



Crafting a **Conical Refraction Polarimeter**

***A low-cost device to
monitor linear polarization***

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Supervisor:

Alex Turpin Avilés

Bachelor's Thesis

Computational Mathematics and Data Analytics

Universitat Autònoma de Barcelona 05/09/2022

O – Motivation

- (a) Polarization of Light**
- (b) Why Linear Polarization?**
- (c) Classical Polarimeter**

I – Conical Refraction and Linear Polarization

- (a) The Theory**
- (b) The Experiment**
- (c) A Snapshot Polarimeter**

II – The Algorithms: from Images to Angles

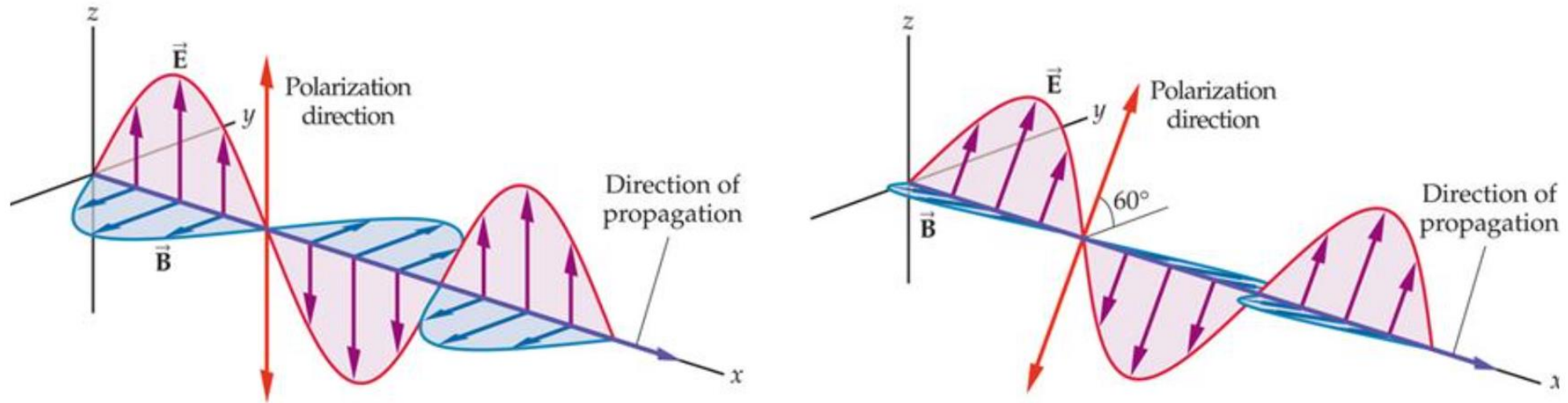
- (a) The Computer Vision task**
- (b) Generated Labelled Datasets**
- (c) Developed Algorithm Examples**
- (d) Algorithm Benchmark**

