1}{2}x^2 - \frac{1}{3}x^3 \right]_0^1 = 2\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{2}{6} = \frac{1}{3} .$$

$$\bar{y} = 2\int_0^1 dx \int_x^1 dy\, y = 2\int_0^1 dx \left[\frac{1}{2}y^2\right]_x^1$$

$$= 2\int_0^1 dx \left(\frac{1}{2} - \frac{1}{2}x^2\right) = \int_0^1 dx \left(1 - x^2\right)$$

$$= \left[x - \frac{1}{3}x^3\right]_0^1 = 1 - \frac{1}{3} = \frac{2}{3}.$$

$$\bar{y} = 2\int_0^1 dy \int_0^y dx\, y = 2\int_0^1 dy\, y\, y$$

$$= 2 \left[\frac{1}{3}y^3\right]_0^1 = \frac{2}{3}.$$