Short-time asymptotics of the diffusion of characteristic functions

We consider the heat flow on a Riemannian manifold with the characteristic function of a compact set as initial data and derive a short time asymptotic expansion for the loss of heat in terms of geometric invariants. The method uses an approximation of heat level-sets through the distance function. As an application we show that this expansion implies the isoperimetric inequality on \mathbb{R}^n