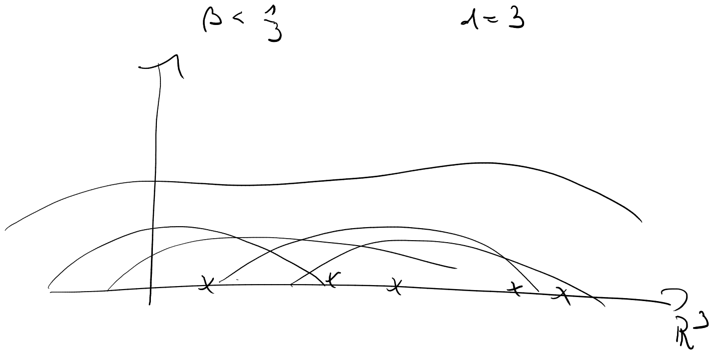
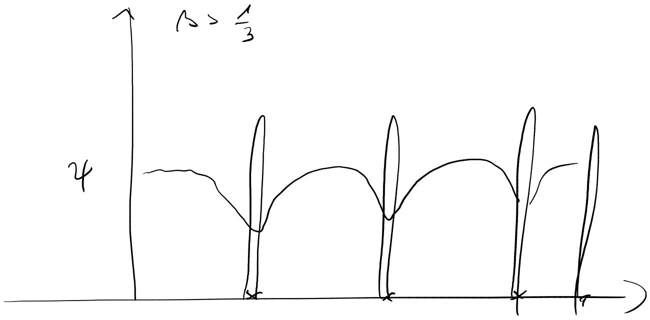
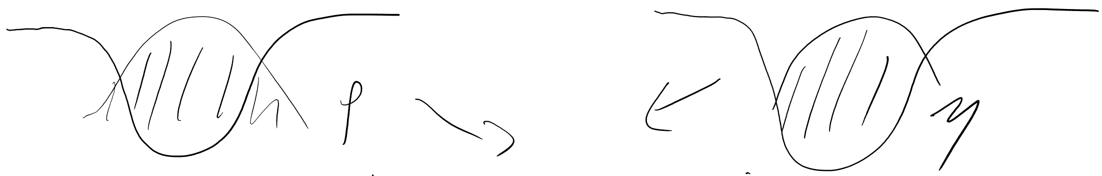




Also in this situation an effective equation can be derived.

Mixtures of Bosons



There are two kinds of mixtures we want to discuss:

1) Two different species of particles
2) Same type of particles with different initial state.

$$H = \sum_{j=1}^{N} -\Delta_{x_j} + \overbrace{\sum_{j=1}^{M} -\frac{\Delta_{y_j}}{m}}^{\text{Different species}} + \frac{1}{N-1}\sum_{\substack{j,k=1\\ j\neq k}}^{N} V(x_j - x_k) + \frac{1}{M-1}\sum_{\substack{j,k=1\\ j\neq k}}^{M} W(y_j - y_k) + \frac{1}{M+N}\sum_{j=1}^{N}\sum_{k=1}^{M} U(x_j - y_k)$$

$$\psi_0(x_1 \dots x_N, y_1 \dots y_M) = \prod_{j=1}^{N} \varphi_0(x_j) \prod_{k=1}^{M} \eta_0(y_k)$$

Effective equation:

$$i\partial_t \varphi_t = \left(-\Delta + V * |\varphi_t|^2 + \frac{M}{M+N} U * |\eta_t|^2\right)\varphi_t$$

$$i\partial_t \eta_t = \left(-\Delta + W * |\eta_t|^2 + \frac{N}{M+N} U * |\varphi_t|^2\right)\eta_t$$

$$\alpha(\psi, \eta, \varphi) := \underbrace{\langle \psi, q_{x_1}^{\varphi_t} \psi \rangle}_{\alpha_1} + \underbrace{\langle \psi, q_{y_1}^{\eta_t} \psi \rangle}_{\alpha_2}$$

as before

$$\frac{d}{dt}\alpha_1(\psi_t, \eta_t, \varphi_t) = i\langle \psi_t, [H - h_1, q_{x_1}^{\varphi_t}]\psi_t\rangle = \boxed{i\langle \psi_t, (V(x_1 - x_2) - V * |\varphi_t|^2)\, q_{x_1}^{\varphi_t} \psi_t \rangle}$$

$$+ \frac{i}{2}\langle \psi_t, \underset{q+p}{} \, U(x_1 - y_1) \, \underset{q+p}{} - U * |\eta_t|^2 \, q_{x_1}^{\varphi_t} \underset{(p_{y_1}^{\eta} + q_{y_1}^{\eta})}{} \psi_t \rangle - c.c.$$

(assuming $M = N$)

