<u>Spin</u>: Actual spin values vs. Actual Positions

$$L^2(\mathbb{R}^3, \mathbb{C}^2) \cong L^2(\underline{\mathbb{R}^3 \times \{1,2\}}, \mathbb{C})$$

$$\psi(\underline{x}) \in \mathbb{C}^2 \qquad\qquad \psi(\underline{x}, s)$$

may suggest $(\underline{Q}, \Sigma)$ Bohm-style theory

$$\mathrm{Prob}(\underline{Q} \in d^3q, \Sigma = s) = |\psi(\underline{q}, s)|^2 d^3q$$

Problem: not rotationally invariant.
Upshot: resist temptation.

Representations of $SO(3)$

$\psi : \mathbb{R}^3 \to \cancel{\mathbb{R}^3}\ \mathbb{C}^3$, change of coo $R \in SO(3)$

$$\tilde{\psi}(\underline{x}) = R\ \psi(R^{-1}\underline{x})$$

tensor-valued: $\psi : \mathbb{R}^3 \to \mathbb{C}^3 \otimes \mathbb{C}^3$

$$\psi_{ab}, \quad \tilde{\psi}_{ab}(\underline{x}) = \sum_{c,d=1}^{3} R_{ac} R_{bd}\ \psi_{cd}(R^{-1}\underline{x})$$

pattern: $\psi : \mathbb{R}^3 \to \mathbb{C}^d$, $\quad \tilde{\psi}_r(\underline{x}) = \sum_{s=1}^{d} M_{rs}(R)\ \psi_s(R^{-1}\underline{x})$ transformation law

composition law $M(R_1)\,M(R_2) = M(R_1 R_2)$, $M(\underline{I}) = \underline{I}$

group representation

gr. rep: gr. homomorphism $SO(3) \to \underline{GL}(\mathbb{C}^d)$

S for spin-$\frac{1}{2}$, one more complication:

$\forall R \in SO(3)$. not $S \to \cancel{S}\, S$ but $\underline{P(S)} \to P(S)$

projective space $P(V) = \{1d \text{ subspaces of } V\}$.

$e^{i\theta}\psi$ equal to ψ in Born rule $\forall$ obs
in BM

'ray' = 1d subsp.

projective mapping $F : P(\mathcal{H}) \to P(\mathcal{H})$

compatible with a linear mapping $M : \mathcal{H} \to \mathcal{H}$

$F(\mathbb{C}\psi) = \mathbb{C}M\psi$ $\quad$ $(\ker M = \{0\})$

projective repr. $F(R_1)\,F(R_2) = F(R_1 R_2)$, $F(I) = I$

linear $\underline{M(R_1)}\,M(R_2) = \underline{\pm} M(R_1 R_2)$

gr. homom. $SO(3) \to$ projective mappings(S)

$M \in GL(\mathbb{C}^2)$, $M \in SU(2)$,

$\varphi: SU(2) \longrightarrow SO(3)$ gr. homom.

φ smooth, 2-to-1, locally diffeomorphism

$\underline{\text{Ex}}$ $\chi: \mathbb{R} \to U(1)$, $\theta \mapsto e^{i\theta}$ is gr. homom., smooth, many-to-one, locally diffeom.

