Literatur:

- H. Heuser: Analysis 2
- O. Forster: "
- H. Fischer, H. Kaul:
  Math. f. Physiker Bd 1.

Analysis 2

Kap. 0: Einleitung

0.1 Funktionen

$f: \mathbb{R}^n \to \mathbb{R}^m$ oder $f: D \to \mathbb{R}^m$
mit $D \subseteq \mathbb{R}^n$

Bspe

- $f: \mathbb{R}^n \to \mathbb{R}$, $f(x_1 \dots x_n) = \max!$
  Optimierung

- Skalarfeld $T(\underline{x})$, $\underline{x} = (x_1 \dots x_n)$

  z.B. Temperatur

- Vektorfeld $\mathbb{R}^3 \to \mathbb{R}^3$

  $\underline{E}(\underline{x})$ el. Feld, $\underline{B}(\underline{x})$ magn. Feld

- $\underline{x}(t)$, $\underline{x} : \mathbb{R} \to \mathbb{R}^3$ parametrisierte Kurve

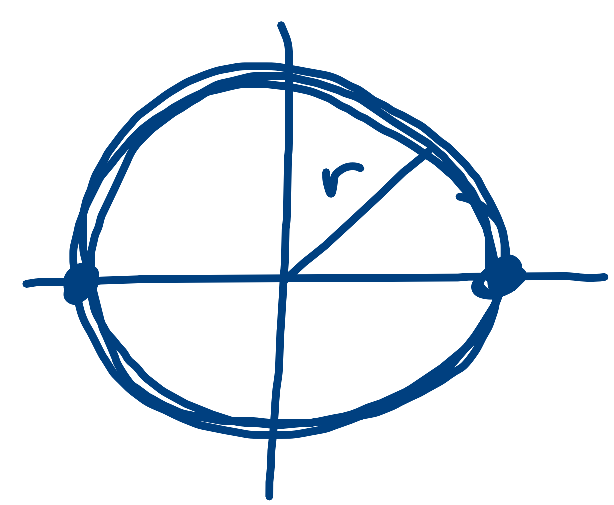
  $\{\underline{x}(t) : t \in \mathbb{R}\} \subset \mathbb{R}^3$ unparametrisierte Kurve $=$ Spur von $\underline{x}$

<u>Bsp</u> $\underline{x} : \mathbb{R} \to \mathbb{R}^2$

$$\underline{x}(t) = (r \cos \omega t,\ r \sin \omega t)$$

$r > 0$, $\omega \in \mathbb{R}$, Umlaufzeit $\frac{2\pi}{\omega}$

$\omega =$ Kreisfrequenz



- parametrisierte Fläche in $\mathbb{R}^3$

$$\phi : \mathbb{R}^2 \to \mathbb{R}^3$$

oder $\phi : D \to \mathbb{R}^3 , \quad D \subset \mathbb{R}^2$

rsp. $\left\{ \phi(u,v) : (u,v) \in D \right\} \subset \mathbb{R}^3$

- n, m

Energie
N Teilchen

$$= \sum_{i=1}^{N} \frac{m_i}{2} \sum_{a=1}^{3} v_{ia}^2 + V(x_{11} \dots x_{N3})$$

$$= E(x_{11} \dots x_{N3}, v_{11} \dots v_{N3}), \qquad 6\underset{\substack{\uparrow \\ 10^{23}}}{N} \text{ Var.}$$

Graphische Darstellungen

- Kurven in $\mathbb{R}^2$, $\mathbb{R}^3$
- $f: \mathbb{R}^2 \to \mathbb{R}$, $\quad$ Graph $= \{(x, y, f(x,y)) : (x,y) \in \mathbb{R}^2\} \subset \mathbb{R}^3$
- $f: \mathbb{R}^2 \to \mathbb{R}$ $\quad$ Konturdiagramm

Niveaulinien $\quad N_z = \{ (x,y) \in \mathbb{R}^2 : f(x,y) = z \}$ $\quad$ Isohypsen

