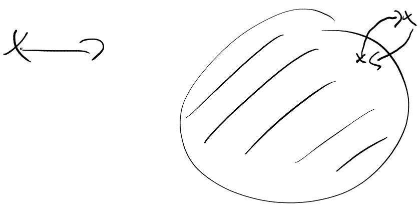
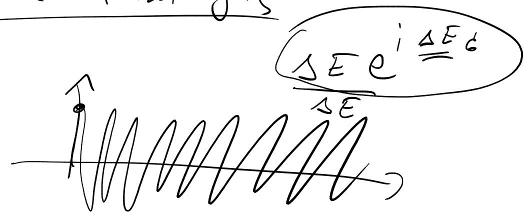
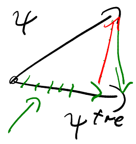
Tracer in the Fermi gas







$\| P\psi_t - P\psi_t^{free} \|_2$

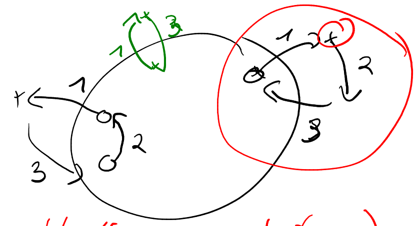
2nd-Order Duhamel $\displaystyle P\int_0^t\int_0^T e^{iH^{free}(t-T)}\, V\, e^{iH^{free}(T-s)}\, V\, e^{iH^{free}(s-0)}\, \psi_0 \, ds\, dT$

$+ \; \ldots \; C \; \ldots \; C \; \ldots$

The V-V-term from second order Duhamel cancels with the C-term in the first order Duhamel (by choice)

Remark: The "constant" C depends in fact on the momentum of the tracer-particle! Thus it changes its dispersion relation!

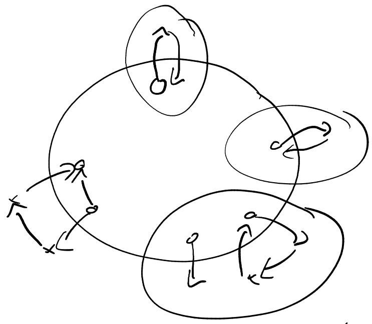
$$-\Delta_y + C_j = \boxed{j^2 + C_j}$$



What about the higher order terms?

$P\int\int\int e^{\sim} V e^{\sim} V e^{\sim} V e^{\sim} \psi_0$ subleading by a factor λ

$+ \; \ldots \; \underbrace{V \;\ldots\; V}_{C_j} \;\ldots\; C_j \ldots \; \psi_0$ cancellation with 2^{nd} order $C_j - C_j$ -term.



If we look at the expression for C_j we find that it is a parabole in j, still $C_j = C_{-j}$. $j^2 + C_j$ will be a parabola, with minimum at zero.

The constant just changes the global phase and has no physical meaning.

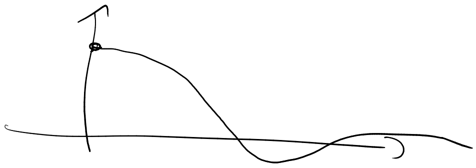
The curvature of $j^2 + C_j$ gives the effective mass of the particle.

Two (or more) tracers

For two tracer particles we will get an indirect interaction between the two mediated via the gas. Leading order: one tracer creates a particle-hole pair, the other tracer immediately annihilates it.

This term can be replaced by an interaction potential!

$$W = \lambda^2 \sum_{\substack{k,l \\ |l|<k_F \\ |k+l|>k_F}} \frac{|\hat{V}(k)|^2 \quad ((j-k)^2 - j^2 - 1)}{(k+l)^2 - l^2 + 1}$$



This effective interaction is mostly attractive!

Fermionic Hartree equation

Goal: Derive an effective equation for N interacting Fermions.

$$H = \sum_{j=1}^{N} -\Delta_j + \lambda \sum_{j \neq k} V(x_j - x_k) \qquad \Psi_0 = \bigwedge_{j=1}^{N} \varphi_j(x_j)$$

(φ_j we call "orbitals")

Question: Will Ψ_t remain close to a $\underbrace{\text{slater-determinant}}_{\text{antisymmetrized wave function}}$?

$$\bigwedge_{j=1}^{N} \varphi_j(x_j) = \begin{vmatrix} \varphi_1(x_1) & \varphi_2(x_1) & \dots \\ \varphi_1(x_2) & \dots & \\ & & \end{vmatrix} \mathcal{N}$$

normalization

By LLN we expect
$$i \frac{d}{dt} \varphi_{j,t} = \underbrace{\Big[-\Delta + \lambda \cdot V \circledast \Big(\overbrace{\sum_{k=1}^{N} |\varphi_{k,t}|^2}^{\rho_t} \Big) \Big]}_{h} \varphi_{j,t}$$

obviously the $\varphi_{j,t}$ will stay ortho-normal.

Remark: a) Having several orbitals but one (Bosonic case) makes things more complicated. See, in particular, the "I-term" below which will not be zero!

b) λ will be not as weak as in the Bose-case: Kinetic energy is large and the potential should keep up if we want to avoid the relatively boring situation of a system whose leading order evolution is free.

But the correlations are suppressed, so LLN should work better.

Let us look at I:
$$p^k = \sum_{j=1}^{N} p_j^k \qquad p_j^k = |\varphi_j(x_k)\rangle\langle\varphi_j(x_k)| \qquad q^k = 1 - p^k$$

$$I = \langle \Psi, p^1 p^2 \left(V(x_1 - x_2) - V^{eff}\right) q^1 p^2 \Psi \rangle$$

$$\underbrace{\begin{matrix} |\varphi_j\rangle\langle\varphi_j|_1 \\ |\varphi_k\rangle\langle\varphi_k|_2 \end{matrix}}_{l \text{ is missing}} V(x_1 - x_2) \underbrace{\begin{matrix} |\varphi_l\rangle\langle\varphi_l|_1 \\ q_2 \end{matrix}}_{k,j \text{ missing}}$$

$j \neq k \neq l$ this can not be cancelled by an V^{eff}

If two of the indices match:
$j = k$ trivial

$$\boxed{\begin{matrix}\langle\varphi_j| \\ \langle\varphi_k|\end{matrix} V \,|\varphi_j\rangle \qquad \qquad \begin{matrix}\langle\varphi_j| \\ \langle\varphi_k|\end{matrix} V \,|\varphi_k\rangle \dots}$$