Thm $A = A^\dagger$, $B = B^\dagger$, $\|\psi\| = 1$, $\sigma_A \sigma_B \geq \frac{1}{2} \left| \langle \psi | [A,B] | \psi \rangle \right|$

Pf Born $\text{distr}_{\psi, A}$, $\langle A \rangle = \langle \psi | A \psi \rangle$ expectation

$$\sigma_A^2 = \langle \psi | (A - \langle A \rangle)^2 | \psi \rangle = \| \phi_A \|^2, \quad \phi_A := (A - \langle A \rangle \overset{I}{}) \psi$$

Cauchy-Schwarz ineq

$$|\langle \phi_A | \phi_B \rangle|^2 \leq \|\phi_A\|^2 \|\phi_B\|^2 = \sigma_A^2 \sigma_B^2$$

Now $\langle \phi_A | \phi_B \rangle = \langle \psi | (A - \langle A \rangle)(B - \langle B \rangle) | \psi \rangle$

$$= \langle \psi | AB - \underline{\langle A \rangle B} - A \langle B \rangle + \langle A \rangle \langle B \rangle | \psi \rangle$$

$$= \langle \psi | AB | \psi \rangle - \langle A \rangle \langle B \rangle - \cancel{\langle A \rangle \langle B \rangle} + \cancel{\langle A \rangle \langle B \rangle}$$

$$=: \underline{\langle AB \rangle} - \langle A \rangle \langle B \rangle$$

$$\underline{|\langle \phi_A | \phi_B \rangle|^2} \geq \left(\mathrm{Im} \langle \phi_A | \phi_B \rangle \right)^2$$

$$= \left| \frac{\langle \phi_A | \phi_B \rangle - \langle \phi_B | \phi_A \rangle}{2\cancel{i}} \right|^2$$

$$= \frac{1}{4} \left| \langle AB \rangle \cancel{- \langle A \rangle \langle B \rangle} - \langle BA \rangle \cancel{+ \langle B \rangle \langle A \rangle} \right|^2$$

$$= \underline{\frac{1}{4} \left| \langle \psi | \; [A, B] \; | \psi \rangle \right|^2} \qquad \square$$

The Measurement Problem

Consider quantum meas. of $A = A^\dagger$, spectrum (A) is finite

object + apparatus "ready state"

$\Psi(x,y) = \psi(x) \quad \phi(y)$, $\quad \Psi(t_1) = \psi \otimes \phi$ at t_1 (begin)

$\Psi(t_2)$ $\quad t_2$ (ends)

$A = \sum \alpha P_\alpha$, $\psi = \sum_\alpha c_\alpha \psi_\alpha$, $\|\psi_\alpha\| = 1$, $\sum_\alpha |c_\alpha|^2 = 1$

First, suppose $\psi = \psi_\alpha$. Born $\Rightarrow$ $\mathbb{P}(Z = \alpha') = \delta_{\alpha\alpha'}$.

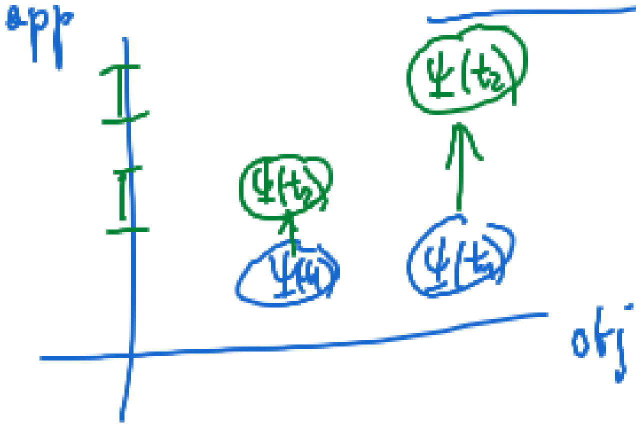
Let $\Psi_\alpha(t_1) = \psi_\alpha \otimes \phi$. Evolves to $\Psi_\alpha(t_2)$ (displays α)

Second, general $\psi = \sum c_\alpha \psi_\alpha$

$$\Psi(t_1) = \psi \otimes \phi = \sum c_\alpha \Psi_\alpha(t_1)$$

Schr. $\Big\downarrow$

$$\boxed{\Psi(t_2) = \sum c_\alpha \Psi_\alpha(t_2)}$$



$\longrightarrow$ 1) In each run, there is a unique outcome.

$\longrightarrow$ 2) The wave fct is the complete description of a system's physical state.

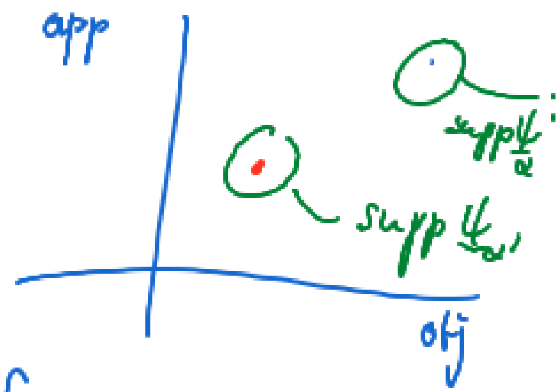
$\longrightarrow$ 3) Time evol. of isolated system is linear.

1) is dropped in the many-worlds theory

2) is dropped in BM.

3) is dropped in GRW.

How BM solves the meas. problem



$$\Psi(t_2) = \sum_\alpha c_\alpha \Psi_\alpha$$

$$\mathbb{P}\left(Q \in \text{support}(\Psi_\alpha)\right) = \int_{\text{support}(\Psi_\alpha)} dq\, |\Psi(q)|^2 = \int_{\mathbb{R}^{3N}} dq\, |c_\alpha \Psi_\alpha(q)|^2$$

$$= |c_\alpha|^2 \|\Psi_\alpha\|^2 = |c_\alpha|^2 \|\Psi_\alpha(t_1)\|^2 = |c_\alpha|^2 \|\psi_\alpha \otimes \phi\|^2$$

$= |c_\alpha|^2$ in agreement with Born's rule for A.

<u>Conseq</u> BM agrees with quantum formalism.
No experiment can distinguish between BM & QF.

Collapse of wf in BM

(1) Ψ_α will never overlap again ("decoherence")

not for $10^{10^{10}}$ yrs.

b/c they have macroscopically disjoint supports.

can collapse

$\Psi = \sum c_\alpha \Psi_\alpha \rightsquigarrow \Psi_{\alpha_0}$

② wf of object

If $\Psi(x,y) = \psi(x)\,\phi(y)$ then ψ.

general <u>Def</u> conditional wf

$$\psi_{cond}(x) = \mathcal{N}\ \Psi(x,Y), \qquad \frac{1}{\mathcal{N}^2} = \int dx\, |\Psi(x,Y)|^2$$

doesn't evolve according to Schr. eq.,

except when $\Psi(x,y) = \psi(x)\,\phi(y)$, and no interaction

$$\Psi(t_2) = \sum c_\alpha \Psi_\alpha, \quad \text{suppose } \Psi_\alpha = \psi_\alpha \otimes \phi_\alpha$$

then $\psi_{cond}(t_2) \simeq \psi_\alpha$