III – The Device

- (a) A Proof-of-concept**
- (b) Commercial Viability**

Motivation

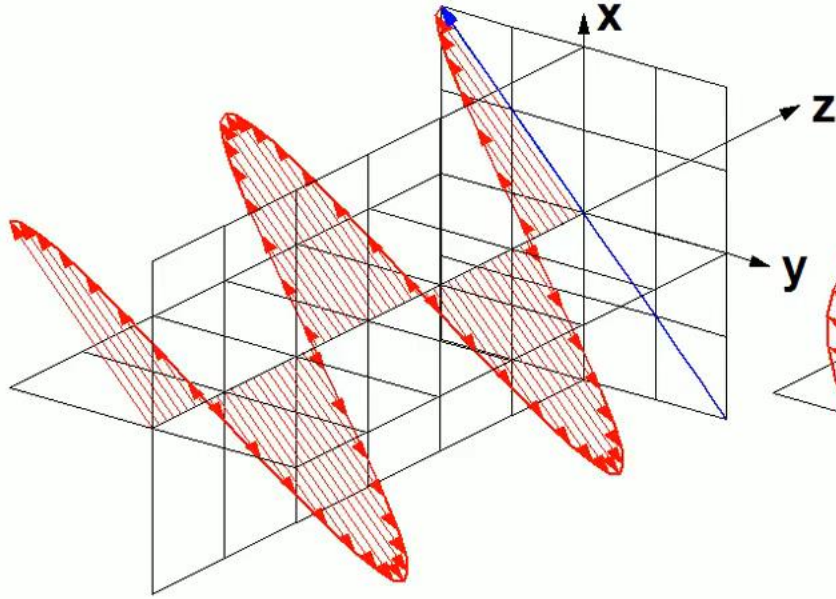
(a) Polarization of Light



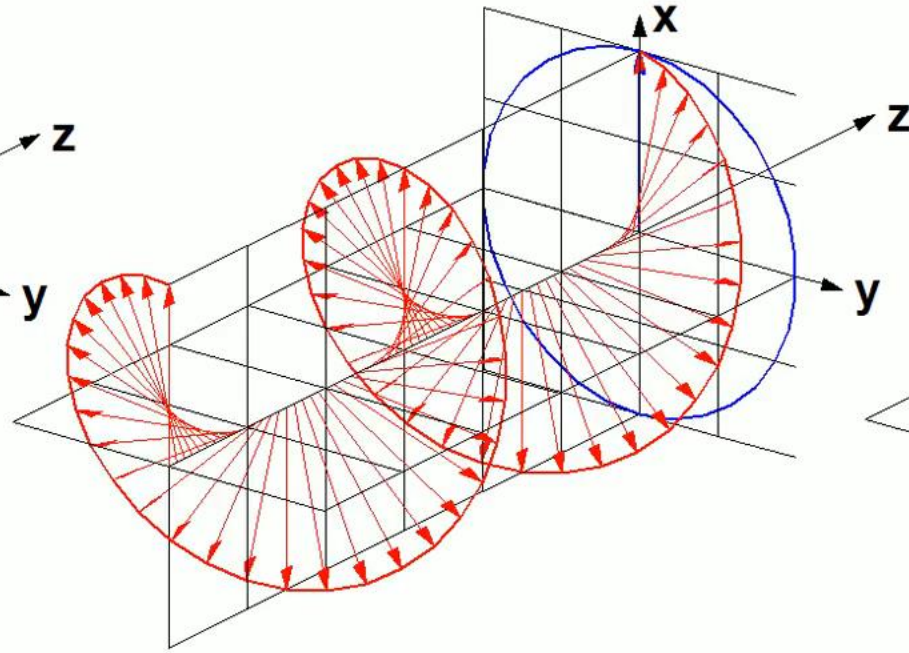
Linear Polarization

(a) Polarization of Light

