Universität Tübingen, Fachbereich Mathematik — Wintersemester 22/23
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Groups and Representations

Homework Assignment 14 (due on 8 February 2023)

Problem 46
We show that the $\mathrm{GL}(N)$ irrep corresponding to the Young diagram $\Theta_\mathrm{a} = \begin{array}{|c|}\hline \\ \hline \\ \hline \vdots \\ \hline \\ \hline \end{array}$ with N rows is given by the determinant:

- First recall that for vectors $|i_1, \ldots, i_N\rangle$ contributing to $e_\mathrm{a} g|\alpha\rangle$ all i_k are different.
- Write these vectors as $p|1, \ldots, N\rangle$ with a permutation p.
- Then calculate $e_\mathrm{a} g|1, \ldots, N\rangle$ for $g \in \mathrm{GL}(N)$.

Which irrep corresponds to Θ_a if we replace $\mathrm{GL}(N)$ by the subgroup $\mathrm{SU}(N) \subset \mathrm{GL}(N)$?

Problem 47
Consider Young diagrams with row lenghts $\lambda = (\lambda_1, \ldots, \lambda_N)$, and $\lambda' = (\lambda_1{+}k, \ldots, \lambda_N{+}k)$, $k \geq 1$. Show that the $\mathrm{SU}(N)$-irreps Γ^λ and $\Gamma^{\lambda'}$ are equivalent.

Hint: Use the Littlewood-Richardson rule and the result of Problem 46.

Problem 48
Let Γ^λ be an $\mathrm{SU}(3)$-irrep with Young diagram λ. Determine how often Γ^λ appears in the product rep $\lambda \otimes \begin{array}{|c|c|}\hline & \\ \hline \\ \cline{1-1} \end{array}$.

Hint: Study separately the cases of rectangular Young diagrams λ (with one or two rows) and of non-rectangular diagrams.

Problem 49
Decompose the product rep $\square \otimes \square \otimes \square$ of $\mathrm{SU}(3)$ into irreps. Use the notation of Problem 28 (e.g. $|uds\rangle = |u\rangle \otimes |d\rangle \otimes |s\rangle \in \square^{\otimes 3}$) and explicitly construct bases for the irreducible invariant subspaces. Compare with the results of Problem 28. What is the relation between the irreducible subspaces with respect to $\mathrm{SU}(3)$ and those with respect to S_3?