A5 $f(x,0) = x^2 \quad \forall x \in \mathbb{R}$ diffbar

$\Rightarrow \exists \, \partial_x f(0,0) = 0$

$f(0,y) = y^2 \quad \forall y \in \mathbb{R}$ diffbar

$\Rightarrow \exists \, \partial_y f(0,0) = 0$

f total diffbar in $(0,0)$ $\Leftrightarrow$

$$f(0+h_x, 0+h_y) = \overbrace{f(0,0)}^{0} + \underset{\substack{\uparrow \\ \partial_x f(0,0)=0}}{a} h_x + \underset{\substack{\uparrow \\ \partial_y f(0,0)=0}}{b} h_y + o(\|h\|)$$

$$= o(\|h\|)$$

In Polarkoo: $f(x,y) = \dfrac{r^4(\cos^4\varphi + \sin^4\varphi)}{r^2}$

$= r^2(\cos^4\varphi + \sin^4\varphi)$ für $(x,y) \neq (0,0)$

$f(h_x, h_y) = o(\|h\|) \Leftrightarrow \underbrace{\dfrac{f(x,y)}{r}}_{} \longrightarrow 0$ wenn $r \to 0$

$$r\underbrace{(\cos^4\varphi + \sin^4\varphi)}_{\in [0,2] \text{ beschr.}} \xrightarrow{r \to 0} 0$$

Alternativ: $x^4 + y^4 \leq x^4 + 2x^2y^2 + y^4 = (x^2+y^2)^2$

$\Rightarrow f(h_x, h_y) = O(\|h\|^2) = o(\|h\|)$.

A13

$F(\alpha,\beta,\gamma,x) := x^3+\alpha x^2+\beta x+\gamma$

$F(0,0,1,-1) = 0 = F(\alpha,\beta,\gamma,g(\alpha,\beta,\gamma))$

$F \in C^1(\mathbb{R}^4,\mathbb{R})$

$\frac{\partial F}{\partial x} = 3x^2+2\alpha x+\beta$

$\frac{\partial F}{\partial x}(0,0,1,-1) = 3(-1)^2 = 3 \neq 0$

Satz über implizite Funktionen:

- g existiert auf einer Umg. von $(0,0,1)$
- g ist dort C^1
- $\cancel{F\partial}$ Berechne ∇g:

$F(\alpha,\beta,\gamma,g(\alpha,\beta,\gamma)) = 0 \qquad | \,\partial_\alpha$

$\frac{\partial F}{\partial \alpha}(\alpha,\beta,\gamma,g(\alpha,\beta,\gamma)) + \frac{\partial F}{\partial x}(\ldots)\,\frac{\partial g}{\partial \alpha}(\alpha,\beta,\gamma) = 0$

$\Rightarrow \frac{\partial g}{\partial \alpha}(0,0,1) = -\underbrace{\left(\frac{\partial F}{\partial x}(0,0,1,-1)\right)}_{3}^{-1}\frac{\partial F}{\partial \alpha}(0,0,1,-1)$

Wegen $\frac{\partial F}{\partial \alpha} = x^2$ ist $\frac{\partial F}{\partial \alpha}(0,0,1,-1) = 1$,

also $\frac{\partial g}{\partial \alpha}(0,0,1) = -\frac{1}{3}$

Entspr. $\frac{\partial F}{\partial \beta} = x$, $\frac{\partial F}{\partial \beta}(0,0,1,-1) = -1$, $\frac{\partial g}{\partial \beta}(0,0,1) = \frac{1}{3}$

$\frac{\partial F}{\partial \gamma} = 1$, $\frac{\partial F}{\partial \gamma}(0,0,1,-1) = 1$, $\frac{\partial g}{\partial \gamma}(0,0,1) = -\frac{1}{3}$

A14 S konvex

$$\Rightarrow f(x) - f(y) = \int_\gamma \nabla f(\underline{u}) \cdot d\underline{u}$$

für $\gamma : [0,1] \to S$, $\gamma(t) = tx + (1-t)y$

also $$f(x) - f(y) = \int_0^1 dt \left\langle \nabla f(\gamma(t)), \underbrace{\dot{\gamma}(t)}_{x-y} \right\rangle$$

$$\Rightarrow |f(x) - f(y)| \leq \int_0^1 dt \, \left| \langle \nabla f(\gamma(t)), x - y \rangle \right|$$

Cauchy-Schwarz-Ungl.

$$\leq \int_0^1 dt \, \underbrace{\|\nabla f(\gamma(t))\|} \, \|x-y\|$$

$$\leq \underbrace{\sup_{\underline{u} \in S} \|\nabla f(\underline{u})\|}_{=:L} < \infty$$ weil ∇f st. und S kompakt (Satz von Weierstraß)

$$\leq \int_0^1 dt \, L \|x-y\| = L\|x-y\|. \qquad \square$$