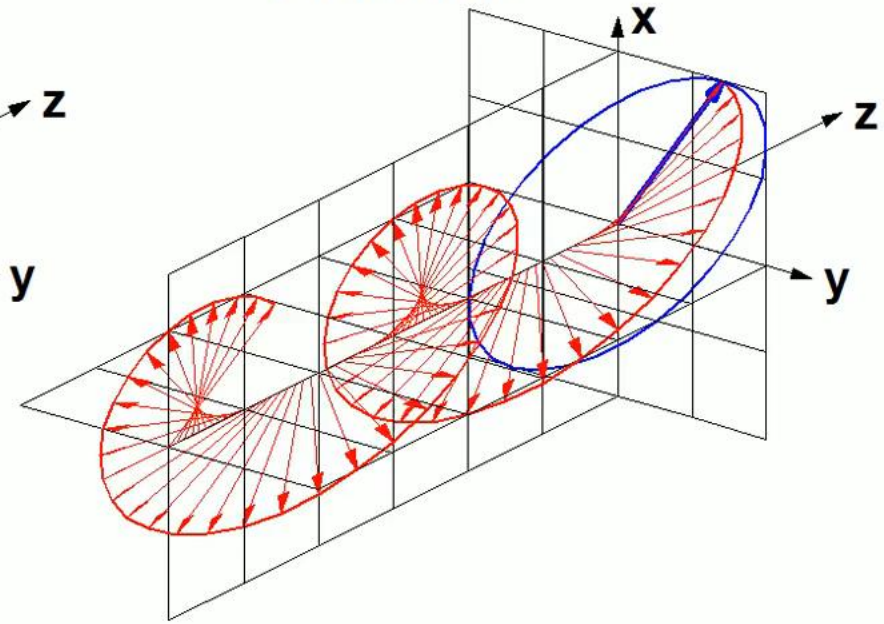
Linear Polarization



Circular (Right Hand) Polarization

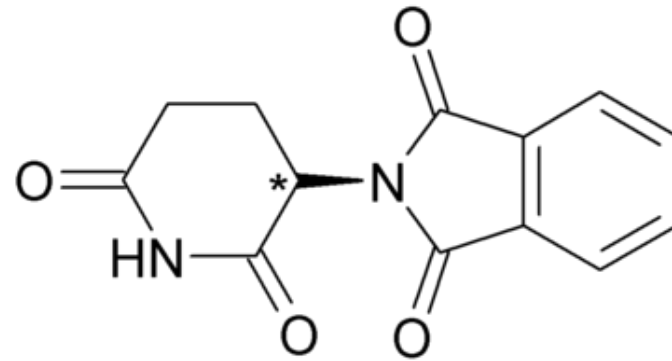


Elliptical (Right Hand) Polarization

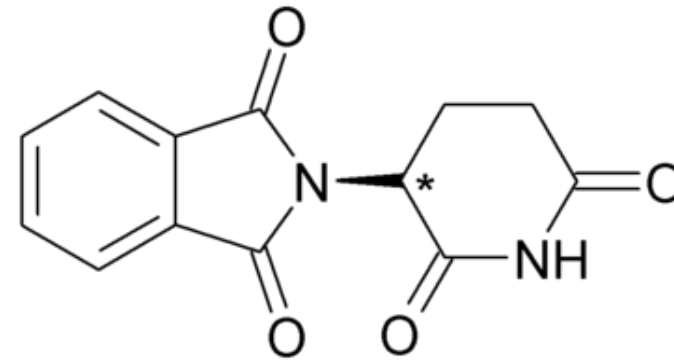


(b) Why Linear Polarization?

