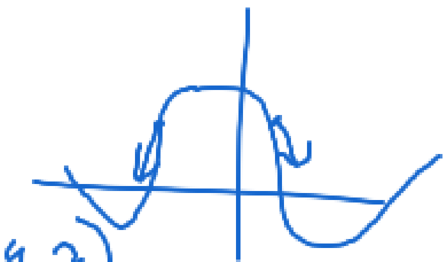
Symmetry of Bohmian trajectories in the 2-slit

$\psi_t(x,y,-z) = \psi_t(x,y,z)$ even in z

f even $\Rightarrow f'$ odd



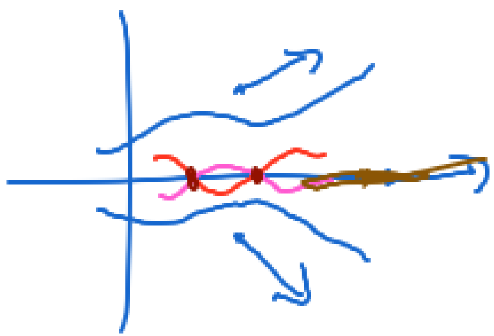
$\partial_z \psi(x,y,-z) = -\partial_z \psi(x,y,z)$

$\partial_x \psi(x,y,-z) = \partial_x \psi(x,y,z)$

$\Rightarrow v = \mathrm{Im} \frac{\nabla \psi}{\psi}$, $v_x(x,y,-z) = v_x(x,y,z)$

