GRW theory

Prob(jump in dt) $= \lambda\, dt$

$\lambda =$ const. $=$ jump rate

$0 < T_1 < T_2 < \dots$

Poisson process with λ

$\mathrm{Prob}(T_1 < dt) = \lambda dt$

$\mathrm{Prob}(T_1 > dt) = 1 - \lambda dt$

$\mathrm{Prob}(T_1 > 2dt) = (1 - \lambda dt)^2$

$\mathrm{Prob}(T_1 > n\, dt) = (1 - \lambda dt)^n$

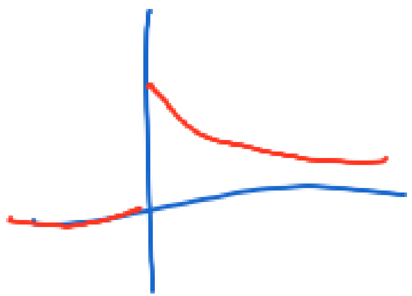
$\underline{\mathrm{Prob}(T_1 > t)} = \lim_{n\to\infty}\left(1 - \frac{\lambda t}{n}\right)^n$

$dt = \frac{t}{n}$

$= \underline{e^{-\lambda t}}$

$$\rho(t)\,dt = \mathbb{P}(t < T_1 < t + dt)$$
$$= \mathbb{P}(t < T_1)\,\mathbb{P}(T_1 < t + dt \,/\, T_1 > t)$$
$$= e^{-\lambda t} \quad \lambda\,dt$$

$$\Rightarrow \underline{\rho(t) = \mathbb{1}_{t>0}\,\lambda e^{-\lambda t}}, \qquad \mathbb{1}_C = \begin{cases} 1 & \text{if } C \\ 0 & \text{if not } C \end{cases}$$



$$T_1 \sim \mathrm{Exp}(\lambda)$$
$$T_2 - T_1 \sim \mathrm{Exp}(\lambda)$$
$$T_{k+1} - T_k \sim \mathrm{Exp}(\lambda)$$

<u>Exer</u> $\mathbb{E}T_1 = \int_{\mathbb{R}} t \, p(t)\, dt = \frac{1}{\lambda}$

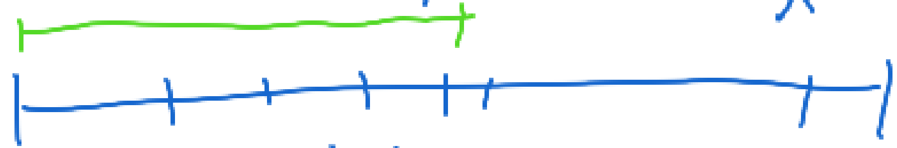
Another repr. is $X_t = \#\{ i \in \mathbb{N} \cup \{0\} : T_i < t \}$

<u>Thm</u> $\mathcal{T} = \{T_1, T_2, \dots\}$ is a Poisson process with rate λ

$\mathcal{T}' = \{T_1', \dots\}$ indep. " λ'

Then $\mathcal{T} \cup \mathcal{T}'$ is a Poisson process with rate $\lambda + \lambda'$.

Thm Choose n points uniformly in $[0, \frac{n}{\lambda}]$.
independently



Joint distr. $\xrightarrow{n \to \infty}$ Poisson proc with rate λ

GRW process Ψ_t

$N=1$. $\Psi_t : \mathbb{R}^3 \to \mathbb{C}$, $T_1, T_2, \dots \sim$ Poisson proc.
between T_k and T_{k+1}, Ψ_t evolves acc. to Schr.
at T_k collapse as if unsharp pos meas. with inacc. $\sigma > 0$.

$\lambda \approx 10^{-16}\, s^{-1}$, $\sigma \approx 10^{-7}\, m$ (GRW 1986)
$\lambda \approx 3 \cdot 10^{-8}\, s^{-1}$, $\sigma \approx 10^{-6}\, m$ (Adler 2007)

$N \in \mathbb{N}$, $\underline{\Psi}_t = \underline{\Psi}_t(\underline{x}_1, \dots, \underline{x}_N)$

N indep. Poisson proc. (λ)

$\Longleftrightarrow$ Poisson $\{T_1, T_2, \dots\}$ $\sim$ rate $N\lambda$

random indices $I_1, I_2, \dots$ $\sim$ ind. uniform $\{1, \dots, N\}$

at T_k: as if unsharp pos meas. on particle I_k.

