Quantum logic

1) math : non-distributive lattice $\mathbb{L}(\mathcal{H})$
2) analogy : $\mathbb{L}(\mathcal{H}) \longleftrightarrow$ logic
3) philosophical hypothesis : revise logic.

$$\mathbb{L}_{\star}(\mathcal{H}) = \{\text{closed subspaces of } \mathcal{H}\}$$

analogy: limited

a) spin-$\frac{1}{2}$ particle, $\psi \in \mathbb{C}^2$, $P :=$ "ψ lies in $\mathbb{C}|up\rangle$"

$\neg P :=$ "ψ lies in $\mathbb{C}^2 \setminus \mathbb{C}|up\rangle$"

$\neq$ "ψ lies in $\underbrace{\mathbb{C}|down\rangle}_{(\mathbb{C}|up\rangle)^{\perp} = \neg(\mathbb{C}|up\rangle)}$"

$\mathbb{L}(\mathcal{H})$

analogy $\mathbb{L}(\mathcal{H}) \longleftrightarrow$ propositions

b) delayed-choice exper.

$I = P_{\text{upper slit}} + P_{\text{lower slit}}$

PVM

$U^\dagger P_{\cancel{\text{upper}}\ \text{lower cluster}} U + U^\dagger P_{\text{upper cluster}} U = I$

"particle passed thru upper slit"

Wheeler's fallacy? PVM

philosophical hypothesis: revise logic.

wrong headed.

Measures

Prob theory, measures $\mu : \mathcal{B} \longrightarrow [0,1]$

$\mu(1) = \mu(\Omega) = 1$, additive

quantum measures: $\hat{\mu} : \mathbb{L}(\mathcal{H}) \longrightarrow [0,1]$

$\hat{\mu}(\mathcal{H}) = 1$, additive:

$$\hat{\mu}\Big(\bigvee_{\substack{n=1 \\ \text{joint}}}^{\infty} A_n\Big) = \sum_{n=1}^{\infty} \hat{\mu}(A_n) \text{ if } A_n \perp A_m \text{ for } n \neq m.$$

$$A \perp B \iff A \leq (\neg B) \iff B \leq (\neg A)$$

$$A \leq C \iff A \vee C = C$$
$$\iff A \wedge C = A$$
$$\iff A \subseteq B$$

<u>Gleason's thm</u> (1957)

Suppose $3 \leq \dim \mathcal{H} \leq \infty$ (countably ∞).
Then the normalized quantum measures are exactly the mappings $\hat{\mu}(A) = \mathrm{tr}(\rho P_A)$
with P_A = proj to A, $\rho \geq 0$, $\mathrm{tr}\,\rho = 1$.

Relativity

Dirac eq: $i\hbar \frac{\partial \psi}{\partial t} = -i\underline{\alpha}\cdot\nabla\psi + mc^2\beta\psi$

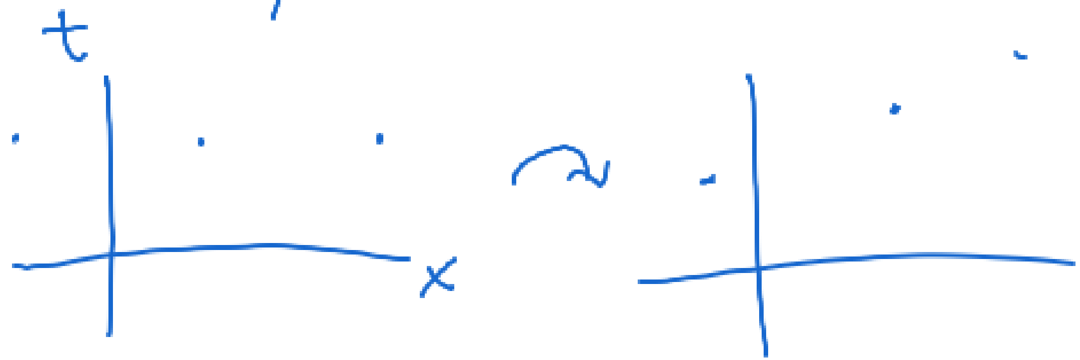
$\beta, \alpha_1, \alpha_2, \alpha_3$ 4×4 matrices

1 particle

$\psi : \mathbb{R}^4 \to \mathbb{C}^4 = S \oplus \overline{S}$

spin $\frac{1}{2}$

N particles, non-rel. $\psi(t, \underline{x}_1, \underline{x}_2, \dots, \underline{x}_N)$ $3N+1$



rel. analog $\quad \phi(x_1 \dots x_N) = \phi(t_1, \underline{x}_1, \dots, t_N, \underline{x}_N)$

$j \in \{1 \dots N\}$

$x_j \in M \qquad M = \mathbb{R}^4 \qquad 4N$

multi-time wave fct.