$v_z(x,y,-z) = -v_z(x,y,z)$

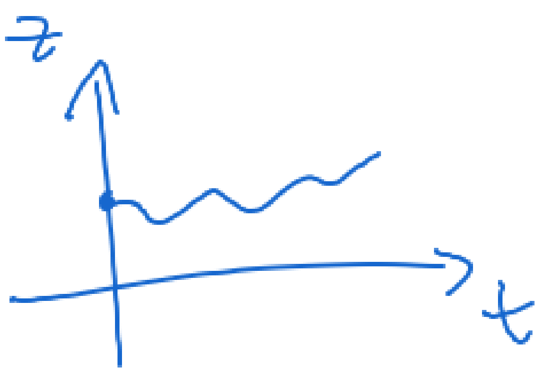
$\Rightarrow v_z(x,y,0) = 0$



Alternative reasoning: ignore y

$v_x = \text{const.}$ $x = t$, model as 1d

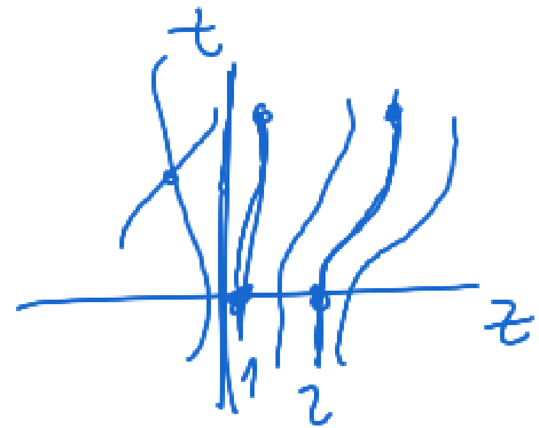
z, t



$\psi_t(z)$ in 1d

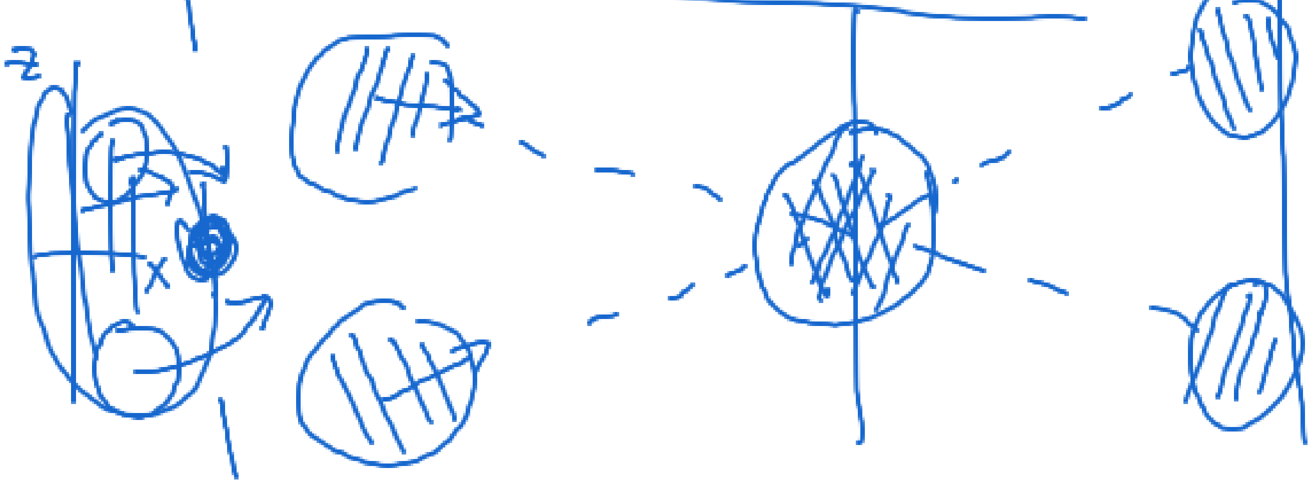
1st-order ODE

by uniqueness



$z(t)=0$ is a ~~sol~~ sol. $\Rightarrow$ if $z(0)>0$ then $z(t)>0\ \forall t$

Delayed-choice experiment (Wheeler 1978)



Wheeler: if screen in interference region,
see interference $\Rightarrow$ el. is a wave
if screen far, ~~el.~~ ~~e~~ see clusters
$\Rightarrow$ el. went through one slit, is a particle.

On top of that, we can choose.

On top of <u>that</u>, we can choose <u>after</u> ~~#~~ the passage

retrocausation.

In BM : (Bell 1980)

- ~~In BM,~~ the BM will account for the experiment
- In BM, $\nexists$ retrocausation

$V(t,x)$, ψ_t

$V_1(t,x), V_2(t,x)$, $V_1(t,x) = V_2(t,x) \ \forall t \le t_0$.

$\Rightarrow \psi_1(t,x) = \psi_2(t,x) \ \forall t \le t_0$

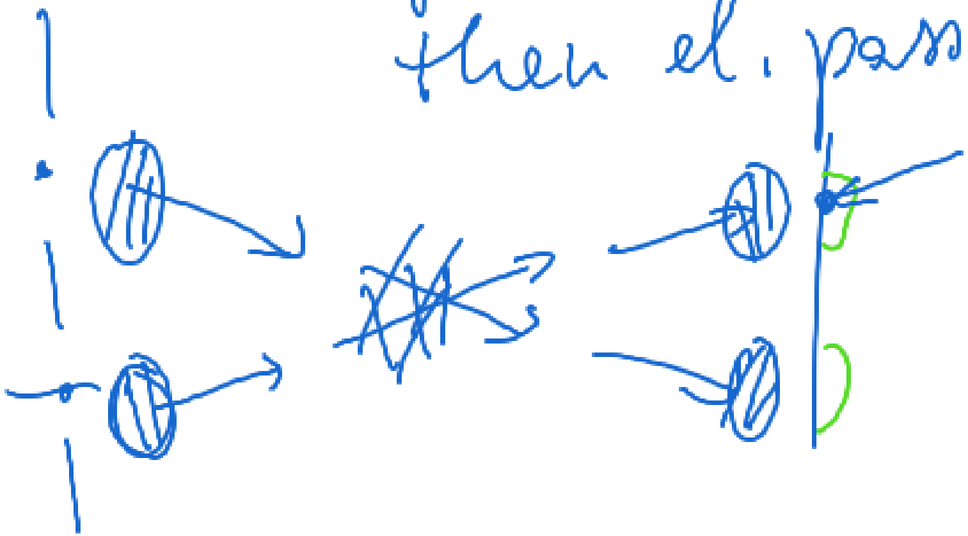
$\Rightarrow Q_1(t) = Q_2(t) \ \forall t \le t_0$

- literal wave + particle

Back to Wheeler:

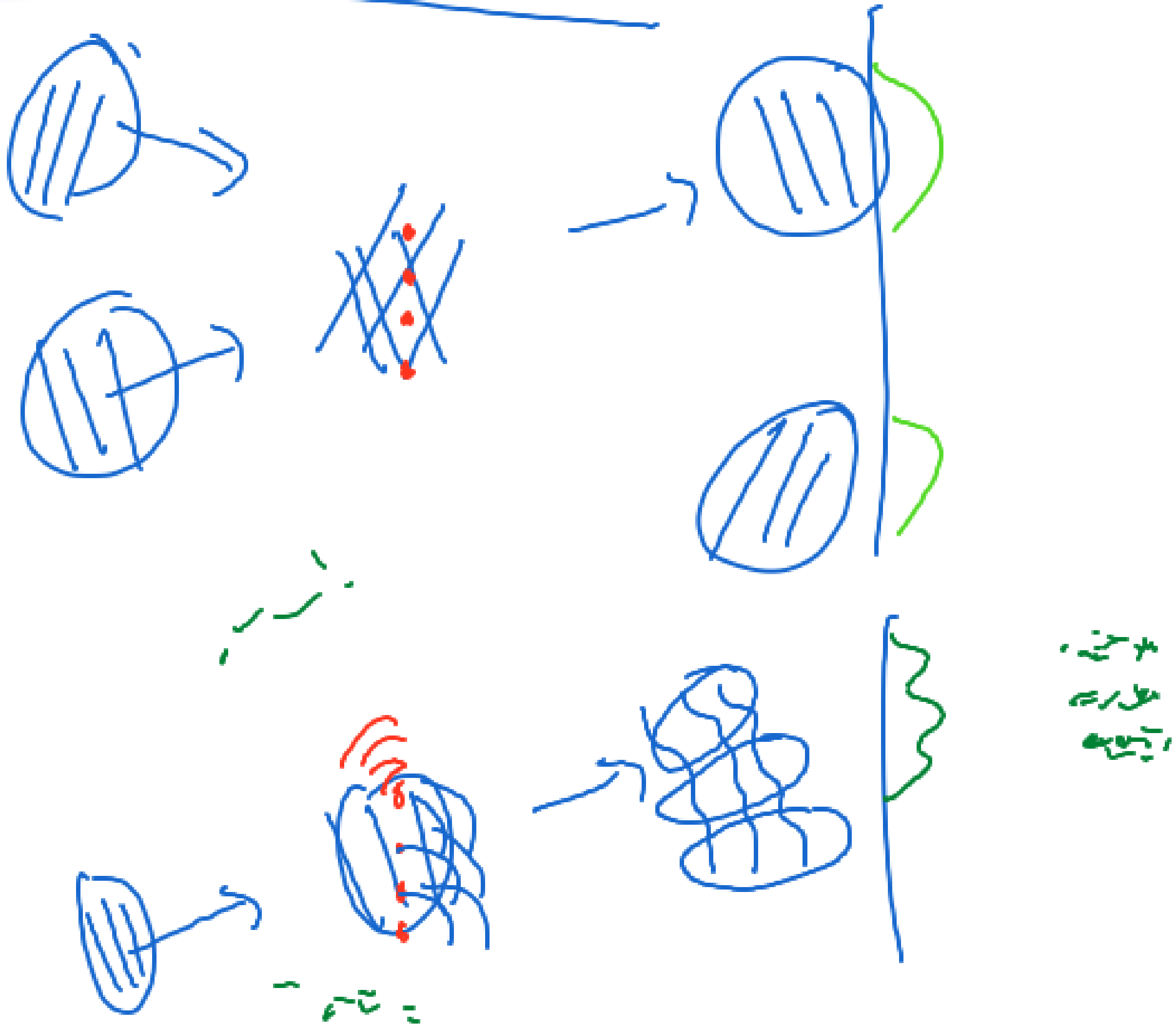
- strange: QM should believe that $\nexists$ fact about which slit.

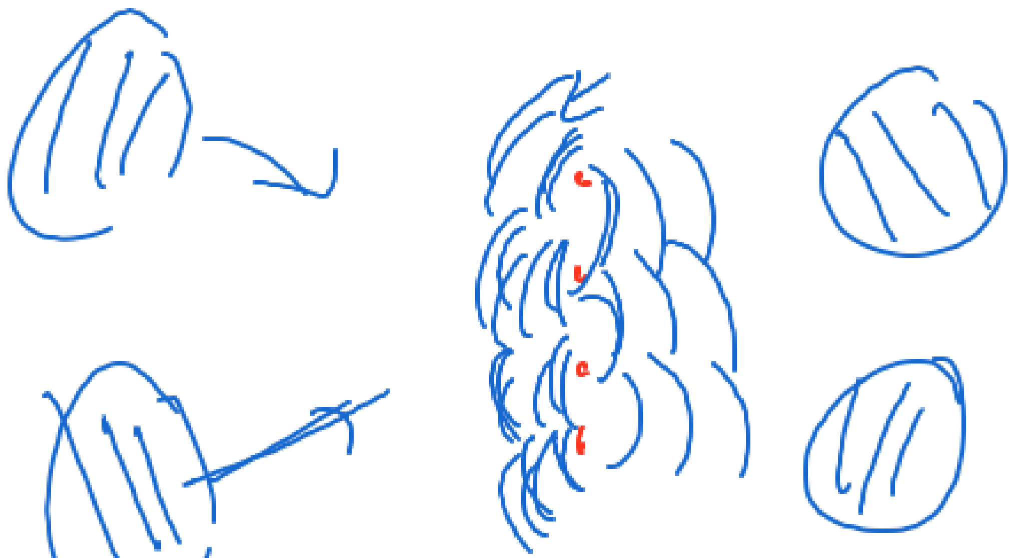
Wheeler: if (far) detected in upper cluster then el. passed thru lower slit



