Interacting Many-Body Systems

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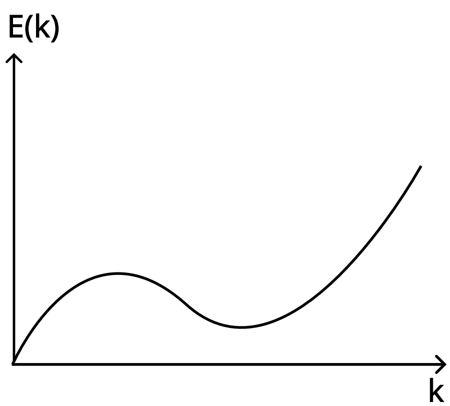
Sheet 7

Exercise 1: Let ϵ_t be a solution of the effective equation we found in class to describe the time evolution of a small perturbation with respect to a Bose gas of constant density. Assume that $\widehat{V}$ is everywhere negative, smooth, radially symmetric and $\lim_{k\to\infty}\widehat{V}(k)=0$.

Argue that ϵ_t is in general unstable. Which momenta lead to instability, which don't?

Exercise 2: (Superfluid Helium-4)



On the left you see a sketch of the dispersion relation for small perturbations in superfluid Helium-4. In the graph, k stands for the momentum of the wave, $E(k)$ for the respective eigenvalue of the Hamiltonian of the effective description. Note, that in contrast to Helium-3, Helium-4 atoms are Bosons.
Give a rough picture of how you expect the interaction between Helium-4-atoms to look like.
Argue that there is a second speed of sound in ultra-cold Helium-4, i.e. a wave that travels with a certain speed and decays slower than usual.

Exercise 3: (Mixture of two condensates of identical particles)

In the following exercise, Ψ_t is a solution of the Schrödinger equation w.r.t. the Hamiltonian

$$H=\sum_{j=1}^{N}-\Delta_j+\frac{1}{N-1}\sum_{j\neg k}V(x_j-x_k)$$

Let ϕ_0,η_0 be two orthogonal and $L^2\left(\mathbb{R}^3\right)$-normalized one-particle wave functions.

In class we have seen, that there is no simple effective description of a mixture of two condensates w.r.t. ϕ_0 respectively η_0. For certain linear combinations of mixtures, however, one can in fact find an effective description.

So consider

$$\Psi_0=\sum_{k=0}^{N}C_k\left(\prod_{i=1}^{k}\phi_0(x_i)\prod_{j=k+1}^{N}\eta_0(x_j)\right)_{sym}$$

for $C_k \in \mathbb{C}$ with $\sum_{k=0}^{N} |C_k|^2 = 1$.

Find a choice of C_k and an evolution equation for ϕ_t and η_t such that Ψ_t is effectively described by

$$\sum_{k=0}^{N} C_k \left(\prod_{i=1}^{k} \phi_t(x_i) \prod_{j=k+1}^{N} \eta_t(x_j) \right)_{sym}$$

Hint: Think of a product of $2^{-1/2}(\phi_0 + \eta_0)$.