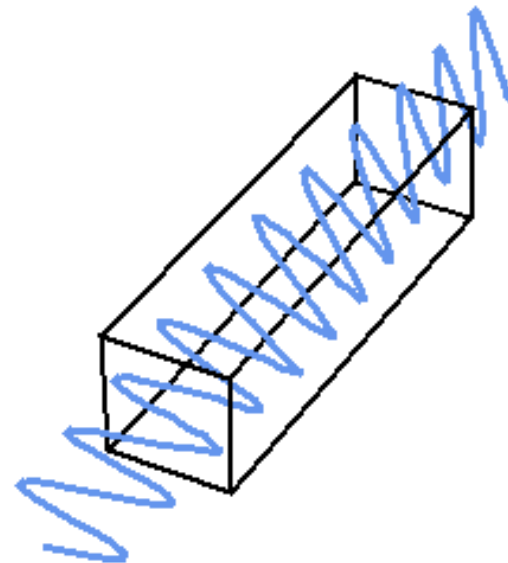
Chemistry



(R)-Enantiomer

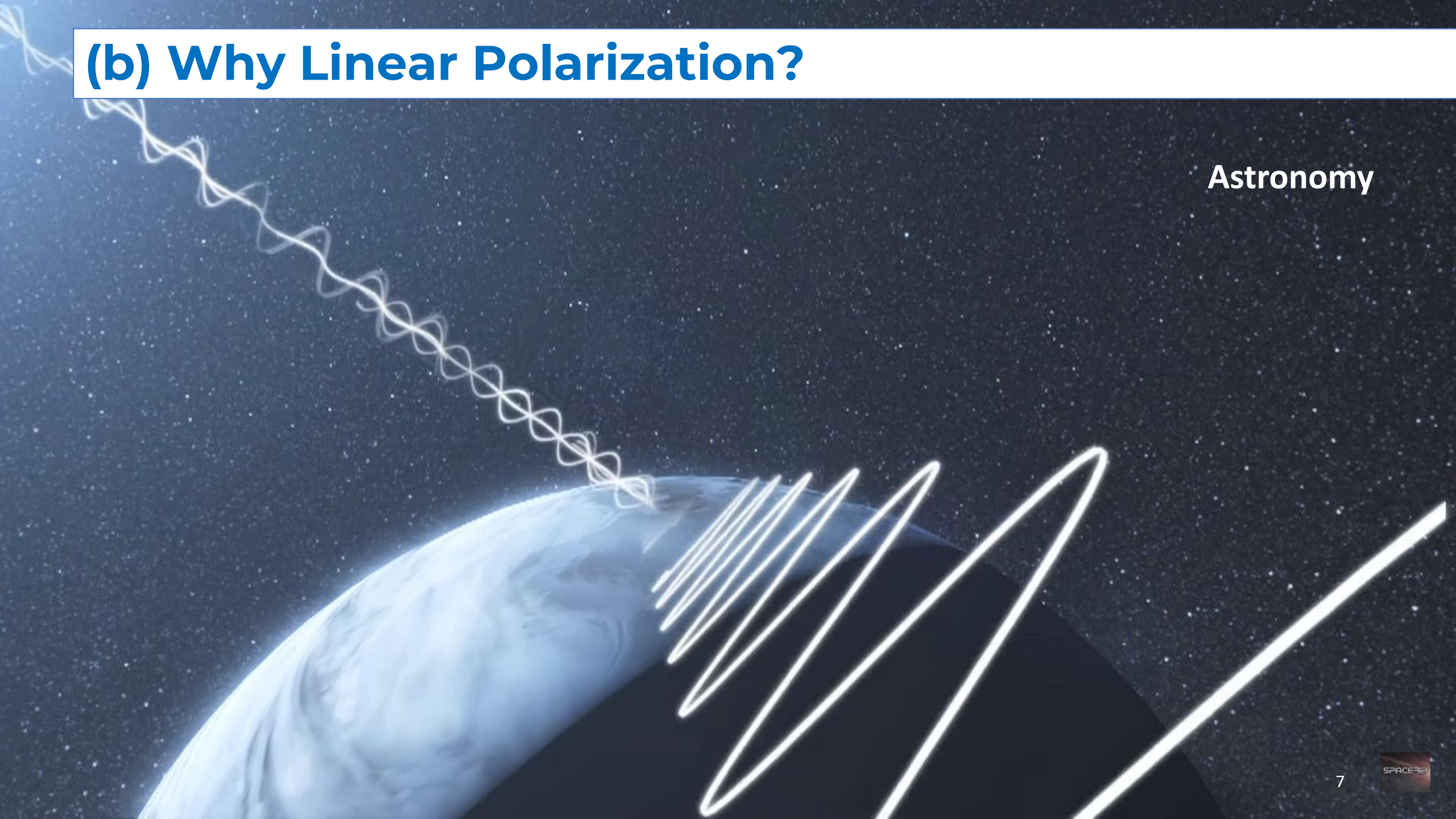


(S)-Enantiomer

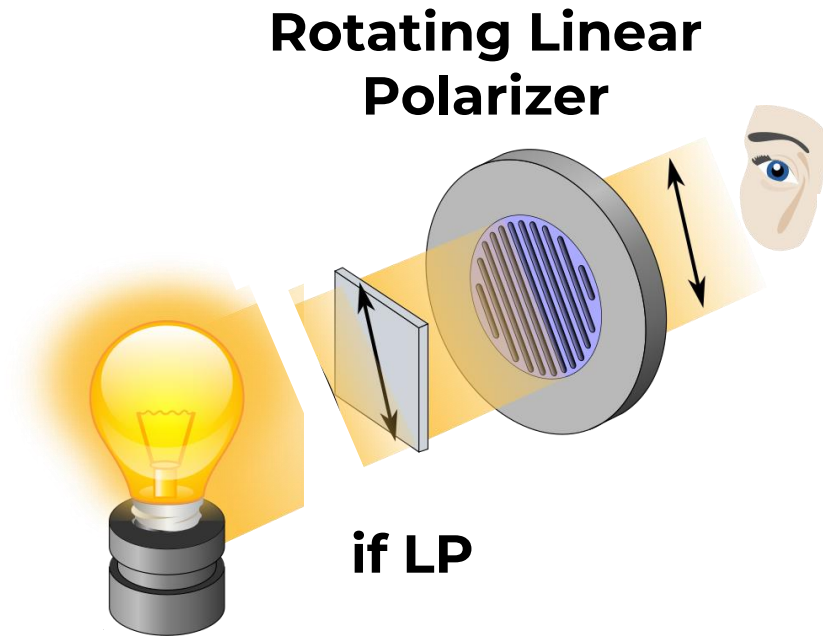


(b) Why Linear Polarization?

