

Groups and Representations

Homework Assignment 10 (due on 7 January 2026)

Problem 40

- a) Determine the Haar measure for $SU(2)$ in axis-angle parametrisation,

$$U = \exp\left(-i\frac{\alpha}{2}\vec{\sigma} \cdot \vec{x}\right),$$

with $0 \leq \alpha \leq 2\pi$ and $\vec{x} \in S^2 \leftrightarrow \mathbb{R}^3$. Normalise s.t. $\text{vol}(SU(2)) = 1$.

HINT: It is convenient to first show $(\vec{x} \cdot \vec{\sigma})(\vec{y} \cdot \vec{\sigma}) = \mathbb{1}\vec{x} \cdot \vec{y} + i\vec{\sigma}(\vec{x} \times \vec{y})$ and to use the unit vectors $\vec{e}_r, \vec{e}_\theta, \vec{e}_\phi$ for spherical coordinates.

- b) Use the result of (a) together with the results of Problem 37 in order to determine the Haar measure for $SO(3)$ in the axis-angle parametrisation.

Problem 41

We consider derived reps $d\Gamma^j$ of irreps Γ^j of $SU(2)$ as in Sec. 6.8 of the lecture.

- a) Determine $d\Gamma^j(\sigma_k/2)$ for $k = 1, 2, 3$ and $j = 0, \frac{1}{2}, 1, \frac{3}{2}$.
b) Verify $[d\Gamma^j(\sigma_1/2), d\Gamma^j(\sigma_2/2)] = i d\Gamma^j(\sigma_3/2)$ for $j = 0, \frac{1}{2}, 1, \frac{3}{2}$.

