Interacting Many-Body Systems

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Sheet 11

Exercise 1: In the derivation of the effective time evolution of the tracer entering the Fermi gas, the V,V term of second order term in the Duhamel expansion of $\Psi_t^{free} - P\Psi_t$ is given by

$$\sum_{|l|<k_F, |l-k|>k_F} \lambda^2 |\widehat{V}(k)| I_{k,l}$$

where

$$I_{k,l} = \int_0^t \int_\tau^t e^{-(E^G+j^2)(t-\tau)} \widehat{V}(-k) e^{-(E^G+(l-k)^2-l^2+(j+k)^2)(\tau-\sigma)} \widehat{V}(k) e^{-(E^G+j^2)\sigma} \Psi_0 d\sigma d\tau$$

where $\Psi_0 = e^{ijy} \bigwedge_{|k|<k_F} e^{ikx}$, describing a system of a tracer with initial momentum j and the filled Fermi sea.

Show that $I_{k,l}$ can be replaced by a term with only one time integration. Explicitly give that term ,respectively the expression for C_j (There was a mistake in the presentation given in class, which was corrected on Jan. 19th.)

HINT: In class we controlled terms of that kind by integration by parts. For this particular term, however, the σ integral can simply be solved.

Argue, that the leading order of C_j calculated in exercise 2 is given by C_0.

Exercise 2: Consider the case of two tracer particles. Give the effective interaction between the two tracers W by finding an expression for $\widehat{W}(k)$ for any k.

Remark: This expression will still include a sum over all l with $|l| < k_F$, $|l - k| > k_F$ which can not be simplified easily.

Exercise 3: Simplify the expression for $\widehat{W}(k)$ you found in exercise 2 in the case of one spatial dimension.