Uniqueness of photon spheres and equipotential photon surfaces in geometrostatic spacetimes via potential theory

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Abstract

Equipotential photon surfaces are timelike hypersurfaces in a static spacetime $(\mathscr{L}^{n+1} = M^n \times \mathbb{R}, \mathscr{g} = -N^2 dt^2 + {}^n g)$, which are null totally geodesic and such that the so called lapse function is constant on the connected components of each time slice. Photon spheres are a special case of equipotential photon surfaces, where the lapse function does not even depend on time, namely N is constant on their connected components.

In this talk I will prove an uniqueness theorem for photon spheres and, more generally, for equipotential photon surfaces in a geometrostatic (static, vacuum and asymptotically flat) spacetime of dimension $n + 1 = 4$. This consists in showing that a geometrostatic spacetime $(\mathscr{L}^4, \mathscr{g})$ that admits such objects must be isometric to the Schwarzschild spacetime of the same ADM-mass. For this purpose, an approach via potential theory developed by Agostiniani & Mazzieri in [1] and [2] will be used, allowing not to assume the regular foliation of the spacetime by the so called lapse function, used in many other methods.

Firstly, I will briefly recall the proof of uniqueness of photon spheres by Cederbaum & Galloway in [4], who adapted the proofs of uniqueness of black holes by Bunting & Masood-ul Alam in [3]. Then I will in fact proceed with a similar attitude, considering the application of potential theory to the proof of uniqueness of black holes by Agostiniani and Mazzieri as a starting point.

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References

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