Formulas $\quad N=1$

$$\underline{\Psi}_{T_k^+} = \frac{C(\underline{X}_k)\,\underline{\Psi}_{T_k^-}}{\| C(\underline{X}_k)\,\underline{\Psi}_{T_k^-} \|}$$

$$C(\underline{X})\,\underline{\Psi}(\underline{x}) = \sqrt{g_{\underline{X},\sigma}(\underline{x})}\,\underline{\Psi}(\underline{x})$$

$$g_{\underline{X},\sigma}(\underline{x}) = \frac{1}{(2\pi\sigma^2)^{3/2}} \exp\left(-\frac{(\underline{x}-\underline{X})^2}{2\sigma^2}\right)$$

$$p(\underline{X}_k = \underline{y} \mid T_1,\dots,T_k,\underline{X}_1,\dots,\underline{X}_{k-1}) = \| C(\underline{y})\,\underline{\Psi}_{T_k^-} \|^2$$

$$\int \rho(\underline{X}_k = \underline{y}) \ldots) \, d^3\underline{y} = \int d^3\underline{y} \, \| C(\underline{y}) \Psi \|^2$$

$$= \int d^3y \int d^3\underline{x} \; \Big| \underbrace{(C(y)\Psi)(\underline{x})}_{\sqrt{g_{\underline{y},\sigma}(\underline{x})}\,\Psi(\underline{x})} \Big|^2$$

$$= \int d^3\underline{y} \, d^3\underline{x} \;\; g_{\underline{y}\sigma}(\underline{x}) \; |\Psi(\underline{x})|^2 = \int d^3\underline{x} \, |\Psi(\underline{x})|^2 = 1.$$

$$N \in \mathbb{N}, \quad \Psi_{T_k+} = = \frac{C_{\underline{I}_k}(\underline{X}_k)\,\Psi_{T_k-}}{\| C_{\underline{I}_k}(\underline{X}_k)\,\Psi_{T_k-} \|}$$

$$C_{\underline{I}}(\underline{X})\,\Psi(\underline{x}_1 \ldots \underline{x}_N) = \sqrt{g_{\underline{X},\sigma}(\underline{x}_I)}\;\Psi(\underline{x}_1 \ldots \underline{x}_N)$$

$$\rho(\underline{X}_k = \underline{y} \,|\, T_1 \ldots T_k, I_1 \ldots I_k, \underline{X}_1, \ldots \underline{X}_{k-1}) = \| C_{I_k}(\underline{X}_k)\,\Psi_{T_k-} \|^2$$

$$\rho = g_{\underline{y},\sigma} * |\Psi(\underline{x}_I)|^2 \quad \text{for } \cancel{N=1} \quad N \in \mathbb{N}$$

$$(f * h)(\underline{x}) = \int d\underline{y}\; f(\underline{y})\, h(\underline{y} - \underline{x})$$

$\Rightarrow$ no-signalling

Primitive ontology

ontology = what exists

primitive on. = part of on. repr. matter in 3d

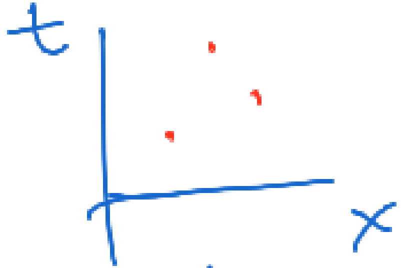
GRWm = matter density on.

GRWf = flash on.

