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Winter semester 2021/22
December 7, 2021

Foundations of Quantum Mechanics: Assignment 7

Exercise 27: Essay question. Describe the measurement problem of quantum mechanics. (Use formulas where appropriate.)

Exercise 28: Exponential distribution
Find the mean and variance of the exponential distribution with parameter λ; it has density $\rho(t) = 1_{t>0}\lambda e^{-\lambda t}$.

Exercise 29: GRW theory
Consider the GRW theory with the constant σ much smaller than the value 10^{-7} m suggested by GRW; say, $\sigma = 10^{-12}$ m. Explain why Heisenberg's uncertainty relation implies that a free electron, after being hit by a GRW collapse, could move very fast. Use the uncertainty relation to compute the order of magnitude of how fast it can be (assuming it was more or less at rest before the collapse); the mass of an electron is about 10^{-30} kg and $\hbar \approx 10^{-34}$ kg m^2 s^{-1}.

Exercise 30: Poisson process
For the Poisson process with rate $\lambda > 0$, determine for any fixed $t_0 > 0$ the distribution of $X_{t_0} = \#\{k \in \mathbb{N} : T_k \le t_0\}$, the number of events up to time t_0. Follow two reasonings:

(a) Heuristically, assume that an event occurs in every infinitesimal time interval $[t, t+dt]$ independently of disjoint intervals with probability $\lambda\, dt$.
Hint: Divide $[0, t_0]$ in $n \gg 1$ subintervals of length $dt = t_0/n$.

(b) Rigorously, assume that the random variables $T_1, T_2, \ldots$ are defined to be $T_k = W_1 + \ldots + W_k$ with all waiting times W_k independent and exponentially distributed with parameter λ, i.e., with density $\rho(w) = 1_{w>0}\,\lambda\, e^{-\lambda w}$.
Hint:

$$\mathbb{P}\big(X_{t_0} \ge 2\big) = \mathbb{P}\Big(W_1 + W_2 < t_0\Big) = \int_0^{t_0} dw_1 \int_0^{t_0 - w_1} dw_2\, \rho(w_1)\, \rho(w_2) \quad \text{and}$$

$$\mathbb{P}(X_{t_0} = k) \;=\; \mathbb{P}\big(X_{t_0} \ge k\big) - \mathbb{P}\big(X_{t_0} \ge k+1\big)\,.$$

Hand in: by Tuesday December 14, 2021, 8:15am via `urm.math.uni-tuebingen.de`

Reading assignment due Thursday December 16, 2021:
Sections 1–3 and 5 of J. Bell: Are There Quantum Jumps? In *Schrödinger. Centenary Celebration of a Polymath.* Cambridge University Press (1987). Reprinted as chapter 22 in J. Bell, *Speakable and Unspeakable in Quantum Mechanics*, Cambridge University Press (1987).