MATHEMATISCH-NATURWISSENSCHAFTLICHE FAKULTÄT DER UNIVERSITÄT TÜBINGEN WiSe 2025/26

# Interacting Many-Body Systems

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## Sheet 9

**Exercise 1:** Show that the difference between $\Psi_t$ and $\Psi_t^{2nd}$ (see Definition in our class) is in fact of order $N^{-1}$ for times of order $N$.

Hint: Use the projectors that project on that part of Hilbert space with an odd (even) number of excitations (particles not in the reference-state).

**Exercise 2:** Let $\phi_j, \eta_j \in L^2(\mathbb{R}^3)$ for any $j \in \mathbb{N}_0$, let $\Psi_t := U_{0,t}^{Bog}\Psi_0$ with $\Psi_0 = \Omega$.

Here $U^{Bog}$ stands for Bogoliubov time-evolution, $\Omega$ is the respective vacuum state.

Show that for every odd $k$ the expression

$$\langle \Psi_t, \prod_{j=1}^{k} \left(a(\phi_j) + a^\star(\eta_j)\right) \Psi_t \rangle = 0$$

for all $t > 0$.

**Exercise 3:** Let $\Psi_t := U_{0,t}^{Bog}\Psi_0$ with $\Psi_0 = \Omega$.

Here $U^{Bog}$ stands for Bogoliubov time-evolution, $\Omega$ is the respective vacuum state, i.e. $a(\phi_k)\Omega = 0$ for all $k \neq 0$ where $\{\phi_k : k \in \mathbb{Z}\backslash\{0\}\}$ is an ONB of the one-particle Hilbert space describing the excitations.

Assume that $V_{kl00} := \langle \phi_k(x_1)\phi_l(x_2)V(x_1 - x_2)\phi_0(x_1)\phi_0(x_2)\rangle = 0$ whenever $k \neq -l$.

Show that there is a time-dependent constant $C_t$ such that

$$\sum_{k \in I} \langle \Psi_t a^*(\phi_k) a(\phi_k) \Psi_t \rangle \leq C_t \ .$$