

Interacting Many-Body Systems

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Sheet 10

Exercise 1: Find all conditions on the Bogoliubov Transformation T that follow from the canonical commutation relations:

$$\begin{aligned}[a(f), a(g)] &= [a^*(f), a^*(g)] = 0 \\ [a(f), a^*(g)] &= \langle f, g \rangle\end{aligned}$$

Exercise 2: Calculate the variance of $\sum_{j=1}^N V(x_j)$ in the case that Ψ is a symmetrized product $\left(\prod_{j=1}^N e_j(x_j)\right)_{sym}$.

compare to the Fermionic and the classical case.

Exercise 3: Given an ONB $\{e_j : j \in I\}$ of the one-particle Hilbert space, let $\Psi := \bigwedge_{j=1}^N e_j(x_j)$. Find a simplified expression for

$$\left\langle \Psi, \left(\sum_{j=1}^N V(x_j) - E \right)^3 \Psi \right\rangle \quad (1)$$

where $E = \langle \Psi, \sum_{j=1}^N V(x_j) \Psi \rangle$.

Assume (like in class) that the e_j of lowest energy are occupied, further that \widehat{V} is bounded and of compact support. Give an estimate for (1) in that case.