Universität Tübingen, Fachbereich Mathematik
Dr. Stefan Keppeler & Saradha Senthil-Velu

Wintersemester 25/26

Groups and Representations

Homework Assignment 13 (due on 28 January 2026)

Problem 47
Consider Young diagrams with row lenghts $\lambda = (\lambda_1, \ldots, \lambda_N)$, and $\lambda' = (\lambda_1+k, \ldots, \lambda_N+k)$, $k \geq 1$. Show that the SU(N)-irreps Γ^λ and $\Gamma^{\lambda'}$ are equivalent.

HINT: Use the Littlewood-Richardson rule and the result of Problem 46.

Problem 48
Let Γ^λ be an SU(3)-irrep with Young diagram λ. Determine how often Γ^λ appears in the product rep $\lambda \otimes$  .

HINT: Study separately the cases of rectangular Young diagrams λ (with one or two rows) and of non-rectangular diagrams.

Problem 49
Decompose the product rep $\square\otimes\square\otimes\square$ of SU(3) into irreps. Use the notation of Problem 35 (e.g. $|uds\rangle = |u\rangle \otimes |d\rangle \otimes |s\rangle \in \square^{\otimes 3}$) and explicitly construct bases for the irreducible invariant subspaces. Compare with the results of Problem 35. What is the relation between the irreducible subspaces with respect to SU(3) and those with respect to S_3?