$\Rightarrow$ I^{old}, II^{old}, III^{old} as before and

$$\mathrm{I}: \quad \langle \psi, P^{\varphi}_{x_1} P^{\eta}_{y_1} U(x_1-y_1) q^{\varphi}_{x_1} P^{\eta}_{y_1} \psi \rangle - \langle \psi, P^{\varphi}_{x_1} P^{\eta}_{y_1} U * |\eta|^2(x_1) q^{\varphi}_{x_1} P^{\eta}_{y_1} \psi \rangle = 0$$

$$P^{\eta}_{y_1} U(x_1-y_1) P^{\eta}_{y_1} = P^{\eta}_{y_1} U * |\eta|^2(x_1) P^{\eta}_{y_1}$$

$$\mathrm{II}: \quad \langle \psi, P^{\varphi}_{x_1} P^{\eta}_{y_1} U(x_1-y_1) q^{\varphi}_{x_1} q^{\varphi}_{y_1} \psi \rangle$$

$$\leq C \left(\frac{1}{N} + \alpha \right)$$

We bring back one summation and get as above a diagonal term which is small due to its limited # of summands. For the offdiagonal we can bring the q on the other side.

$$\left(\sqrt{\alpha} \left(\sqrt{\alpha} + \frac{1}{N} \right) = \alpha + \sqrt{\alpha} \frac{1}{\sqrt{N}} \leq 2\alpha + \frac{1}{N} \right)$$

$ab \leq a^2 + b^2$

$$\mathrm{III}a: \quad \langle \psi, P^{\varphi}_{x_1} q^{\eta}_{y_1} U(x_1-y_1) - U * |\eta|^2(x_1) \, q^{\varphi}_{x_1} q^{\eta}_{y_1} \psi \rangle$$

$$\mathrm{III}_b: \quad \langle \psi, q^{\varphi}_{x_1} P^{\eta}_{y_1} U(x_1-y_1) q^{\varphi}_{x_1} q^{\eta}_{y_1} \psi \rangle$$

$$|\mathrm{III}_a| \leq \| q^{\eta}_{y_1} \psi \| \cdot \| P^{\eta}_{y_1} U(x_1-y_1) - U * |\eta|^2(x_1) \|_{op} \, \| q^{\varphi}_{x_1} \|_{op} \, \| q^{\eta}_{y_1} \psi \| \leq \alpha_2 \cdot C$$

$$|\mathrm{III}_b| \leq \dots \qquad \dots \qquad \leq \alpha_1 C$$

There might be an additional term:

$$\mathrm{IV}: \quad \langle \psi, q^{\varphi}_{x_1} P^{\eta}_{y_1} U(x_1-x_2) P^{\varphi}_{x_1} q^{\eta}_{y_1} \psi \rangle \qquad \left(\leq C \sqrt{\alpha_1} \sqrt{\alpha_2} \leq C(\alpha_1 + \alpha_2) \right)$$

Using the same weight function for α_1 and α_2 this expression drops.
If one has different weight functions it has to be controlled.

Mixing one type of particles

$$H = \sum_{j=1}^{M+N} -\Delta_j + \frac{1}{M+N} \sum_{j \neq k} V(x_j - x_k) \qquad \psi_0(x_1 \dots x_N) = \left(\prod_{j=1}^{N} \varphi(x_j) \prod_{k=1}^{N} \eta(x_k) \right)_{sym}$$

(without "sym" it is the upper case with $U = V = W$ and $m = 1$)

Some (wrong) idea might lead to assume

$$\psi_t \approx \left(\prod \varphi_t(x_j) \prod \eta_t(x_j) \right)_{sym} \qquad \begin{aligned} i\partial_t \varphi &= \left(-\Delta + V * |\varphi|^2 + V * |\eta|^2 \right) \varphi_t \\ i\partial_t \eta &= \dots \end{aligned}$$

However, there are problematic terms in deriving α.

Good guess: $\alpha := \langle \psi, q^{\varphi,\eta}_1 \psi \rangle$

$$P^{\varphi,\eta}_j = \underbrace{|\varphi\rangle\langle\varphi|_j}_{p^a} + \underbrace{|\eta\rangle\langle\eta|_j}_{p^b}$$

$$q_j = 1 - P_j$$

$$\mathrm{I}: \quad \langle \psi, P_1 P_2 V_{12} q_1 P_2 \psi \rangle = \dots$$

$P_1^a + P_1^b \dots$

Problematic: $\langle \psi, P_1^a P_2^a V_{12} q_1 P_2^b \psi \rangle$ does not drop!

The $P_1^a P_2^a V(x_1 - x_2) q_1 P_2^b$ is an operator which changes the state of both particles. (#1 bad + $\eta \to \varphi + \varphi$)

This can ~~not~~ be cancelled by a term coming from the effective description:
The eff. desc is generated by one-body operators, the generator only touches one particle. No matter which eff. descr. you choose, there is no cancellation.
On the other hand there is only one q! The term is of leading order!