Prof. Dr. Roderich Tumulka — Winter semester 2023/24
Eberhard Karls University Tübingen — February 7, 2024

Practice Exam Foundations of Quantum Mechanics

The exam takes 2 hours. During the exam, it is not allowed to use calculators, electronic devices, books, or notes taken before the exam.

Problem 1: (worth 3 points out of 100)
In a Poisson process, the waiting time between two events has (check one)
☐ Gaussian distribution ☐ uniform distribution ☐ exponential distribution

Problem 2: True or false?
(a) (3 points) In the presence of interaction, an initially entangled wave function becomes disentangled.

(b) (6 points) Any proof of nonlocality must involve an *entangled* wave function of *at least two* particles. Explain.

Problem 3: (6 points) What does the spectral theorem for self-adjoint operators say?

Problem 4: (6 points) What are the 3 premises that lead to a contradiction in the quantum measurement problem? For each premise, give an example of a theory that denies it.

Problem 5: (12 points) Describe the quantum Zeno effect. (You can use formulas and drawings. No need for proofs.)

Problem 6: Consecutive quantum measurements (14 points)
Let A_1, A_2 be self-adjoint operators in $\mathscr{H}$ whose spectra $\sigma(A_k)$ are purely discrete (i.e., countable), so that

$$A_k = \sum_{\alpha\in\sigma(A_k)} \alpha\, P_{k,\alpha}$$

with $P_{k,\alpha}$ the projection to the eigenspace of A_k with eigenvalue α. Consider a quantum system with initial wave function $\psi_0 \in \mathscr{H}$ with $\|\psi_0\| = 1$ at time t_0. At times $t_1 < t_2$, ideal quantum measurements of A_1, A_2 (respectively) are carried out with outcomes $Z_1, Z_2 \in \mathbb{R}$ ($t_0 < t_1$).

(a) Compute the joint probability distribution of Z_1, Z_2.

(b) Show that there is a POVM E on $\mathbb{R}^2$ such that

$$\mathbb{P}\Big((Z_1, Z_2) \in B\Big) = \langle\psi_0|E(B)|\psi_0\rangle$$

and give an explicit expression for $E(B)$.

Problem 7: Uncertainty relation (12 points)
Compute both sides of the generalized uncertainty relation

$$\sigma_A\,\sigma_B \geq \frac{1}{2}\Big|\langle\psi|[A,B]|\psi\rangle\Big|$$

for $A = \sigma_1$ (Pauli matrix), $B = \sigma_2$, and $\psi = |z\text{-down}\rangle$.

Problem 8: GRW theory (12 points)
Consider the GRW theory with the constant σ much smaller than the value 10^{-7} m suggested by GRW; say, $\sigma = 10^{-12}$ m. Explain why Heisenberg's uncertainty relation implies that a free electron, after being hit by a GRW collapse, could move very fast. Use the uncertainty relation to compute the order of magnitude of how fast it can be (assuming it was more or less at rest before the collapse); the mass of an electron is about 10^{-30} kg and $\hbar \approx 10^{-34}$ kg m^2 s^{-1}.

Problem 9: Quantile rule (12 points)
Show that in Bohmian mechanics in 1 dimension, if Q_0 is the α-quantile of $\rho = |\psi_0|^2$, then Q_t is the α-quantile of $\rho = |\psi_t|^2$. Recall that for a probability density $\rho(x)$, the α-quantile for $0 < \alpha < 1$ is the point x_α where

$$\int_{-\infty}^{x_\alpha} \rho(x)\, dx = \alpha.$$

Problem 10: Galilean relativity of GRWf theory (14 points)
Recall that a Galilean transformation is a change of space-time coordinates given by

$$\boldsymbol{x}' = \boldsymbol{x} + \boldsymbol{v}t\,, \quad t' = t\,. \tag{1}$$

Suppose the potential V is translation invariant. In this problem, we verify that GRWf is Galilean invariant, for simplicity only for $N = 1$ particle. We know already that if $\psi(t, \boldsymbol{x})$ is a solution of the Schrödinger equation, then so is

$$\psi'(t', \boldsymbol{x}') := \exp\left[\tfrac{im}{\hbar}(\boldsymbol{x}' \cdot \boldsymbol{v} - \tfrac{1}{2}\boldsymbol{v}^2 t')\right] \psi\Big(t', \boldsymbol{x}' - \boldsymbol{v}t'\Big). \tag{2}$$

(a) Show that if $\boldsymbol{X}' = \boldsymbol{X} + \boldsymbol{v}T$ and $T' = T$ are the transformed coordinates of the first flash $(T, \boldsymbol{X})$, then the distribution of $(T', \boldsymbol{X}')$ is given by the same formula in terms of the transformed wave function ψ' as that of $(T, \boldsymbol{X})$ in terms of ψ.

(b) Show that, given that a flash occurs at the space-time point $(T, \boldsymbol{X})$ (which has new coordinates $(T', \boldsymbol{X}')$), the collapsed of the Galilean transformed wave function equals the Galilean transformed of the collapsed one.

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