- Wheeler's fallacy: wrong retrodiction. non sequitur

Afshar's experiment (2005)





Some Observables

Fourier Transform and Momentum

$$e^{i\underline{k}\cdot\underline{x}} \quad \underbrace{e^{-i\hbar \underline{k}^2 t/2m}}_{e^{-iEt/\hbar}\,,\; E = \frac{\hbar^2 \underline{k}^2}{2m}}$$

Schr. eq. linear : $\sum_{\underline{k}} c_{\underline{k}} \, e^{i\underline{k}\cdot\underline{x}}$, $c_{\underline{k}} \in \mathbb{C}$

will evolve to

$$\sum_{\underline{k}} c_{\underline{k}} \, e^{-i\hbar \underline{k}^2 t/2m} \, e^{i\underline{k}\cdot\underline{x}}$$

continuous linear comb.

$$\psi(\underline{x}) = \int_{\mathbb{R}^3} d^3\underline{k}\, \underbrace{c(\underline{k})}_{\hat{\psi}(\underline{k})}\, e^{+i\underline{k}\cdot\underline{x}} \quad \text{evolves to} \int_{\mathbb{R}^3} d^3\underline{k}\, c(\underline{k})\, e^{-i\underline{k}^2 t} \quad e^{i\underline{k}\cdot\underline{x}}$$

<u>Def</u> For given $\psi : \mathbb{R}^d \to \mathbb{C}^m$,

$$\hat{\psi}(\underline{k}) = \frac{1}{(2\pi)^{d/2}} \int_{\mathbb{R}^d} d^d\underline{x}\ \psi(\underline{x})\, e^{-i\underline{k}\cdot\underline{x}}$$

is called the Fourier transform $\hat{\psi} = \mathcal{F}\psi$ of ψ.

<u>Thm</u> (inverse Fourier transform)

$$\psi(\underline{x}) = \frac{1}{(2\pi)^{d/2}} \int_{\mathbb{R}^d} d^d\underline{k}\ \hat{\psi}(\underline{k})\, e^{+i\underline{k}\cdot\underline{x}}.$$

Math remarks

- need $\psi \in L^1(\mathbb{R}^d, \mathbb{C}^n) = \{ f : \mathbb{R}^d \to \mathbb{C}^n :$

$$\|f\|_{L^1} = \int_{\mathbb{R}^d} |f(\underline{x})| \, d^d\underline{x} < \infty \}$$

then $\hat{\psi} \in L^\infty(\mathbb{R}^d, \mathbb{C}^n) = \{ f : \mathbb{R}^d \to \mathbb{C}^n : \text{bdd} \}$

$$\|f\|_{L^\infty} = \operatorname{ess.sup}_x |f(x)|$$

triangle ineq.

b/c $|\hat{\psi}(\underline{k})| \le \frac{1}{(2\pi)^{d/2}} \|\psi\|_{L^1}$ b/c $\left|\int f\right| \le \int |f|$

$$\left\| \sum_{i=1}^r \underline{u}_i \right\| \le \sum_i \|\underline{u}_i\|$$

- If $\psi \in L^1 \cap L^\infty$ then $\exists\, \hat{\psi} \in L^1 \cap L^\infty$ and inversion formula is valid.
- Schwartz space $\mathcal{S}$ of rapidly decaying fcts

$$= \{ f : \mathbb{R}^d \to \mathbb{C}^m ;\ \forall n \in \mathbb{N}\ \forall \underline{\alpha} \in \mathbb{N}_0^d\ \exists\, C_{n,\underline{\alpha}} > 0 : \\ |\partial^{\underline{\alpha}} f| < C_{n,\underline{\alpha}} |x|^{-n} \quad \forall x \in \mathbb{R}^d$$

$\underline{\alpha} = (\alpha_1, \ldots, \alpha_d)$, $\partial^{\underline{\alpha}} = \partial_1^{\alpha_1} \partial_2^{\alpha_2} \cdots \partial_d^{\alpha_d}$

$\mathcal{S} \subset L^1 \cap L^\infty$, $\mathcal{S} \subset L^2$ dense subspace

$\mathcal{F}\mathcal{S} = \mathcal{S}$, Gaussian $\in \mathcal{S}$ $\quad \Longrightarrow\ \mathcal{F} : L^2 \to L^2$.