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October 19, 2021

Foundations of Quantum Mechanics: Assignment 1

Exercise 1: Essay question. What got you interested in the foundations of quantum mechanics?

Exercise 2: Plane waves.
Show that for every constant vector $\boldsymbol{k} \in \mathbb{R}^3$ (the *wave vector*), there is a unique constant $\omega \in \mathbb{R}$ so that

$$\psi(t, \boldsymbol{x}) = e^{i\boldsymbol{k}\cdot\boldsymbol{x}} e^{-i\omega t} \tag{1}$$

satisfies the free (i.e., $V = 0$) Schrödinger equation (2.1) for $N = 1$. Specify ω in terms of $\boldsymbol{k}$. Remark: Since for every t, $\|\psi_t\| = \infty$, this function (called a plane wave) is not square-integrable (not normalizable) and thus not physically possible as a wave function; but it is a useful toy example.

Exercise 3: Time reversal invariance.
Show that if $\psi(t, q)$ is a solution of the Schrödinger equation (2.1) with real-valued potential V, then so is $\psi^*(-t, q)$.

Exercise 4: Gaussian wave packet.
(a) Show that the function

$$\psi(\boldsymbol{x}, t) = (2\pi\lambda_t^2\sigma^2)^{-3/4} e^{i\boldsymbol{k}\cdot(\boldsymbol{x}-\hbar\boldsymbol{k}t/2m)} e^{-\frac{(\boldsymbol{x}-\hbar\boldsymbol{k}t/m)^2}{4\lambda_t\sigma^2}} \tag{2}$$

with

$$\lambda_t = 1 + \frac{i\hbar t}{2m\sigma^2} \tag{3}$$

and arbitrary constants $\boldsymbol{k} \in \mathbb{R}^3$, $\sigma > 0$, is a solution of the free Schrödinger equation of a single particle in 3 dimensions,

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi\,. \tag{4}$$

The main difficulty here is to organize the calculation so as to make it manageable.

(b) Show that the probability density $\rho_t(\boldsymbol{x})$ is, at every t, Gaussian and specify its mean and standard deviation.

Hand in: Tuesday October 26, 2021, in class or by 8:15 am via `http://urm.math.uni-tuebingen.de` .

Reading assignment due Thursday October 28, 2021: R. Feynman, *Feynman Lectures on Physics vol. 3*, chapter 1.