POVMs

What is an observable?

<u>Def</u> 2 experiments $\mathcal{E}_1, \mathcal{E}_2$ <u>equivalent in law</u>

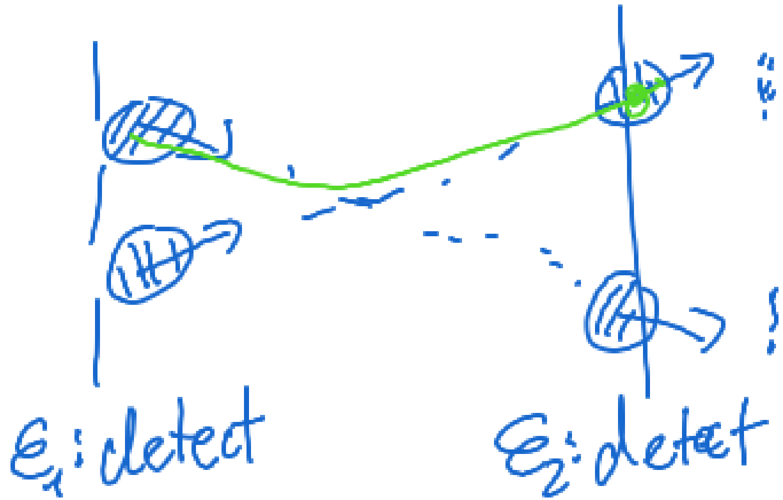
$\Longleftrightarrow$ $\forall \psi \in \mathcal{S}(\mathcal{H})$: $\mathbb{P}_\psi(Z_1 \in B) = \mathbb{P}_\psi(Z_2 \in B) \; \forall B$

An <u>observable</u> is an equivalence class of experiments.

Contextuality: ~~It~~ $\mathcal{E}_1, \mathcal{E}_2$ same observable
may yield different outcomes, $Z_1 \neq Z_2$,
when applied to the same object.

rolling a die

Wheeler's fallacy

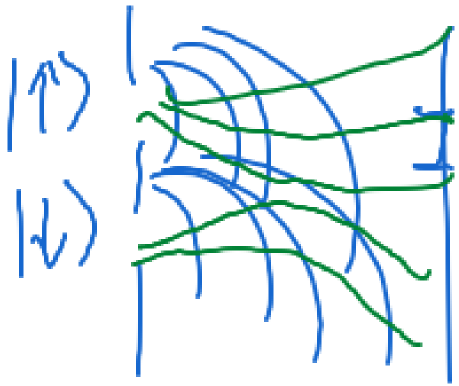


$$\psi = c_1 |\text{upper slit}\rangle + c_2 |\text{lower slit}\rangle$$

$$\psi(t_f) = c_1 |\text{lower cluster}\rangle + c_2 |\text{upper clustr}\rangle$$

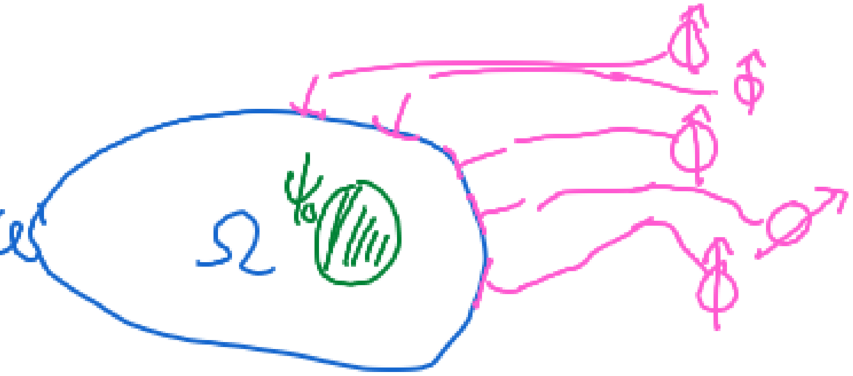
$$E_1(+1) = |\text{upper slit}\rangle\langle\text{upper slit}|, \quad E_1(-1) = |\text{lower slit}\rangle\langle \dots|$$

$$E_2(+1) = |\text{lower cluster}\rangle\langle\text{lower cluster}|, \quad E_2(-) = \text{upper}$$


