Microscopic Derivation of Vlasov type equations

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Abstract

I present a probabilistic technique for the proof of the mean-field limit and propagation of chaos of a N-particle system in three dimensions with highly singular interactions. The two recent results lead to a derivation of the Vlasov-Poisson equation one the one hand and to a derivation of the Vlasov-Dirac-Benney equation on the other hand, depending on the pair interaction potential. In the first application we prove for typical initial data convergence of the empirical distributions to solutions of the Vlasov-Poisson system for Coulomb interaction and cut-off size much smaller than the typical inter particle distance. More precisely, the interaction fulfils $f^N(q) = \pm\frac{q}{|q|^3}$ for $|q| > N^{-\frac{5}{12}+\sigma}$ and has a cut-off at $|q| = N^{-\frac{5}{12}+\sigma}$ where $\sigma > 0$ can be chosen arbitrarily small.
In the second application we prove also for typical initial data, convergence of the empirical distributions to solutions of the Vlasov-Dirac-Benney system with compactly supported pair potentials of the form $N^{3\beta-1}\phi(N^\beta x)$ for $\beta \in \left[0, \frac{1}{7}\right)$ and $\phi \in L^\infty(R^3) \cap L^1(R^3)$. Thus our result leads to a derivation of the Vlasov-Dirac-Benney equation from the microscopic N-particle dynamics with a strong short range force.
In particular, for typical initial data, we show convergence of the Newtonian trajectories to the characteristics of the Vlasov type equations.

References

[1] M. Feistl, P. Pickl, *Microscopic derivation of Vlasov equation with compactly supported pair potentials.*

[2] M. Feistl, P. Pickl, *On the mean-field limit for the Vlasov-Poisson system.*