HINTS FOR THE EXAM

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The exam will consist of exercises in the style of the homework problems. The problems posed at the exam will be a subset of the types of problems listed below. Examples of such problems either appeared in the lecture or in the problems of the exercise class. References to such problems are given (with numbering according to the handwritten lecture notes and the exercise classes).

- Show that a given set together with charts defines a smooth manifold (Example 1.8, Problem 1)
- Show that a given map between manifolds is smooth (Problem 5)
- Show that a given subset is a submanifold and compute its tangent space (Problem 9, Example 2.22, Example 9.22)
- Compute coordinate changes of vector fields, 1-forms and tensor fields (Problems 7, 13, Example 4.22 (d))
- Compute the Hodge dual and the exterior derivative of a differential form (Problems 17, 20, 21, Example 5.20)
- Compute the volume form of a given Riemannian metric (Definition 6.18, Problem 15)
- Compute the integral curve of a vector field (Problems 24,26, Example 7.12, 7.15). The functions in Definition 14.13 may be also be useful.
- Compute the pullback of a tensor field along a map (Problems 27, 32, 33)
- Determine whether an object defines a tensor field (Def 10.12, Lemma 13.1) and compute its local coefficients (Problems 28, 36, 43)
- Do calculations with tensors involving the covariant derivative, contractions, tensor products, rasing and lowering indices (Problem 37, Lemma 13.30, Problem 48)
- Be able to use computational rules for the Lie-derivative and the covariant derivative (Definition 10.12, Definition + Lemma 10.15, Lemma 13.1)
- Find coordinate vector fields of a given chart of a submanifold of $\mathbb{R}^n$ (Example 7.15). Compute components of the metric in that chart (Remark 9.23)
- Compute Christoffel symbols and formulate the geodesic equation in given coordinates (Problem 42)
- Determine whether a given curve is a geodesic in a submanifold (Example 12.25)
- Compute the curvature of some explicit pseudo-Riemannian manifolds (Problems 45, 47), use the formula for the Riemann tensor for constant curvature metrics (Definition 13.20)