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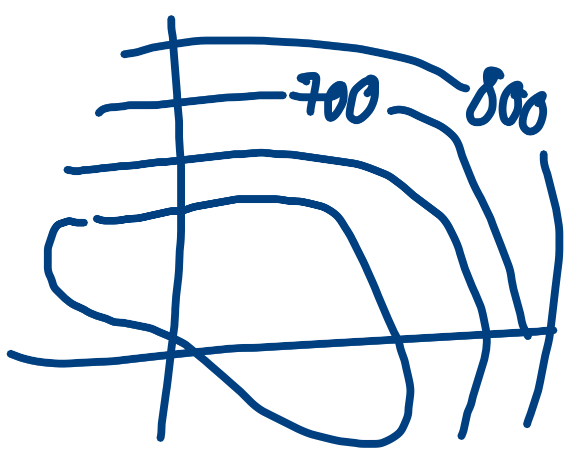
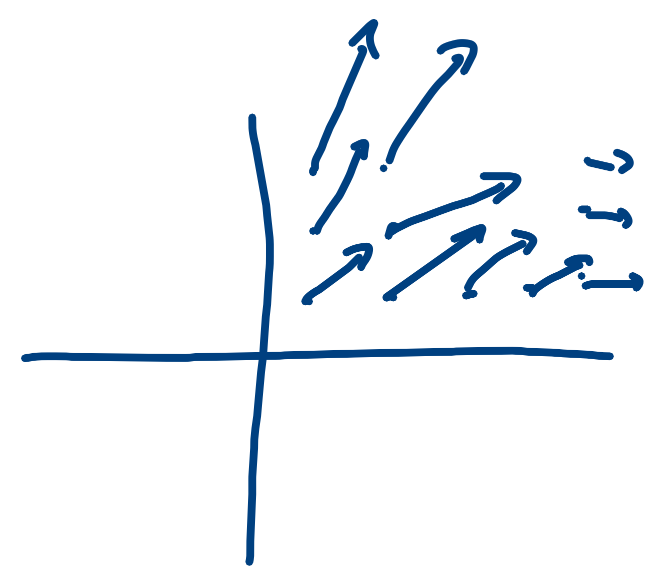
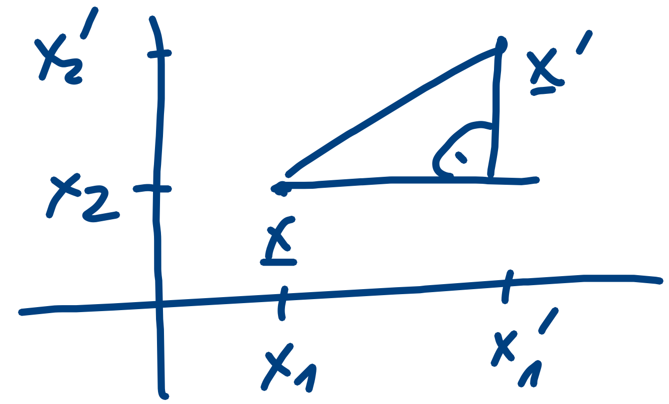


- $f: \mathbb{R}^3 \to \mathbb{R}$ $\quad$ Niveaumengen
- Vektorfeld: $\quad f: \mathbb{R}^2 \to \mathbb{R}^2$