Astronomy



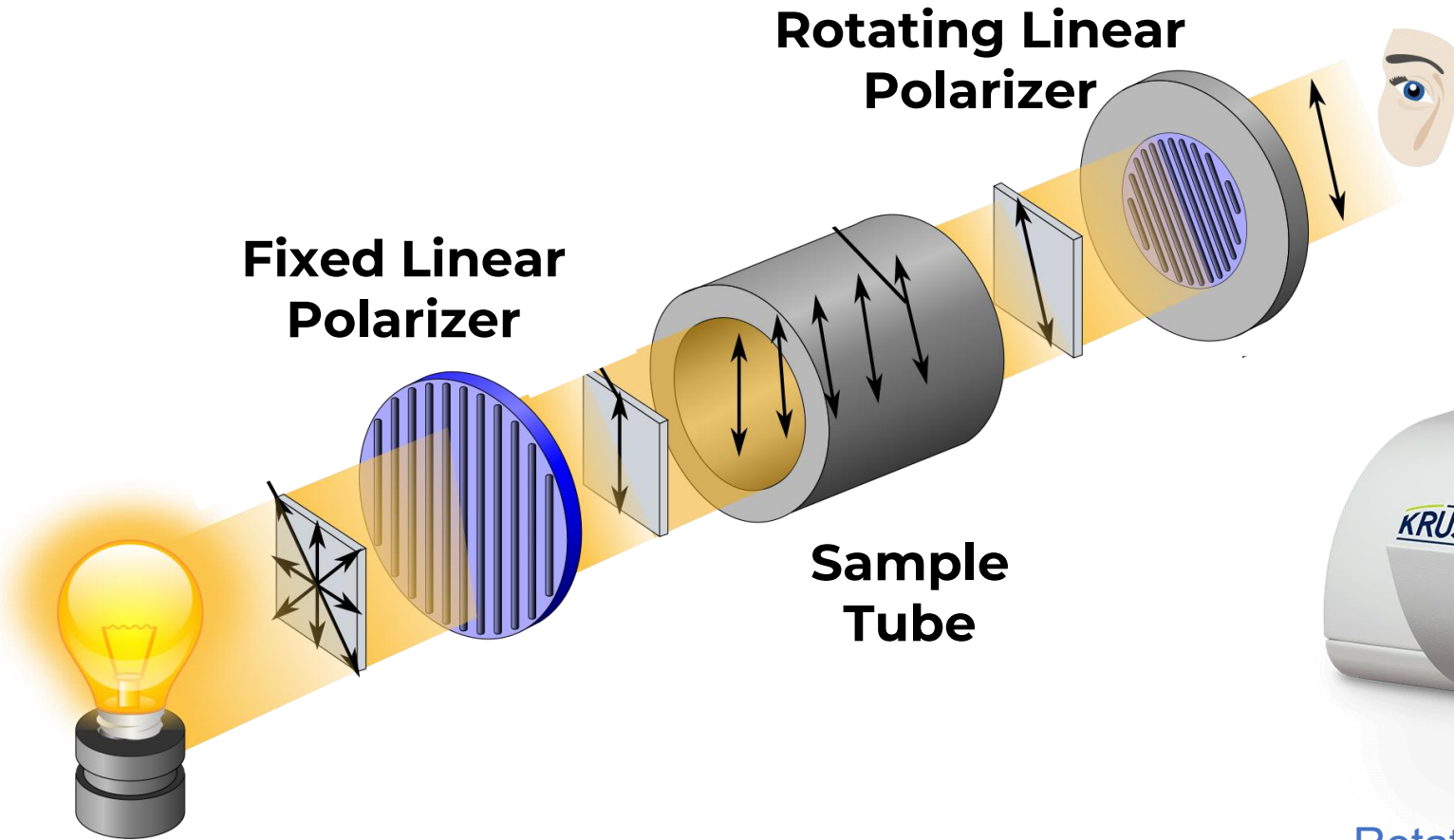
(c) Classical Polarimeter



Rotating-Polarizer-Based Polarimeter

Ex. Krüss P3000
~ 8.000 euro

(c) Classical Polarimeter



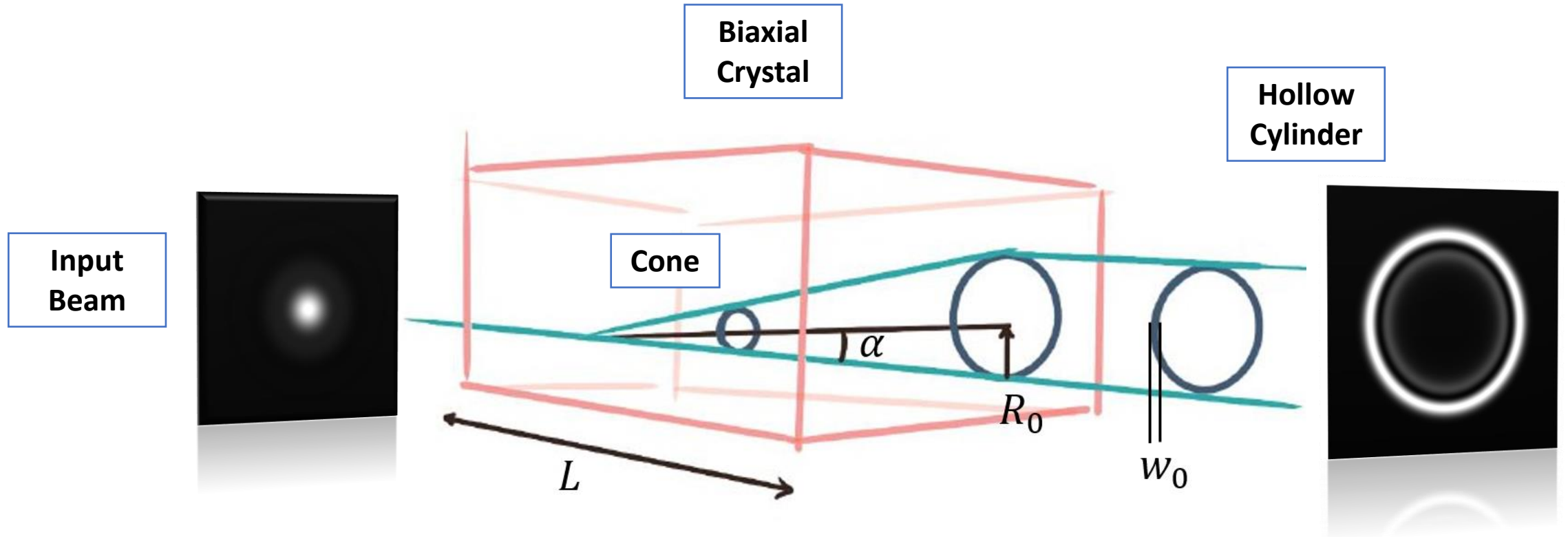
Rotating-Polarizer-Based Polarimeter

Ex. Krüss P3000
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Conical Refraction and Linear Polarization

