Universität Tübingen, Fachbereich Mathematik
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Groups and Representations

Homework Assignment 2 (due on 29 October 2025)

Problem 5
Let G be a finite group acting on the set M; for $m \in M$ let $G_m = \{g \in G : gm = m\}$. Show:

a) For each $m \in M$ the set G_m is a subgroup of G.

b) If $n \in Gm$ then $G_n \cong G_m$.

c) $|Gm| \cdot |G_m| = |G|$ (orbit-stabiliser theorem).

Problem 6
Let W be the symmetry group of a cube. (We consider only rotations, no reflections.) Determine $|W|$, the order of W, by considering the action of W on corners, edges or faces of the cube and applying the orbit-stabiliser theorem.

Problem 7
Let G be a group. For every $g \in G$ conjugation with g is defined by the map $\hat{g} : G \to G$, $x \mapsto gxg^{-1}$. Show:

a) Conjugation defines an action, $(g, h) \mapsto \hat{g}(h)$, of G on itself.

b) G is abelian iff every orbit of this action has length one.

c) The number of elements of a conjugacy class divides $|G|$.

Problem 8
Let $\varphi : G \to H$ be a group homomorphism with kernel K and image B. Show:

a) K is a normal subgroup of G.

b) φ induces an isomorphism $\hat{\varphi} : G/K \to B$.

Problem 9
We denote by $\mathrm{Inn}(G)$ the set of all inner automorphisms of the group G, i.e. the isomorphisms $\varphi : G \to G$ such that there exists an $h \in G$ with $\varphi(g) = hgh^{-1}\ \forall\, g \in G$. Show that $\mathrm{Inn}(G)$ is a normal subgroup of $\mathrm{Aut}(G)$, the group of all isomorphisms $G \to G$ (under composition).