Bohmian mech

$$\frac{d\underline{Q}_i(t)}{dt} = \frac{\hbar}{m_i} \operatorname{Im} \frac{\nabla_i \Psi}{\Psi} \Big(t, \underbrace{\underline{Q}_1(t), \dots, \underline{Q}_N(t)}_{=: Q(t)}\Big)$$

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi .$$

Born's rule: $Q(t) \sim |\Psi_t|^2$ $\qquad X \sim \mu$

(equivariance thm)

deterministic, state (Q_t, Ψ_t)

eq. of motion

$$\frac{d\underline{Q}_i}{dt} = \frac{\underline{j}_i}{\rho}\Big|_{q=Q_t} = \frac{\text{prob current}}{\text{prob density}}$$

$$\rho = |\Psi|^2$$

$$\underline{j}_i = \frac{\hbar}{m_i} \operatorname{Im}\left(\underline{\Psi}^*(t,q)\, \nabla_i \Psi(t,q) \right)$$

$z = |z| e^{i\varphi}$

∘ another rewrite of eq. of motion.

$$\Psi(t,q) = R(t,q)\, e^{iS(t,q)/\hbar} \Rightarrow \frac{d\underline{Q}_i}{dt} = \frac{1}{m_i} \nabla_i S$$

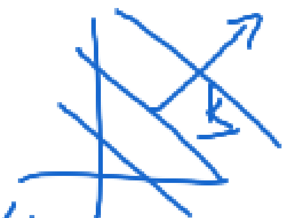
Pf $\quad \dfrac{dQ_i}{dt} = \dfrac{\hbar}{m_i}\,\mathrm{Im}\,\dfrac{\nabla_i \Psi}{\Psi}$

$$\nabla_i \Psi = \nabla_i \left(R e^{iS/\hbar} \right) = (\nabla_i R)\, e^{iS/\hbar} + R\, \nabla_i S \left(\frac{i}{\hbar}\right) e^{iS/\hbar}$$

$$\frac{\hbar}{m_i}\,\mathrm{Im}\,\frac{\nabla_i \Psi}{\Psi} = \underbrace{\cancel{\frac{\nabla_i R}{R}}}_{\text{real}} + \frac{\cancel{i}^{\,1}}{\cancel{\hbar}\, m_i} \nabla_i S .$$

cf. de Broglie relation $\quad \underline{p} = \hbar \underline{k} \quad$ (1924)

$e^{i\underline{k}\cdot\underline{x}}$ plane wave ; wave vector $\underline{k}$



↓ locally plane wave, local wave vector $\nabla S/\hbar$.

History

"guiding field"	A. Einstein	1928
	J. Slater	1924
"pilot wave"	L. de Broglie	1927, abandoned
	N. Rosen	1945, abandoned
	D. Bohm	1952

$$\begin{cases} m_i \dfrac{d^2 Q_i(t)}{dt^2} = \ldots\ldots \\ \text{constraint} \quad \dfrac{dQ_i(0)}{dt} = \dfrac{\hbar}{m_i} \operatorname{Im} \dfrac{\nabla_i \Psi}{\Psi} \end{cases}$$

"<u>Hidden variables</u>"

any variables in addition to ψ

<u>Equivariance</u>

<u>Claim</u> $\rho_t = |\Psi_t|^2 \ \forall t$ if $\rho_0 = |\Psi_0|^2$

i.e. diagram commutes

$$\begin{tikzcd} \Psi_0 \arrow[r, "| \, |^2"] \arrow[d, "U_t"'] \arrow[dr, phantom, "\%"] & \rho_0 \arrow[d, "\text{transport by Bohmian motion}"] \\ \Psi_t \arrow[r, "| \, |^2"'] & \rho_t \end{tikzcd}$$

Pf $\quad \frac{dQ}{dt} = v_t(Q_t)$ $\qquad v_t : \mathbb{R}^{3N} \to \mathbb{R}^{3N}$

$$\underline{v}_i = \frac{\hbar}{m_i} \operatorname{Im} \frac{\nabla_i \Psi}{\Psi}$$

prob. transport:

v_t known, ρ_0 known, $\rho_t = ?$

cont. eq. $\quad \frac{\partial \rho_t}{\partial t} = -\nabla \cdot (\rho_t v_t)$.

Claim 2: If $\rho_t = |\Psi_t|^2$ then $\frac{\partial \rho}{\partial t} = \frac{\partial |\Psi|^2}{\partial t}$.

Pf: We know Schr eq $\Rightarrow \frac{\partial |\Psi|^2}{\partial t} = -\nabla \cdot j$, $\quad \underline{j}_i = \frac{\hbar}{m_i} \operatorname{Im} (\Psi^* \nabla \Psi)$

$v_t = \frac{j_t}{|\Psi_t|^2}$, so if $\rho_t = |\Psi_t|^2$ then $\rho_t v_t = j_t$

$\Rightarrow$ Claim 2.

Uniqueness of sol. of cont. eq.

$\Rightarrow$ if $\rho_0 = |\Psi_0|^2$ then $\rho_t = |\Psi_t|^2$. $\square$

<u>Thm</u> (Dürr et al. 1995) For a ~~sm~~ large class of V's and a dense subspace of ψ_0, $t \mapsto Q_t$ exists globally (i.e., $\forall t \in \mathbb{R}$) with prob 1, and equivariance holds:

$$\mathbb{P}(Q_t \in B) = \int_B |\psi_t(q)|^2 \, dq. \qquad \forall B \subseteq \mathbb{R}^{3N}.$$

Double slit in BM

1 particle (as if alone in the universe).

$$Q_t^{\text{Bohm}} \neq Q_t^{\text{Newton}}$$