Interacting Many-Body Systems

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Sheet 2

Exercise 1: Let ϕ be a one-particle wave function, i.e. a vector on the Hilbert space of square integrable complex functions on $\mathbb{R}^3$ and $V : \mathbb{R}^3 \to \mathbb{R}$ an integrable function. Show that

$$p_1 V(x_1 - x_2) p_1 = V \star |\phi|^2(x_2) p_1 \ .$$

Here, $\star$ stands for the convolution: $f \star g(x) := \int f(x-y) g(y) d^3y$.

Exercise 2: Let Ψ, χ, Θ, η and ϕ be the objects defined in exercise 2 from sheet number one. Let $p_j = |\phi(x_j)\rangle\langle\phi(x_j)|$. Calculate $\langle \Psi, p_1 \Psi \rangle$, $\langle \chi, p_1 \chi \rangle$ and $\langle \Theta, p_1 \Theta \rangle$ as well as $\|\mu_\Psi - p_1\|_{op}$, $\|\mu_\chi - p_1\|_{op}$, $\|\mu_\Theta - p_1\|_{op}$, $\|\mu_\Psi - p_1\|_{tr}$, $\|\mu_\chi - p_1\|_{tr}$, an $\|\mu_\Theta - p_1\|_{tr}$.

Exercise 3: Let $(\Psi_N)_{N\in\mathbb{N}}$ be a sequence of Bosonic N-particle wave functions, i.e. for each N Ψ_N is an element of the Hilbert space of square integrable complex functions on $\mathbb{R}^{3N}$ that are symmetric under the exchange of variables.

Let ϕ be a one-particle wave function and $p_j = |\phi(x_j)\rangle\langle\phi(x_j)|$.

Show that

(a)
$$\lim_{N\to\infty} \langle \Psi_N, p_1 \Psi_N \rangle = 1 \Leftrightarrow \lim_{N\to\infty} \langle \Psi_N, p_1 p_2 \Psi_N \rangle = 1$$

(b)
$$\lim_{N\to\infty} \langle \Psi_N, p_1 \Psi_N \rangle = 1 \Leftrightarrow \lim_{N\to\infty} \left\| \mu_\Psi^{(2)} - p_1 p_2 \right\|_{op} = 0$$

where $\mu_\Psi^{(2)}$ is the reduced two-particle density matrix of the system.

(c)
$$\lim_{N\to\infty} \|\mu_\Psi - p_1\|_{op} \Leftrightarrow \lim_{N\to\infty} \left\| \mu_\Psi^{(2)} - p_1 p_2 \right\|_{op} = 0$$