# Nonlocality

BM, GRW, CI, MW

1) BM
$$\frac{dQ_i}{dt} = v_i(Q_1 .. Q_N)$$

$$v_i = \mathrm{Im}\,\frac{\nabla_i\psi(Q_1 .. Q_N)}{\psi(Q_1 .. Q_N)}$$

If $\psi(Q_1 .. Q_N) = \psi_1(Q_1) \cdots \psi_N(Q_N)$ then

$v_i = v_i(Q_i)$ indep of $Q_1 .. Q_{i-1}, Q_{i+1} .. Q_N$

BM: $(\psi, Q)$

choose particular experiments, order

$Z_{1\underline{\alpha}}, \; Z_{2\underline{\alpha}\underline{\beta}}$

12 random var.s

EPR: $Z_{i\underline{\alpha}}$

$i = 1,2, \; \underline{a} \in \mathbb{R}^3, |\underline{\alpha}| = 1$

$Z_{1\underline{\alpha}}, \; Z_{2\underline{\beta}}$

6 random var.s

GRW

$$\psi_t(\underline{x}_1, \underline{x}_2)$$

GRWf: Einstein's boxes

GRWm: Einstein's boxes

Copenhagen: Einstein's boxes

Many worlds: Sm is nonlocal, too.

Rem: $\lambda$ BM: $\lambda = (\psi, Q)$

GRW: $\lambda = \psi$

misconceptions about Bell's proof.

# General Observables: POVMs

POVM = positive-operator-valued measure

= generalized observable

<u>Def</u> An operator $A$ is <u>positive</u> $\Leftrightarrow$ self-adj, ("positive semi-definite") spectrum $= \sigma(A) \subseteq [0, \infty)$

Bounded $A: \mathcal{H} \to \mathcal{H}$ positive $\Leftrightarrow$ $\forall \psi \in \mathcal{H}: \langle \psi | A \psi \rangle \geq 0$.

<u>Ex</u> proj. is positive

Def (first) POVM = family of positive op.s $E_z$ with

$$\sum_z E_z = I.$$

Ex 1) $E_1 = \begin{pmatrix} 1/2 & \\ & 1/3 \end{pmatrix}$, $E_2 = \begin{pmatrix} 1/2 & \\ & 2/3 \end{pmatrix}$

In general, $\sigma(E_z) \subseteq [0,1]$ because if $E_\zeta \psi = \eta \psi$

then $\langle \psi | \psi \rangle = \langle \psi | I \psi \rangle = \langle \psi | \sum E_z \psi \rangle = \langle \psi | E_\zeta \psi \rangle +$

$+ \sum_{z \neq \zeta} \underbrace{\langle \psi | E_z \psi \rangle}_{\geq 0} \geq \langle \psi | E_\zeta \psi \rangle = \eta \langle \psi | \psi \rangle$ so $\eta \leq 1$.

Ex 2) $E_1 = \begin{pmatrix} 1 & \\ & 0 \end{pmatrix}$, $E_2 = \begin{pmatrix} 0 & \\ & 1 \end{pmatrix}$

$E_z$ = proj. projection-valued measure (PVM)

($\Rightarrow$ mutually orthogonal)

3) A self-adjoint matrix defines a PVM

$z \in \sigma(A)$, $E_\alpha$ = proj. to eigenspace of A w/ e.v. $\alpha$

$\Rightarrow \sum_{\alpha \in \sigma(A)} E_\alpha = I$ spectral PVM. E.g. $A = \sigma_3$

4) $E$ POVM, $\psi \in S(\mathcal{H}) = \{\phi \in \mathcal{H} : \|\phi\| = 1\}$
define a prob. distr.

$$P_\psi(z) = \langle \psi | E_z \psi \rangle$$

on $Z = \{z\}$.

5) fuzzy position observable

$$E_z \psi(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-z)^2}{2\sigma^2}} \psi(x)$$

$E_z$ is positive b/c $\langle \psi | E_z \psi \rangle = \int dx \, \psi^*(x) \frac{e^{-\cdots}}{\cdots} \psi(x) \geq 0$

Here, $\int_{\mathbb{R}} dz \, E_z = I$ b/c

$$\int_{\mathbb{R}} dz \, \frac{e^{-\frac{(x-z)^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}} \psi(x) = \psi(x) \int_{\mathbb{R}} dz \, \frac{e^{-\frac{(x-z)^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}} = \psi(x).$$

<u>Def</u> $Z$ set, $\sigma$-algebra $\mathcal{B}$ of subsets of $Z$

$(\emptyset \in \mathcal{B},\ B \in \mathcal{B} \Rightarrow B^c = Z \setminus B \in \mathcal{B},\ B_1, B_2, \ldots \in \mathcal{B} \Rightarrow \bigcup_{j=1}^{\infty} B_j \in \mathcal{B})$

$\mu : \mathcal{B} \to [0,1]$ is a (normalized) <u>measure</u> iff

$$\mu(Z) = 1$$

$$\mu\left(\bigcup_{j=1}^{\infty} B_j\right) = \sum_{j=1}^{\infty} \mu(B_j) \quad \text{if } B_i \cap B_k = \emptyset \ \forall i \neq k.$$

"$\sigma$-additive"

<u>Def</u> (2nd) <u>POVM</u> $E : \mathcal{B} \longrightarrow \{\text{bdd op.s on } \mathcal{H}\}$

such that $E(B)$ is positive, $E(Z) = I$, and

$$E\left(\bigcup_{j=1}^{\infty} B_j\right) = \sum_{j=1}^{\infty} E(B_j) \quad \text{if } B_i \cap B_k = \emptyset \ \forall i \neq k.$$

(in operator norm)

Conseq $E$ POVM, $\psi \in S(\mathcal{H})$ define a measure

$$\mathbb{P}_\psi(B) = \langle \psi | E(B) \psi \rangle$$

If $Z$ is countable and $\mathcal{B} = \{\text{all subsets}\}$, then

$$\mu(B) = \sum_{z \in B} \mu(\{z\})$$

$$E(B) = \sum_{z \in B} \underbrace{E(\{z\})}_{E_z}$$

fuzzy position POVM

$$E(B)\psi(x) = \int_B dz \, \frac{e^{-\frac{(x-z)^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}} \psi(x)$$

$$1_B * g_\sigma$$