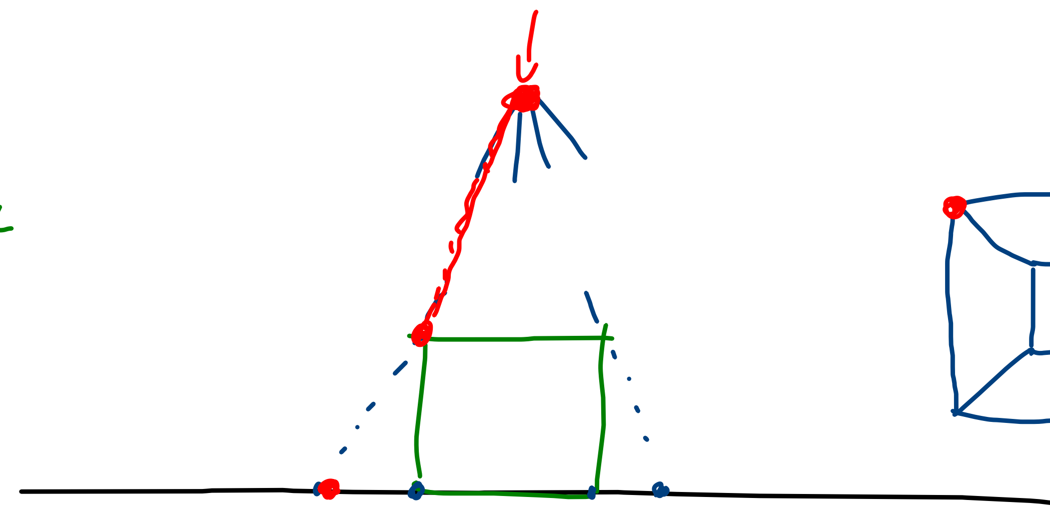
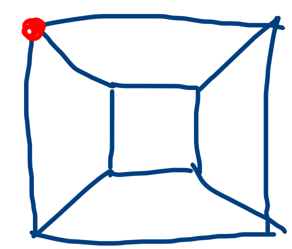
Rep. Analysis 2

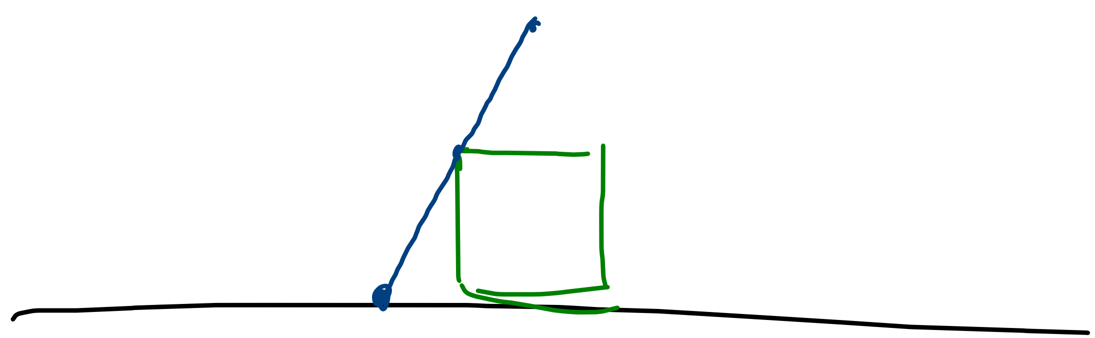
- Software: Maple, Mathematica (Wolfram), MatLab, Octave, ...