unfold $U(1)$, get $\mathbb{R}$

unfold $SO(3)$, get $SU(2)$

'unfold' = universal covering space

$\forall$ curve in $SO(3)$ starting from I

$\exists_1$ curve in $SU(2)$ starting from I

called the lift

$SO(3)$

$SU(2)$

2 ways: proj. gr. repr. $SO(3) \longrightarrow P(\mathbb{C}^2)$

or gr. repr. $SU(2) \longrightarrow GL(\mathbb{C}^d)$

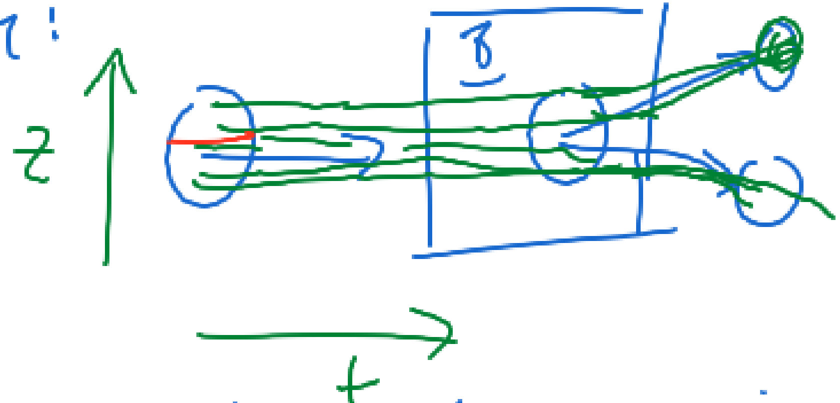
S is equipped with a particular repr of $\widehat{SO(3)} \cong SU(2)$ / proj. repr. of $SO(3)$.

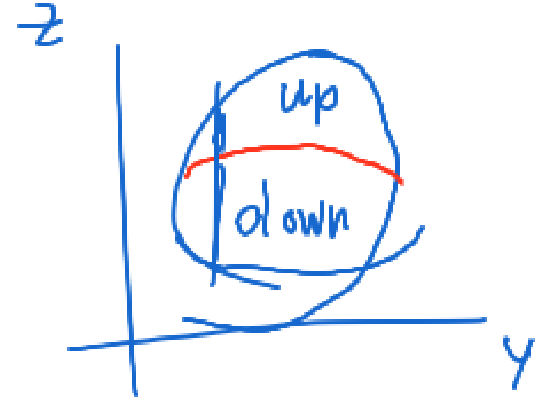
Def S is the repr. space of $\widehat{SO(3)}$ of the "spin-$\frac{1}{2}$ repr.".

Inverted SG magnet and contextuality

SG in +z direction, $\psi_s(\underline{x}) = \phi_s \chi(\underline{x})$

BM:

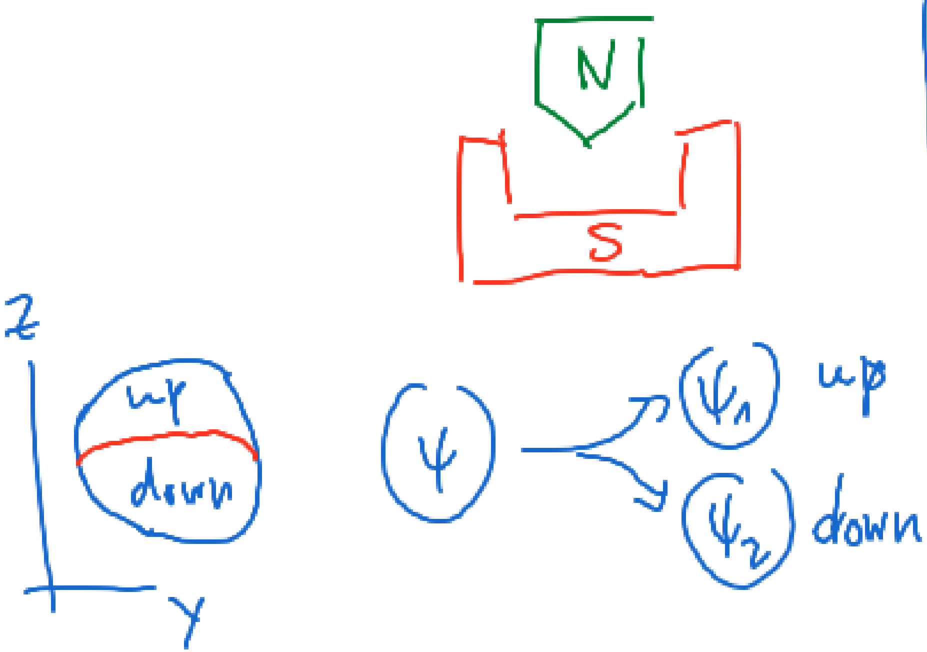


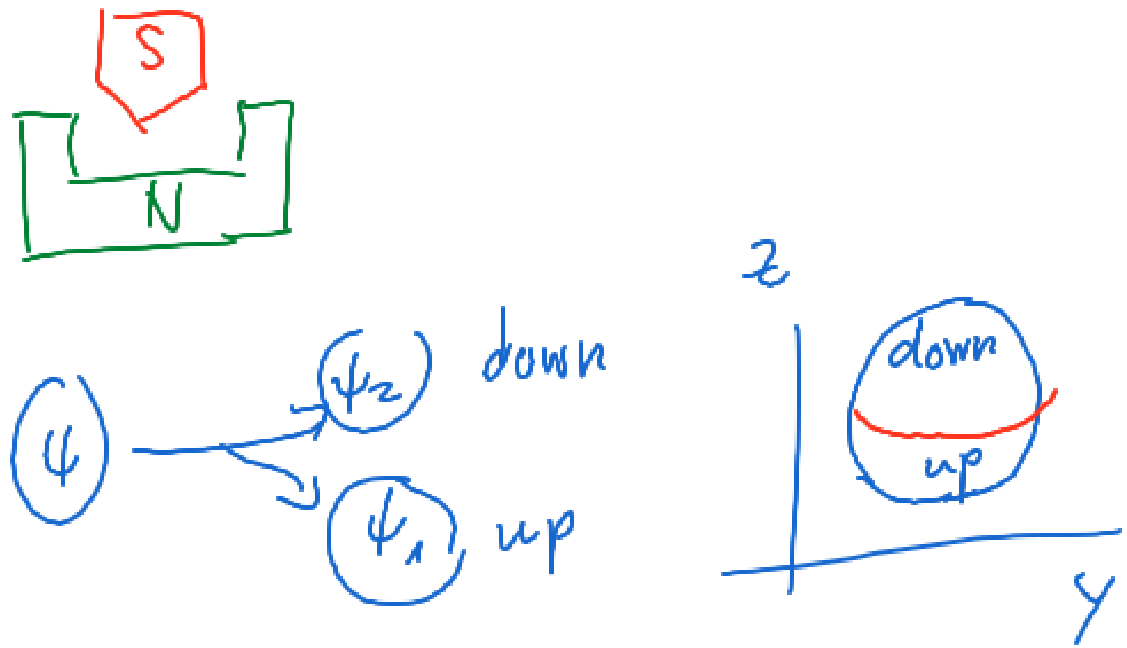


The result of the experiment is determined by $\underline{Q}(0)$ and $\psi(0)$

In general, the result is <u>not</u> determined by $\underline{Q}(0)$, $\psi(0)$, and σ_3.

Another experiment





σ_3 meas. $\qquad$ σ_3 meas.

$$Z_{SG}(Q_0, \psi_0) \neq Z_{ISG}(Q_0, \psi_0)$$

contextuality

Collapse and Measurement

The Projection Postulate

Notation: Dirac $|\psi\rangle = \psi \in \mathcal{H}$

$|\phi_n\rangle = |n\rangle$ ket

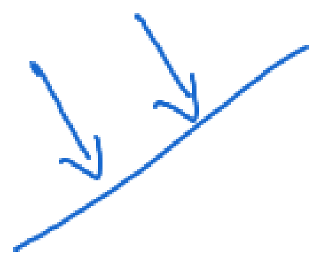
$\langle\phi| : \mathcal{H} \to \mathbb{C} : \psi \longmapsto \langle\phi|\psi\rangle$

bra $\quad \langle\phi| \; |\psi\rangle = \langle\phi|\psi\rangle$

$\langle\phi|A\psi\rangle = \underline{\langle\phi|A|\psi\rangle}$ particularly for self-adj. A.

$|\phi\rangle\langle\phi| : \mathcal{H} \to \mathcal{H}$

$|\phi\rangle\langle\phi|\psi\rangle$, if $\|\phi\| = 1$, then this is the proj. to $\mathbb{C}\phi$.



orth. proj.

$\otimes$ notation: $\underline{\Psi(x,y)} = \psi(x)\,\phi(y)$

$$\Longleftrightarrow \quad \Psi = \psi \otimes \phi$$

$L^2(x,y) =: L^2(x) \otimes L^2(y)$ tensor product

$\mathbb{C}^2 \otimes L^2(\mathbb{R}^3, \mathbb{C}) = L^2(\mathbb{R}^3, \mathbb{C}^2)$

Notation: $f(t-) = \lim_{s \nearrow t} f(s)$, $f(t+) = \lim_{s \searrow t} f(s)$