

Groups and Representations

Homework Assignment 1 (due on 22 October 2025)

Problem 1 (rearrangement lemma)

For the group table of a finite group prove that no two elements within one row (or column) can be the same.

Problem 2

List all possible groups (up to isomorphism) of order 3 and 4 by explicitly constructing their group tables.

Problem 3

Show that the map $\exp : (\mathbb{R}, +) \rightarrow (\mathbb{C} \setminus \{0\}, \cdot)$, $t \mapsto e^{2\pi it}$ is a group homomorphism. Determine kernel and image of $\exp$, and check whether these are subgroups of $(\mathbb{R}, +)$ and $(\mathbb{C} \setminus \{0\}, \cdot)$, respectively.

Problem 4

Let

$$\mathcal{B} = \{f : \mathbb{R} \rightarrow \mathbb{R} \mid f \text{ bijective}\} \quad \text{and} \\ \mathcal{A} = \{f : \mathbb{R} \rightarrow \mathbb{R} \mid \exists a, b \in \mathbb{R}, a \neq 0 : f(x) = ax + b\} .$$

Show that $(\mathcal{B}, \circ)$, with $\circ$ the composition of functions, is a group and that $\mathcal{A}$ is a subgroup of $\mathcal{B}$. Is $\mathcal{B}$ or $\mathcal{A}$ abelian?

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