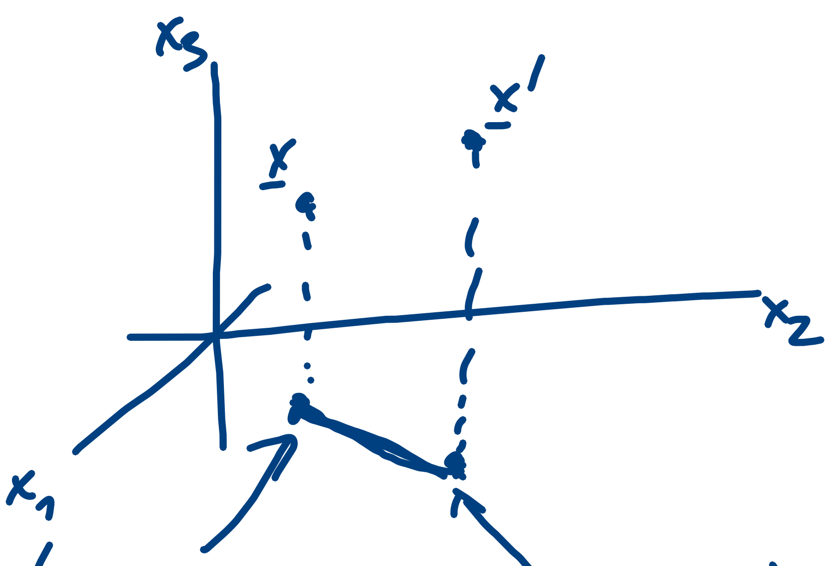


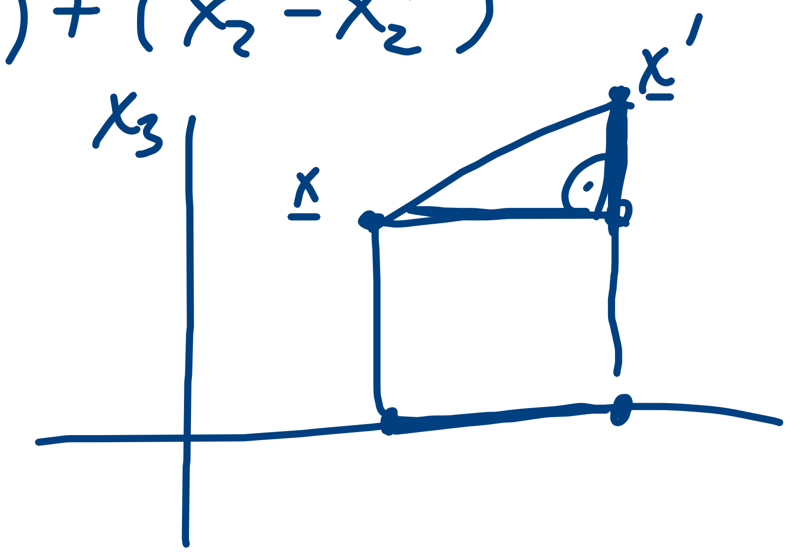
0.2 Abstand

Pythagoras



$$d(\underline{x},\underline{x}') = \sqrt{(x_1-x_1')^2+(x_2-x_2')^2}$$





$$d\Big((x_1,x_2,0),\ (x_1',x_2',0)\Big) = \sqrt{(x_1-x_1')^2+(x_2-x_2')^2}$$

$$d(\underline{x},\underline{x}') = \sqrt{(x_1-x_1')^2+(x_2-x_2')^2+(x_3-x_3')^2}$$

Es lässt sich zeigen: In $\mathbb{R}^n$,

$$d(\underline{x},\underline{x}') = \sqrt{\sum_{i=1}^{n} (x_i - x_i')^2}$$

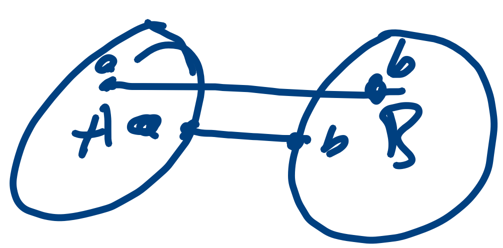
Abstand von $\underline{x}$ zum Ursprung

<u>Def</u> Norm $\quad \|\underline{x}\| := d(\underline{x},\underline{0}) = \sqrt{\sum_{i=1}^{n} x_i^2}$

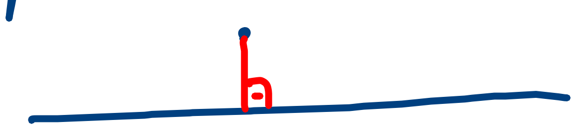
Euklidische Norm

<u>Def</u> Abstand 2er Mengen: $A, B \subseteq \mathbb{R}^n$, $A \neq \emptyset \neq B$,

$$d(A,B) := \inf\{ d(a,b) : a \in A,\ b \in B \} \geq 0$$

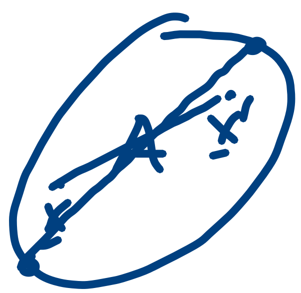


<u>Bsp</u> $A \cap B \neq \emptyset \Rightarrow d(A,B) = 0$



Def Durchmesser von $A \subseteq \mathbb{R}^n$, $A \neq \emptyset$

$$\operatorname{diam}(A) := \sup\{d(\underline{x}, \underline{x}') : \underline{x}, \underline{x}' \in A\}$$



sup (nicht nach oben beschr.)