$|\uparrow\rangle$
$|\downarrow\rangle$

Time of Detection

$\Omega \subset \mathbb{R}^3$, $\Sigma = \partial\Omega$ detecting surface



T = time of detection

$\underline{X}$ = place of detection

$Z = (T, \underline{X})$

1) feasible

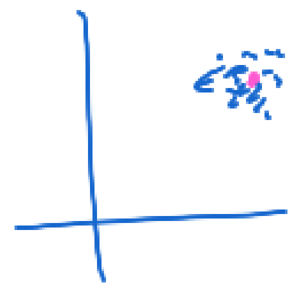
2) POVM E_{real}, E_{ideal}

Problem:

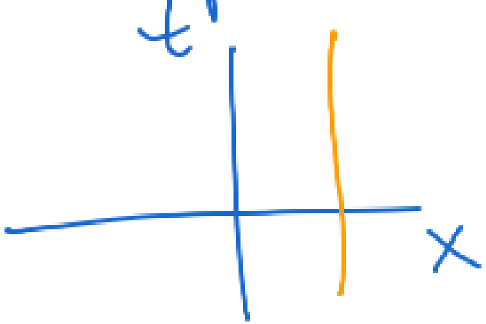
$\mathbb{P}(Z \in B) = ?$

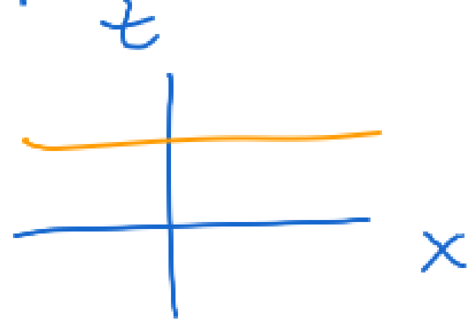
$B \subseteq \mathcal{Z} = [0,\infty) \times \Sigma \cup \{\infty\}$

$Z = \infty$

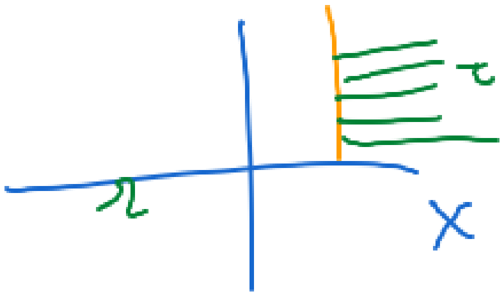


time of detection vs. pos. meas.



