Micro : $H = \sum_{j=1}^{N} (i\nabla_j + \lambda\hat{A}(x_j))^2 + \frac{1}{N}\sum_{j \neq k} v(x_j - x_k) + \sum_k |k| a_k^* a_k$ $\qquad i\partial_t \psi = H\psi_t$

Macro : $i\frac{d}{dt}\varphi_t = \underbrace{\left((i\nabla + \lambda A)^2 + V * |\varphi_t|^2\right)}_{h^{mf}} \varphi_t$

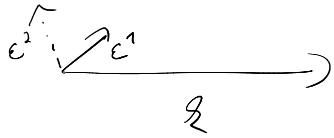
$\nabla A = 0$ ✓
$\partial_t A = -E_\perp$ ✓
$\partial_t E_\perp = -\Delta A - j_\perp$
$j = \operatorname{Im}\left(\varphi_t(\nabla - iA)\varphi_t^*\right)$

$$\alpha := \underbrace{\langle \psi, q_1 \psi \rangle}_{\alpha_a} + \underbrace{\lambda^2 \langle \psi, (\hat{A} - A)^2 \psi \rangle}_{\alpha_b} + \underbrace{\lambda^2 \langle \psi, (\hat{E}_\perp - E_\perp)^2 \psi \rangle}_{\alpha_c} \qquad \lambda = \frac{1}{\sqrt{N}}$$

$\Rightarrow \hat{A} = \sum_{\kappa=1}^{2} \sum_k \frac{\vec{\varepsilon}_k^{\,\kappa}}{\sqrt{|k|}} \left(e^{ikx} \cdot a^*(k,\kappa) + e^{-ikx} a(k,\kappa)\right)$

$\vec{\varepsilon}_k^{\,1/2}$ are unit vectors, orthogonal to $\vec{k}$ and to each other

$\hat{E}_\perp = \sum_{\kappa=1}^{2} \sum_k \vec{\varepsilon}_k^{\,\kappa} \cdot \sqrt{|k|}\, i \left(e^{ikx} a^*(k,\kappa) - e^{-ikx} a(k,\kappa)\right)$



The time derivative of quantum mech. operators are given by the commutator with H.

Theorem : "Assuming that ∇ is bounded" (replace $\hat{\nabla} = k$ by $\frac{k}{1+|k|}$) and introducing an IR and UV-cutoff we can show that α fulfills a Grönwall inequality:

$$\dot{\alpha} \leq C \cdot \alpha + o(1) \qquad \lim_{N\to\infty} o(1) = 0.$$

Proof: We proof that $\alpha_{a,b,c} \leq C \cdot \alpha + o(1)$ for a, b, c seperately.

a) $\left| \frac{d}{dt}\alpha_a \right| = \left| \langle \psi_t \left[(H - H^{mf}), q_1\right] \psi_t \rangle \right|$ $\qquad H^{mf} = \sum_{j=1}^{N} h_j^{mf}$

$\leq C \cdot (\mathrm{I} + \mathrm{II} + \mathrm{III}) + \left| \langle \psi_t, \left[(i\nabla_1 + \hat{A}(x_1))^2 - (i\nabla_1 + A(x_1))^2, q_1\right] \psi_t \rangle \right|$

$|\langle \psi_t, [\sum v(x_1 - x_j) - v * |\varphi|^2, q_1] \psi \rangle|$

old

$\leq C\left(\alpha_a + \frac{1}{N}\right) + 2\left| \langle \psi_t, \left(2\nabla_1 \lambda(\hat{A} - A) + \underbrace{(\lambda^2\hat{A}^2 - \lambda^2 A^2)}_{(\lambda\hat{A} - \lambda A)(\lambda\hat{A} + \lambda A)}\right) q_1 \psi_t \rangle \right|$

$\leq C\left(\alpha_a + \frac{1}{N}\right) + C \lambda \underbrace{\|(\hat{A} - A)\psi\|}_{\sqrt{\alpha_b} \leq \sqrt{\alpha}} \cdot \underbrace{\|q_1 \psi\|}_{\sqrt{\alpha_a} \leq \sqrt{\alpha}}$ $\quad \leq \boxed{C\left(\alpha + \frac{1}{N}\right)}$

under our assumptions ∇_1 and $(\hat{A} + A)$ are bounded operators.