$:= \infty$

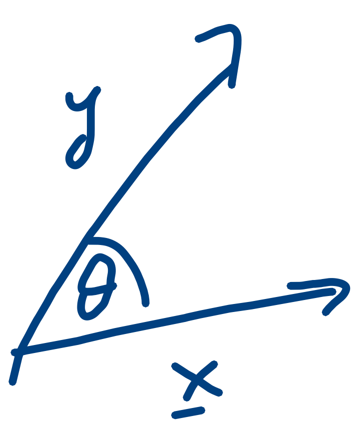
$\sup \in \mathbb{R} \cup \{\infty\}$

$$\operatorname{diam}(A) \in [0, \infty] := [0, \infty) \cup \{\infty\}$$

Skalarprodukt

$\underline{x}, \underline{y} \in \mathbb{R}^n$

$$\underline{x} \cdot \underline{y} := \sum_{i=1}^{n} x_i y_i$$



$$= \|\underline{x}\| \, \|\underline{y}\| \cos\theta$$

0.3 Besondere Arten von Funktionen

Bspe $\quad \frac{x}{y}, \quad \frac{\sin(x+y)}{\cos(xy)}, \quad \sqrt{x^3+e^{xy}}$

Polynom in $x_1 \dots x_n$

$$P(x_1 \dots x_n) = \sum_{\alpha_1 \dots \alpha_n = 0}^{r} c_{\alpha_1 \dots \alpha_n} x_1^{\alpha_1} x_2^{\alpha_2} \cdots x_n^{\alpha_n}$$

$\alpha_i \in \mathbb{N}_0 = \{0, 1, 2 \dots\}$ (endl. Summe)

Grad eines Monoms $\alpha_1 + \dots + \alpha_n$ (d.h. $\operatorname{Grad}(x^3y^4) = 7$)

$\operatorname{Grad}(P) = \max.\ \operatorname{Grad}.$

P heißt homogen, falls alle Monome darin denselben Grad haben.

Bsp $x^3 y^4 + 19 x^2 y^5 - 3x^7$

hom. vom Grad 7.

Def $f: \mathbb{R}^n \to \mathbb{R}$ permutations-symmetrisch $\iff$

$$\forall \sigma \in S_n \ \forall \underline{x} \in \mathbb{R}^n : \ f(x_{\sigma(1)}, x_{\sigma(2)}, \dots, x_{\sigma(n)}) = f(x_1, \dots, x_n)$$

Def $f: \mathbb{R}^n \to \mathbb{R}$ rotations-symmetrisch $\iff$

$$\exists g : [0,\infty) \to \mathbb{R} : \forall \underline{x} \in \mathbb{R}^n , \ f(\underline{x}) = g(\|\underline{x}\|)$$

"f hängt von $\underline{x}$ nur durch $\|\underline{x}\|$ ab."

Def $f : \mathbb{R}^n \to \mathbb{R}$ heißt **gerade** $\Longleftrightarrow$

$\forall \underline{x} \in \mathbb{R}^n \quad f(-\underline{x}) = f(\underline{x})$

ungerade $\Longleftrightarrow$

$f(-\underline{x}) = -f(\underline{x})$.

Bsp $h, k : \mathbb{R} \to \mathbb{R}$

$f(x,y) = h(x) + k(y)$

$g(x,y) = \underline{h(x)\,k(y)}$

$N, \dim\{f\} = N^2, \quad \dim\{h(x) + k(y)\} \leq 2N$

$= 2N - 1$

0.4 Ableitung im $\mathbb{R}^n$

$$\underline{x} : \mathbb{R} \to \mathbb{R}^n$$

$$\frac{d\underline{x}}{dt}(t_0) = \lim_{h\to 0} \frac{\underline{x}(t_0+h) - \underline{x}(t_0)}{h}$$