$\phi : \underset{\mathbb{R}^{4N}}{M^N} \longrightarrow (\mathbb{C}^4)^{\otimes N}$, or $\phi : \mathcal{S}_N \longrightarrow (\mathbb{C}^4)^{\otimes N}$

$x \times y :\Longleftrightarrow$ spacelike separated

$\mathcal{S}_N = \{$ spacelike config.s $\}$

$= \{ (x_1 \dots x_N) \in M^N : \forall i \neq j,\ x_i \times x_j \}$

time evol.: PDE

$$i\hbar \frac{\partial \phi}{\partial t_1} = H_1 \phi$$

$$\vdots$$

$$i\hbar \frac{\partial \phi}{\partial t_N} = H_N \phi$$

H_j partial Hamiltonians

$$\psi(t, \underline{x}_1, \dots, \underline{x}_N) = \phi(t, \underline{x}_1, \dots, t, \underline{x}_N)$$

$$\Rightarrow i\hbar \frac{\partial \psi}{\partial t} = i\hbar \sum_{j=1}^{N} \frac{\partial \phi}{\partial t_j}\Big|_{t_1 = \dots = t_N = t} = \sum_{j=1}^{N} H_j \phi \Big|_{t_1 = \dots = t_N = t}$$

$$= H\psi, \text{ so } H = \sum_{j=1}^{N} H_j \Big|_{t_1 = \dots = t_N = t}$$

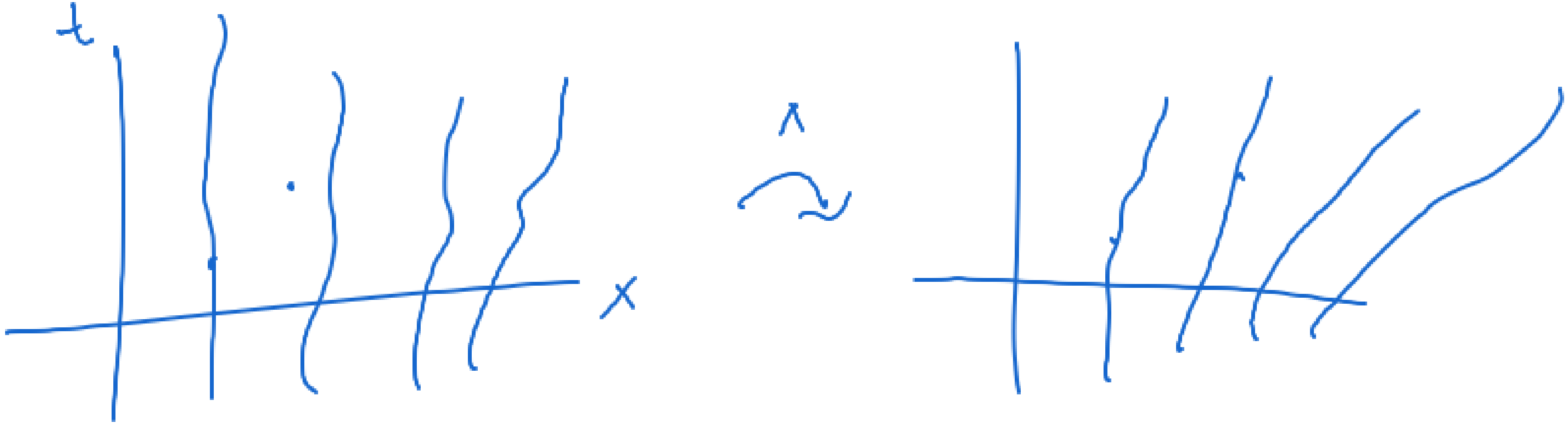
Simple ex. of multi-time eq.s $\qquad \phi_{s_1 \dots s_N}(x_1 \dots x_N)$

N non-interacting particles

$$\begin{cases} i\hbar \dfrac{\partial \phi}{\partial t_1} = H_1^{free} \phi \\ \vdots \\ i\hbar \dfrac{\partial \phi}{\partial t_N} = H_N^{free} \phi \end{cases}, \qquad H_j^{free} = -i \underline{\alpha}_j \cdot \nabla_j + \beta_j$$

$$\Rightarrow \phi(t_1, \cdot, t_2, \cdot, \dots t_N, \cdot) = e^{-iH_1 t_1} \dots e^{-iH_N t_N} \underbrace{\phi_0(0, \underline{x}_1, 0, \underline{x}_2, \dots, 0, \underline{x}_N)}$$

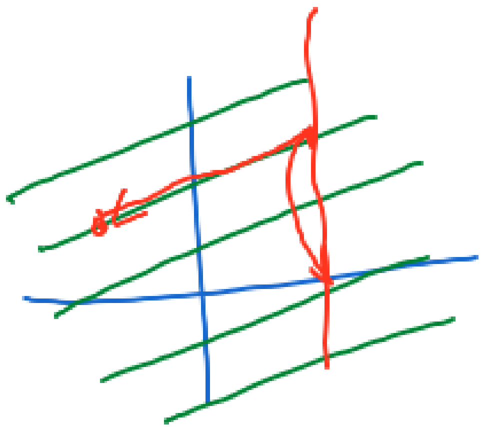
$$\phi : M^N \to (\mathbb{C}^4)^{\otimes N}$$



Lorentz inv

$\Lambda W = W$

set of possible history



Upshot: for nonlocal theories, Lorentz inv. is a very weak cond.