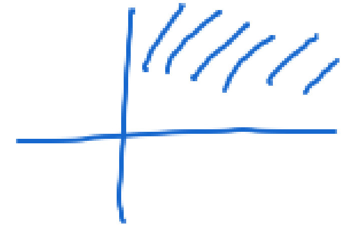


quantum Zeno effect



$\tau \to 0$

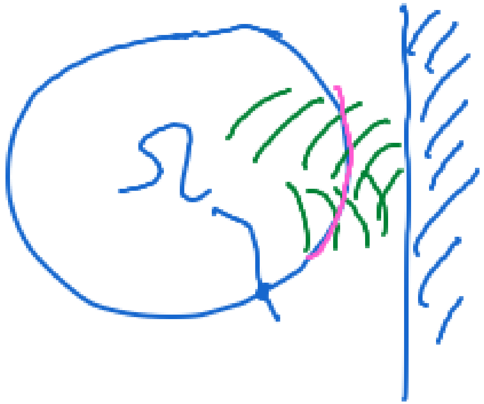
$\mathbb{P}(Z = \infty) = 1$



soft detectors, imaginary potentials

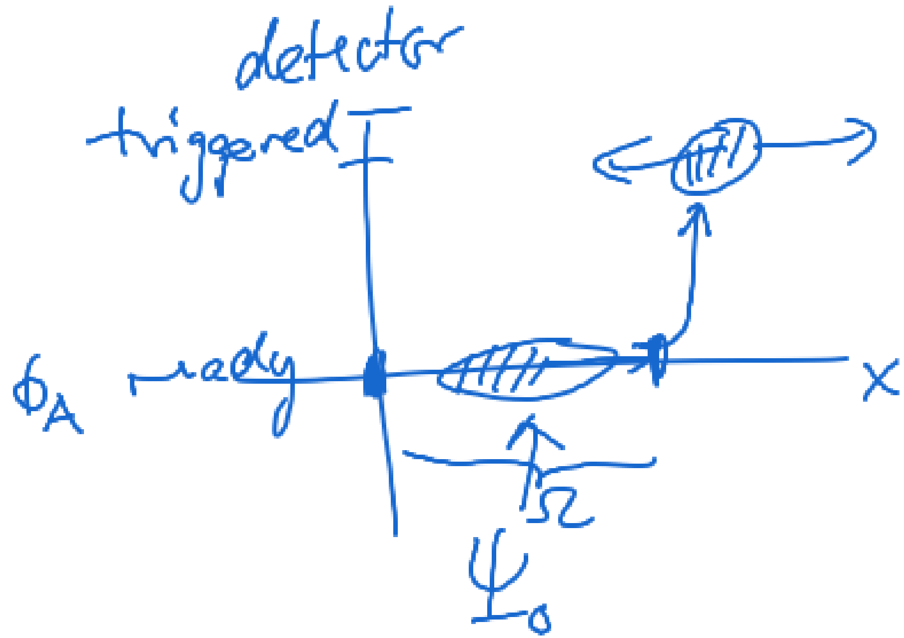
$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi - i v \, \mathbb{1}_{\substack{x>0 \\ \Omega^c}} \psi$$

Allcock's paradox

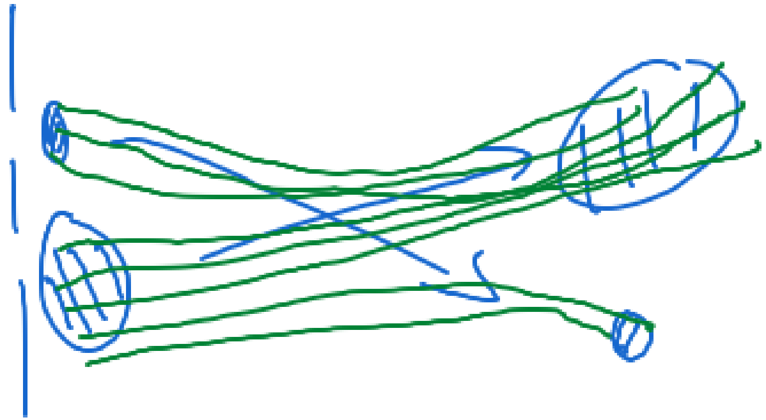


$$\Psi_0 = \psi \otimes \phi_A$$

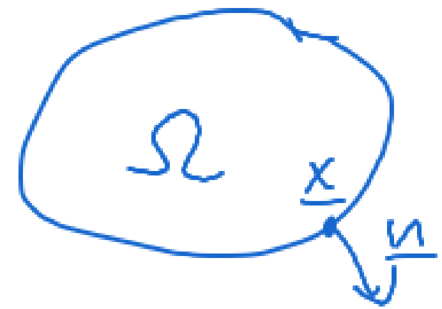
in Ω



$\psi, \psi, \psi, \psi, \psi, \psi$



Absorbing Boundary Rule



To compute $\mathbb{P}(Z \in B)$, :

$$i\hbar \frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi + V\psi \qquad \text{in } \Omega$$