- Diagramme
  - 4d Würfel $[0,1]^4$

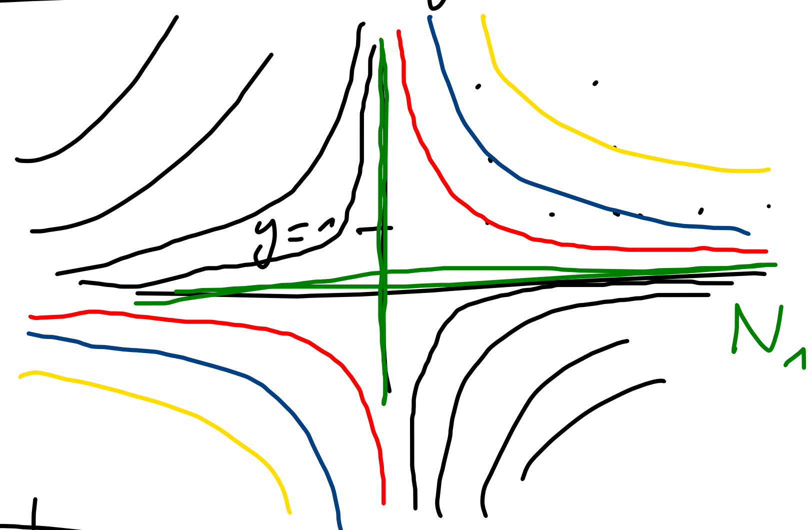
    

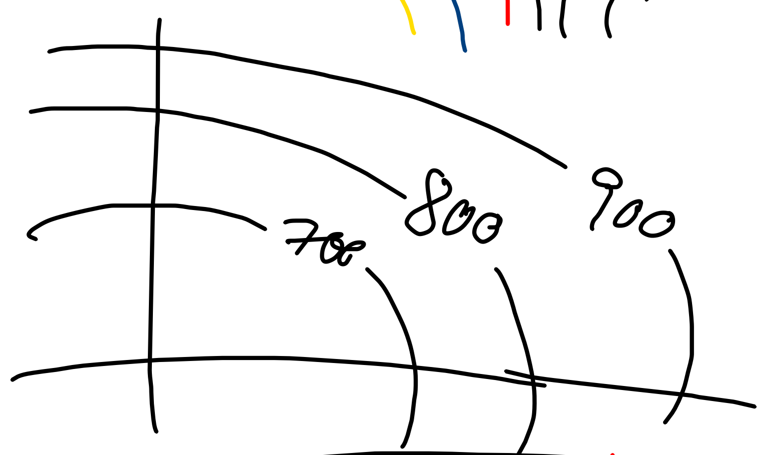
    

  - 3d Würfel

Konturdiagramme





$f(x,y) = e^{xy} > 0$

$f(1,1) = e$

$f(2,1) = e^2$

$f(3,1) = e^3$

$f(x,y) = 0, \quad N_0 = \emptyset$

$f(x,y) = 1$

$\Leftrightarrow \quad e^{xy} = 1$

$\Leftrightarrow \quad xy = 0$

$\Leftrightarrow \quad \begin{cases} x = 0 \text{ oder} \\ y = 0 \end{cases}$

$f(x,y) = e^2 \qquad \frac{1}{e}$

$\Leftrightarrow \quad e^{xy} = e^2$

$\Leftrightarrow \quad xy = 2 \qquad -1$

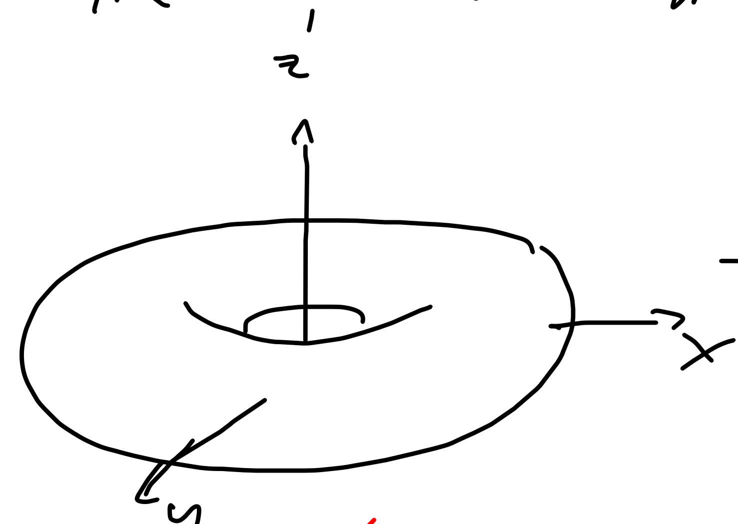
$\Leftrightarrow \quad y = \frac{2}{x} \qquad y = -\frac{1}{x}$