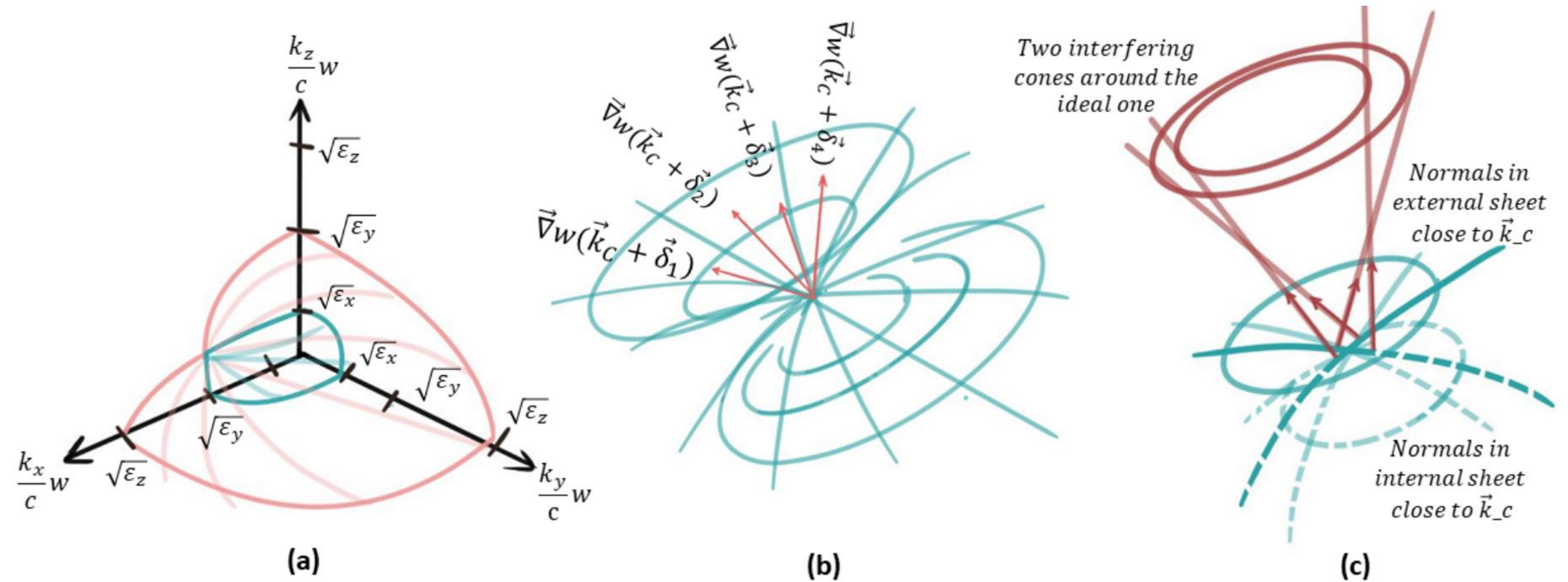
(a) Conical Refraction: The Theory

(i) Phenomenological Description



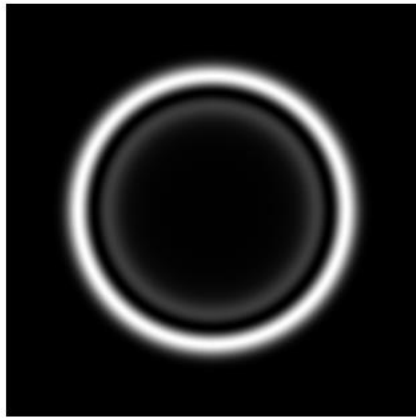
(a) Conical Refraction: The Theory

(ii) Fresnel Surface

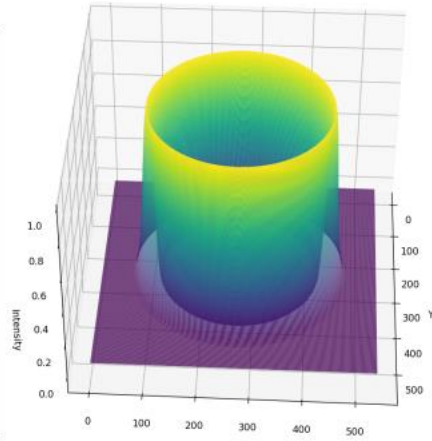