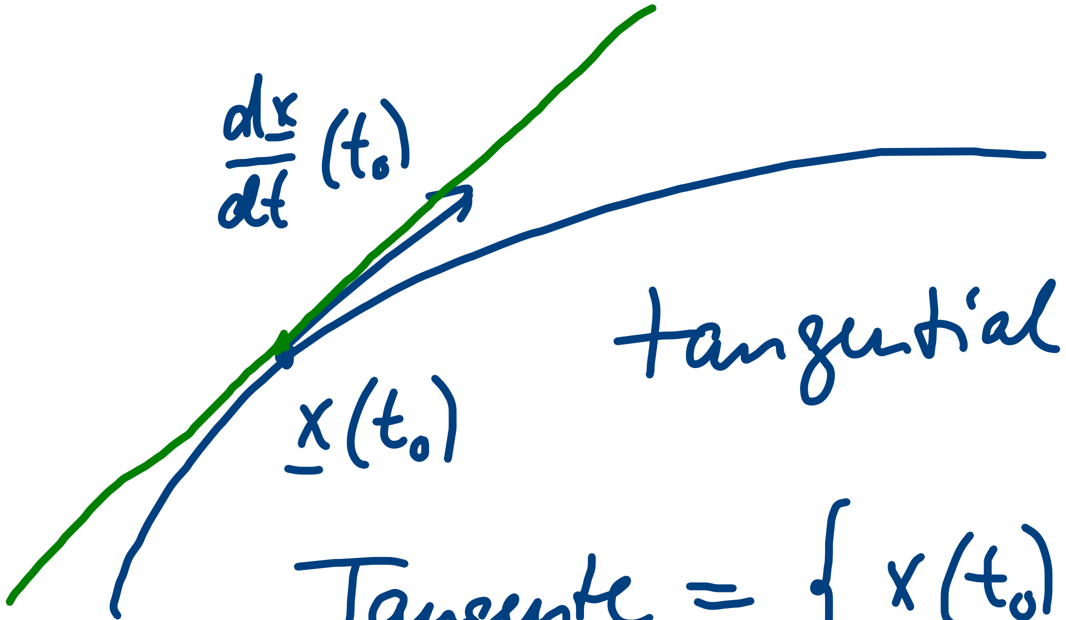
$(\underline{x}_m)_{m\in\mathbb{N}}$ Folge in $\mathbb{R}^n$

$$\lim_{m\to\infty} \underline{x}_m \overset{?}{=}$$

<u>Vorläufige Def</u>: $\lim_{m\to\infty} x_{mi} = x_i \quad \forall i \in \{1 \ldots n\}$

dann $\lim_{m\to\infty} \underline{x}_m = \underline{x} = (x_1 \ldots x_n)$

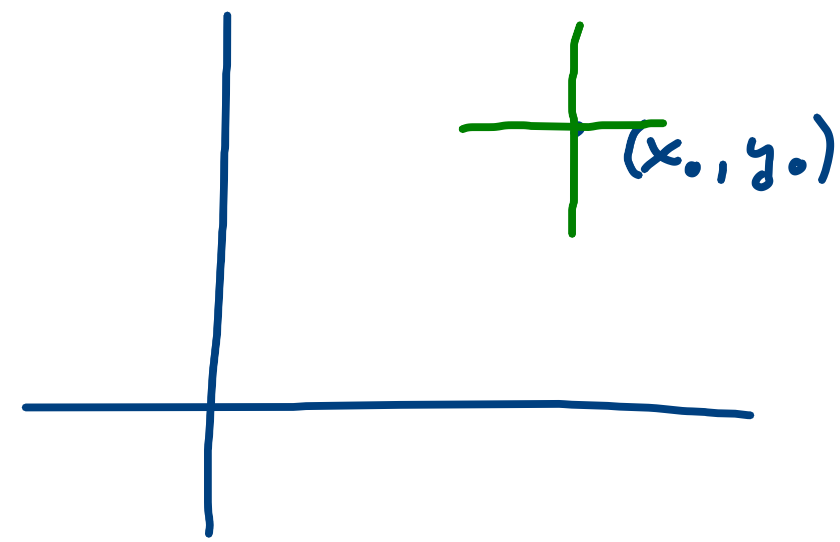
phys. Bed. von $\frac{dx}{dt}(t) =$ Geschw. zur Zeit t_0.



$$\text{Tangente} = \left\{ \underline{x}(t_0) + s\,\frac{d\underline{x}}{dt}(t_0) : s \in \mathbb{R} \right\}$$

falls $\frac{d\underline{x}}{dt}(t_0) \neq \underline{0}$.

Partielle Ableitung



$f(x,y) = \max!$ bei (x_0, y_0)

$g(x) := f(x, y_0)$

g ist max. bei x_0

$$\Rightarrow g'(x_0) = 0 \Rightarrow \frac{\partial f}{\partial x}(x_0,y_0) = \lim_{h \to 0} \frac{f(x_0+h, y_0) - f(x_0, y_0)}{h} = 0$$