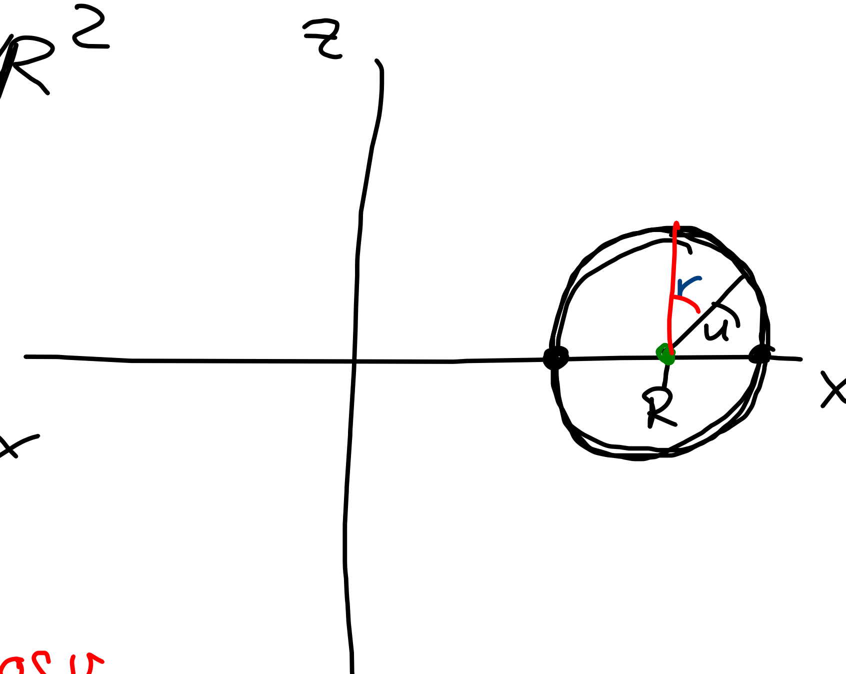
Parametrisierte Flächen

$$\phi : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$$

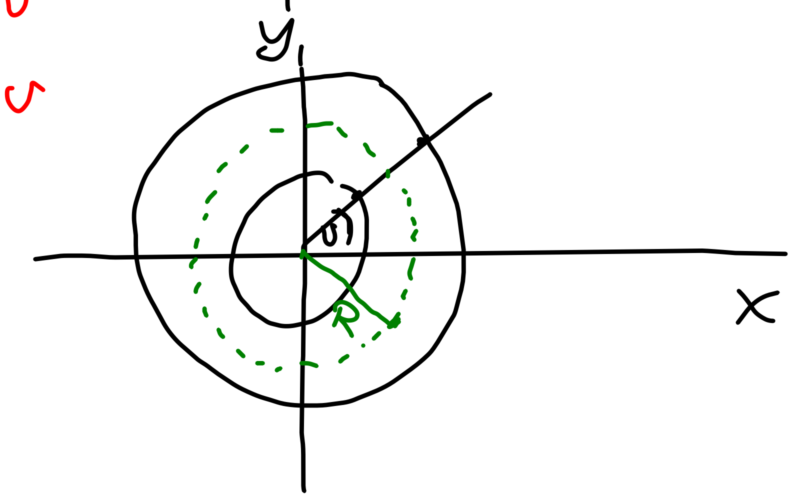
$$\phi : D \longrightarrow \mathbb{R}^3, \qquad D \subseteq \mathbb{R}^2$$

$$\phi(u,v)$$





$$\phi(u,v) = \begin{pmatrix} \cancel{R\cos v + r\sin u} \\ \cancel{R\sin v + r\sin u} \\ r\sin u \end{pmatrix} \qquad \begin{matrix} (R + r\cos u)\cos v \\ (R + r\cos u)\sin v \end{matrix}$$



Kreis $\psi(u) = \begin{pmatrix} r\,\overset{\cos}{\cancel{\sin}}\,u + R \\ r\,\overset{\sin}{\cancel{\cos}}\,u \end{pmatrix} = \begin{pmatrix} R \\ 0 \end{pmatrix} + \begin{pmatrix} r\cos u \\ r\sin u \end{pmatrix}$

$x_0 > 0 \qquad \begin{pmatrix} x_0 \\ 0 \end{pmatrix} \longmapsto \begin{pmatrix} x_0\cos v \\ x_0\sin v \end{pmatrix}$

Permutationsinv. Funktionen

Def $f: \mathbb{R}^n \to \mathbb{R}$ perm. inv. $\iff$

$$\forall \pi \in S_n \; \forall x \in \mathbb{R}^n : \; f(x_{\pi(1)} \dots x_{\pi(n)}) = f(x_1 \dots x_n).$$

Wahr oder falsch?