(a) Conical Refraction: The Theory

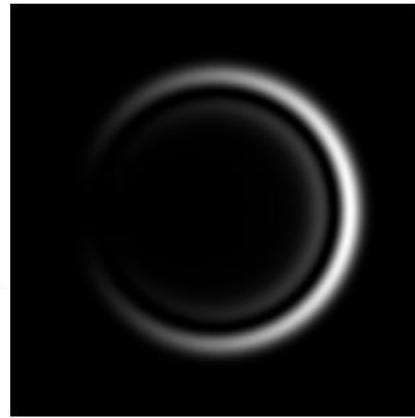
(iii) Belsky-Khapalyuk-Berry Theory



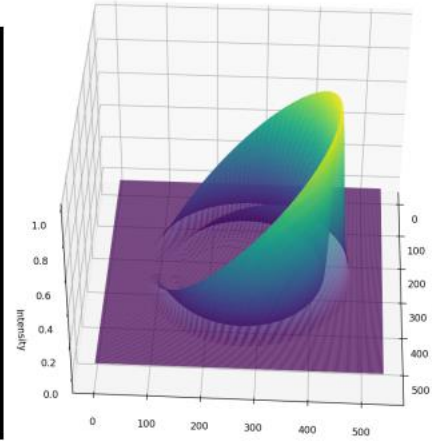
(a) CP



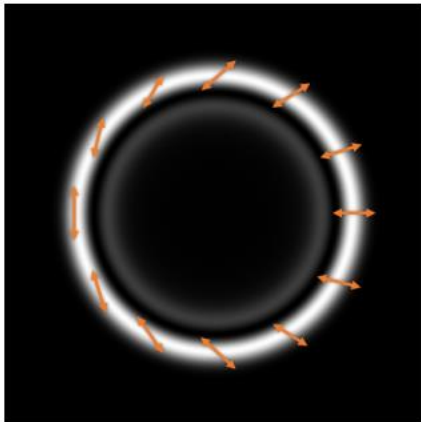
(b) CP



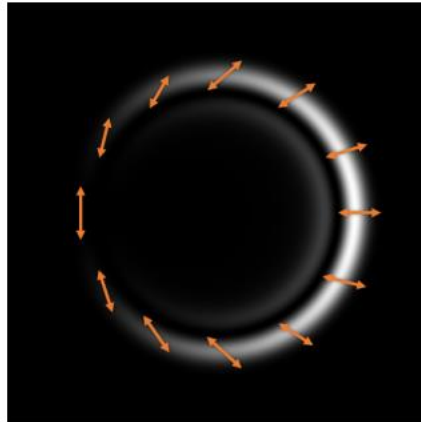