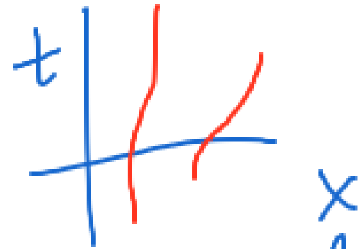
GRWm matter density $m(\underline{x},t)$

$$m(\underline{x},t) = \sum_{i=1}^{N} m_i \int d^3\underline{x}_1 \cdots \widehat{d^3 x_i} \cdots d^3\underline{x}_N \left| \Psi_t(\underline{x}_1, \ldots, \underline{x}_N) \right|^2 \Big|_{\underline{x}_i = \underline{x}}$$

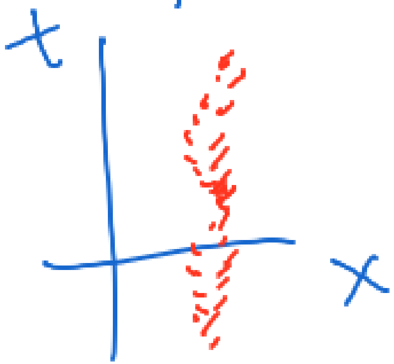
GRWf



flash



particle

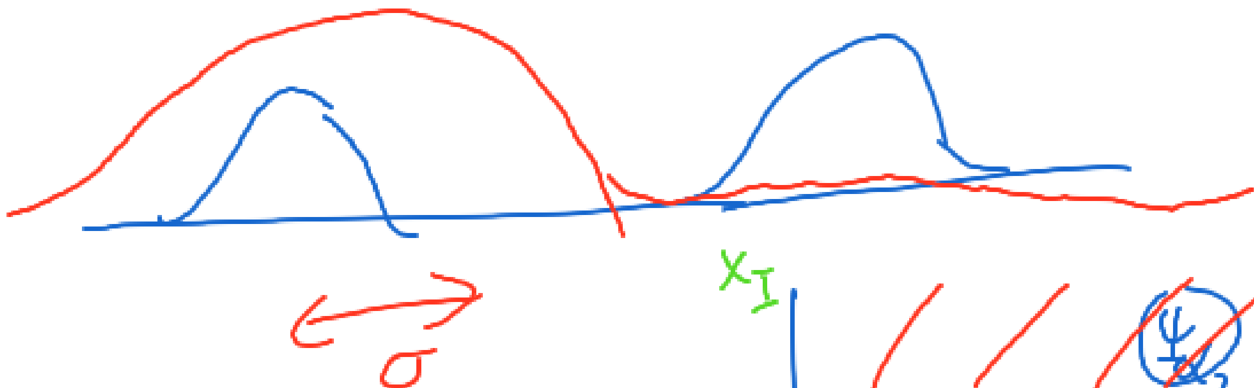


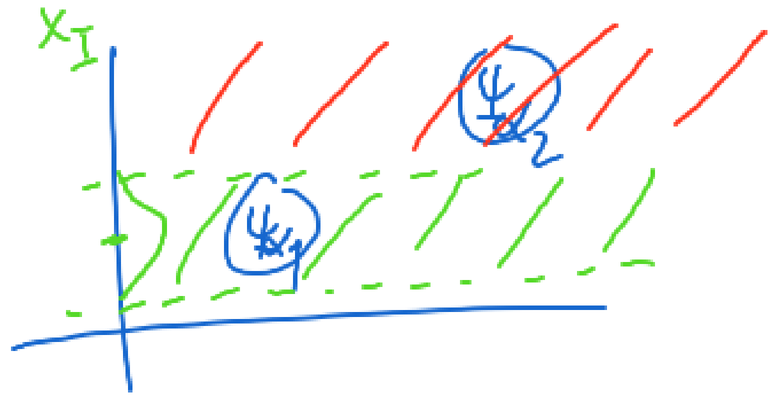
$$F = \{ (\underline{X}_1, T_1, I_1), (\underline{X}_2, T_2, I_2), \dots \}$$

$N = 10^{23} \Rightarrow 10^7$ flashes/s.

beable (Bell)

How GRW theories solve the meas. problem





$$\Psi = \sum c_\alpha \Psi_\alpha$$

$$\mathbb{P}(\alpha_1) \simeq |c_{\alpha_1}|^2$$

upshot

GRWf & GRWm give intuitively reasonable pictures of reality, empirically adequate.