$\lambda \hat{A}$ can be bounded: Due to the IR. cutoff the number of Photons is bounded by the total energy $\sim N$.

b) $\left| \frac{d}{dt} \langle \psi_t, (\hat{A} - A_t)^2 \psi_t \rangle \right| \leq \left| \langle \psi_t, 2(-\dot{A})(\hat{A} - A)\psi_t \rangle - i \langle \psi_t, [H, (\hat{A} - A)^2] \psi_t \rangle \right|$

$[A, B^2] = AB^2 - B^2 A = [A,B]B + B[A,B]$

$\leq 2 \left| \langle \psi_t, \left[-\dot{A}(\hat{A} - A) - i[H, \hat{A}](\hat{A} - A)\right] \psi_t \rangle \right|$

$$\leq 2 \left| \langle \psi_t, \left(E_\perp - i \left[\sum |\ell| a_\ell^* a_\ell, \hat{A} \right] \right) (\hat{A} - A) \psi_t \rangle \right| = \circledast$$

$$\left[\cancel{\sum} |\ell| a_\ell^* a_\ell, \sum \frac{\varepsilon}{\sqrt{|\ell|}} \left(e^{i\ell x} a^* + e^{-i\ell x} a \right) \right] = \sum \overbrace{|\ell|}^{\sqrt{|\ell|}} \frac{\varepsilon}{\sqrt{|\ell|}} \left(a^* e^{i\ell x} - a e^{-i\ell x} \right)$$

$$-i\, E_\perp$$

$$\circledast = 2 \left| \langle \psi_t, (E_\perp - \hat{E}_\perp) (\hat{A} - A) \psi_t \rangle \right| \leq 2 \| (E_\perp - \hat{E}_\perp) \psi \| \cdot \| (\hat{A} - A) \psi \| \leq \frac{2\sqrt{\alpha_b}\sqrt{\alpha_c}}{\lambda^2} \leq \frac{2\alpha}{\lambda^2}$$

$$\Rightarrow \frac{d}{dt} \alpha_b \leq 2\alpha \quad \checkmark$$

c) $\dot{\alpha}_c = \lambda^2 \langle \psi_t, 2(-\dot{E}_\perp) \cdot (\hat{E} - E_\perp) \psi_t \rangle - i \langle \psi_t [H, (\hat{E} - E)^2] \psi_t \rangle$

$$|\dot{\alpha}_c| \leq \frac{2}{N} \left| \langle \psi_t \left(-\dot{E}_\perp - i [H, (\hat{E} - E)] \right) (\hat{E} - E) \psi_t \rangle \right|$$

$$i[H, \hat{E}] = i\left[\sum |\ell| a^*(\ell) a(\ell), \hat{E} \right] + i\left[\sum_j \left[(i\nabla_j + \hat{A}(x_j)) \right]^2, \hat{E} \right]$$

$$i[H, E] = i\left[\sum_j (i\nabla_j + A(x_j))^2, E \right]$$

$$\dot{E}_\perp = -\Delta A - j_\perp$$

$$i\left[\sum \ell a^*(\ell) a(\ell), \hat{E} \right] = i\left[\cancel{\sum} |\ell| a^*(\ell) a(\ell), \sum \sqrt{|\ell|}\, \varepsilon \cdot i \left(e^{i\ell x} a^*(\ell) - e^{-i\ell x} a(\ell) \right) \right]$$

$$= i \sum \frac{\ell^2}{\sqrt{|\ell|}} \varepsilon \cdot i \left(e^{i\ell x} a^* + e^{-i\ell x} a(\ell) \right) = -\Delta \hat{A}$$

Assum the Δ is bounded the green parts almost cancel : $C \cdot \| (\hat{A} - A) \psi \| \cdot \| (\hat{E} - E) \psi \|$
$\leq C \cdot \alpha.$