$f(x,y) = g(x,y) + g(y,x)$ ist für ~~g~~ jedes $g: \mathbb{R}^2 \to \mathbb{R}$ perm.-inv.

Wahr: $f(y,x) = g(y,x) + g(x,y) = g(x,y) + g(y,x) = f(x,y)$

Wahr oder falsch?

$f(x,y,z) = g(x,y,z) + g(y,z,x) + g(z,x,y)$ ist für jedes $g: \mathbb{R}^3 \to \mathbb{R}$ perm.-inv.

Falsch. Beweis: $g(x,y,z) = \frac{x}{1+y^2}$

$$\Rightarrow f(x,y,z) = \frac{x}{1+y^2} + \frac{y}{1+z^2} + \frac{z}{1+x^2}$$

$$f(y,x,z) = \frac{y}{1+x^2} + \frac{x}{1+z^2} + \frac{z}{1+y^2} \quad \square$$

Alternativ: $g(x,y,z) = x + 2y + 3z$

$$\Rightarrow f(x,y,z) = x + 2y + 3z + y + \cancel{3}2z + 3x + z + 2x + 3y$$

$$= 6x + 6y + 6z$$

Gerade und ungerade Fktnen

<u>Def</u> f gerade $\Longleftrightarrow$ $f(-x) = f(x)$

f ungerade $\Longleftrightarrow$ $f(-x) = -f(x)$

Wahr oder falsch?

- Die Summe 2er gerader Fktnen ist stets gerade.

Wahr. Beweis: $g(x) = g(-x)$, $h(x) = h(-x)$, $f(x) = g(x) + h(x)$

$$f(-x) = g(-x) + h(-x) = g(x) + h(x) = f(x).$$

- Die Summe 2er unger. Fktnen ist stets ungerade.

$x+1$,
$-x+1$,

~~$f(x)$~~ $g(x) = x^3$, $h(x) = x$

$$f(x) = g(x) + h(x) = x^3 + x$$

$$f(-x) = (-x)^3 + (-x) = -(x^3 + x)$$

Wahr: $g(x) = -g(-x)$, $h(x) = -h(-x)$, $f(x) = g(x) + h(x)$,

$f(-x) = g(-x) + h(-x) = -g(x) - h(x) = -f(x)$. $\square$

- Das Produkt 2er ungerader Fktnen ist stets ~~ungerade~~ gerade.

Falsch. Gegenbeispiel: x^3, x^5, Prod $= x^8$

Wahr: $g(-x)\,h(-x) = (-g(x))(-h(x)) = g(x)\,h(x)$.

x^n gerade $\Leftrightarrow$ n gerade

x^n unger. $\Leftrightarrow$ n unger.

Rotations-inv. Fktionen

Def $f: \mathbb{R}^n \to \mathbb{R}$ rot.-inv. $\Longleftrightarrow$

$$\exists g: [0,\infty) \to \mathbb{R}\ \forall x \in \mathbb{R}^n: f(x) = g(\|x\|).$$

Eukl. Norm

Bem rot. inv. $\Longleftrightarrow$ $\forall A \in O(n): f(Ax) = f(x)$.

Wahr oder falsch?

1) ∘ Jede rot.-inv. Fkt ist perm.-inv.

2) ∘ Jede rot.-inv. Fkt ist gerade.

2) Wahr. Bew: $f(x) = g(\|x\|) = g(\|-x\|) = f(-x)$

1) Wahr. Bew: $f(x_{\pi(1)} \dots x_{\pi(n)}) = g\left(\left\| \begin{pmatrix} x_{\pi(1)} \\ \vdots \\ x_{\pi(n)} \end{pmatrix} \right\|\right) = g(\|x\|) = f(x).$ □