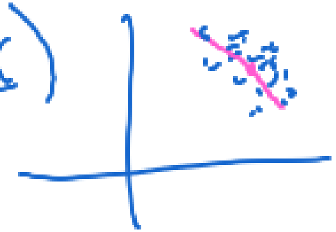
with boundary condition

$$\kappa > 0$$

$$\frac{\partial\psi}{\partial n}(\underline{x}) = i\kappa\psi(\underline{x}) \qquad \forall \underline{x}\in\partial\Omega \qquad E_{ideal} = E\kappa$$

$$\left(\frac{\partial\psi}{\partial n} = \underline{n}(\underline{x})\cdot\nabla\psi\right) \qquad B\subseteq\partial\Omega$$

Then $\mathbb{P}(t_1\le T<t_2, \underline{X}\in B) = \int_{t_1}^{t_2}dt\int_B d^2\underline{x}\;\underline{n}(\underline{x})\cdot\underline{j}(\underline{x})$



$$\underline{j}(\underline{x},t) = \frac{\hbar}{m} \operatorname{Im}[\psi^* \nabla\psi]$$

$$\mathbb{P}(Z=\infty) = 1 - \int_0^\infty dt \int_{\partial\Omega} d^2\underline{x}\ \underline{n}(\underline{x})\cdot\underline{j}(\underline{x})$$

$\square$

<u>Properties:</u> 1) $\underline{j}(\underline{x})$ is outward-pointing,

$\psi_t \in L^2(\Omega)$ $\qquad$ $\underline{n}(\underline{x})\cdot\underline{j}(\underline{x}) \geq 0$

Pf: $\underline{n}(\underline{x})\cdot\underline{j}(\underline{x}) = \frac{\hbar}{m}\operatorname{Im}\Big[\psi^* \underbrace{\underline{n}(\underline{x})\cdot\nabla\psi}_{\partial\psi/\partial n}\Big] \overset{BC}{=} \frac{\hbar}{m}\operatorname{Im}\big[\psi^* i\kappa\psi\big]$

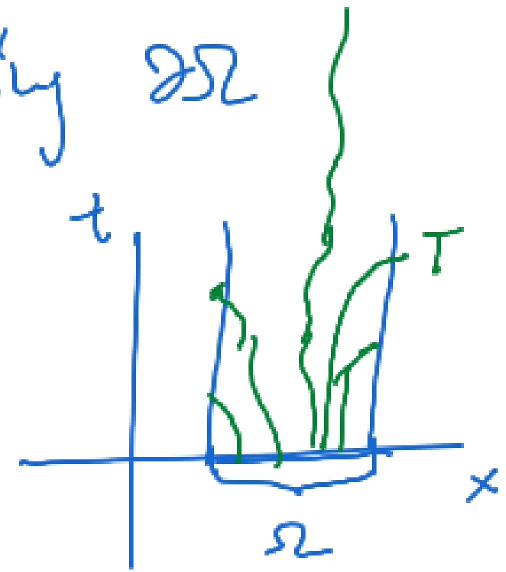
$= \frac{\hbar}{m}|\psi|^2\kappa \geq 0.$

2) <u>Thm</u> Time evolution is well defined, $W_t\psi_0 = \psi_t$

3) Bohmian traj.:

$$Q_0 \sim |\psi_0|^2$$

$\underline{n}(\underline{x}) \cdot \underline{j}(\underline{x},t) =$ prob density of Q hitting $\partial\Omega$ at $(\underline{x},t)$



4) $Q_t \sim |\psi_t|^2$

$$\|\psi_t\|^2 = \int_\Omega d^3\underline{x}\; |\psi_t(\underline{x})|^2 \le 1$$

$=$ prob. of $\{T>t\}$ decreasing fct of t

$-\frac{d}{dt}\|\psi_t\|^2 =$ density of T, $\mathbb{P}(t<T<t+dt) = -\frac{d\|\psi_t\|^2}{dt}dt$

W_t is not unitary, $W_t = e^{-iHt}$

H not self-adj.

defined as $-\frac{\hbar^2}{2m}\Delta + V$

on domain satisfying BC.

$\|W_t \psi\| \leq \|\psi\|$, W_t contraction

$W_s W_t = W_{s+t}$, $W_0 = I$ semigroup

$\underline{\underline{s,t \geq 0}}$