(c) LP



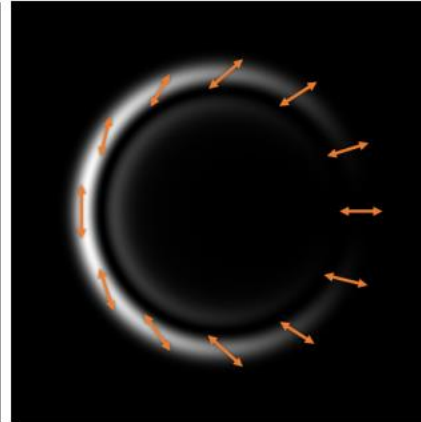
(d) LP



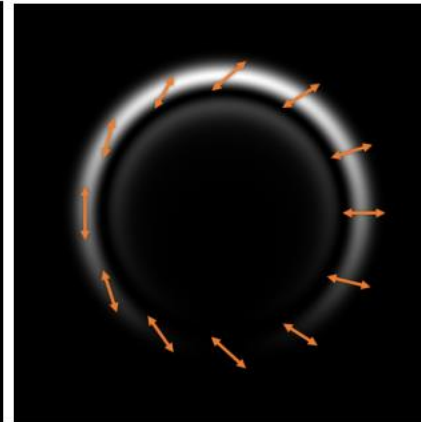
(a) CP



(b) LP 0 rad

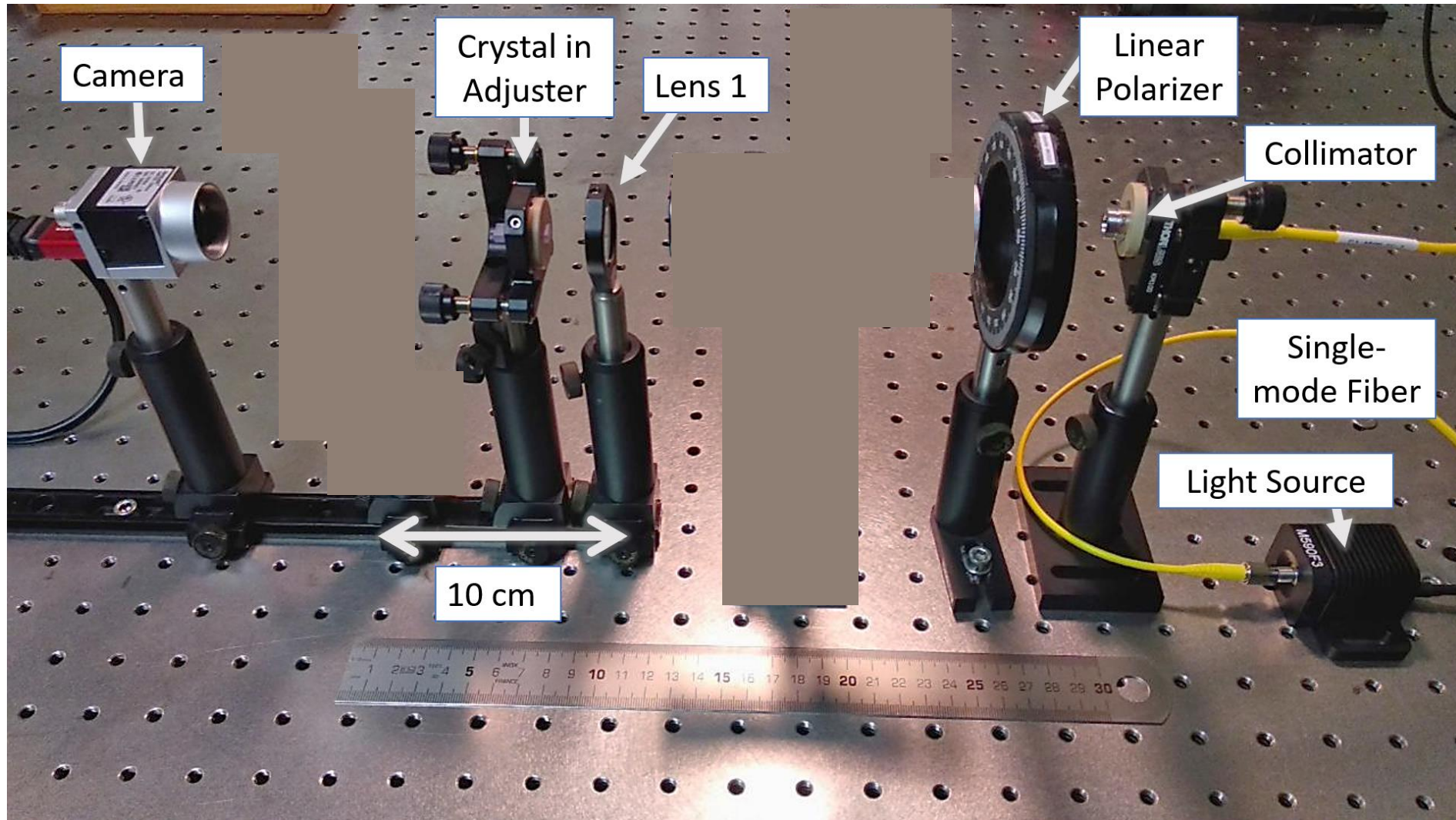


(c) LP $\pi/2$ rad

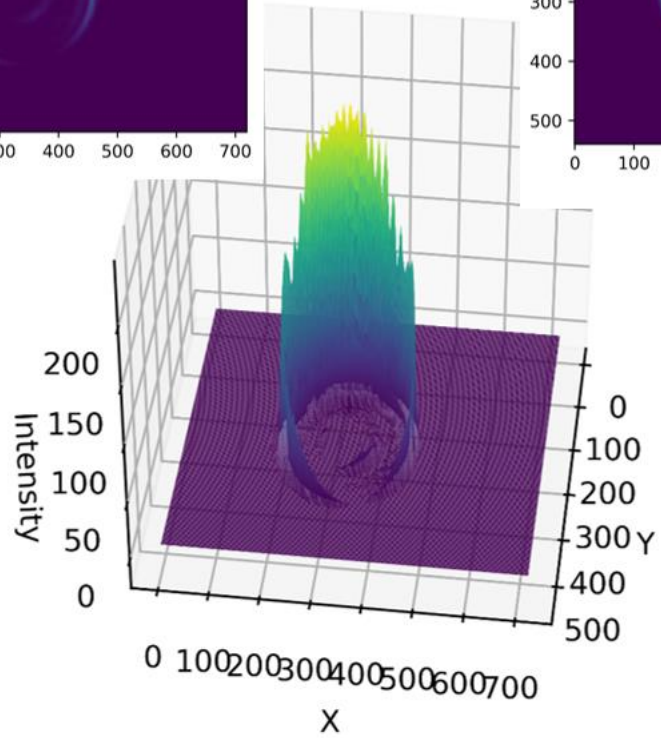
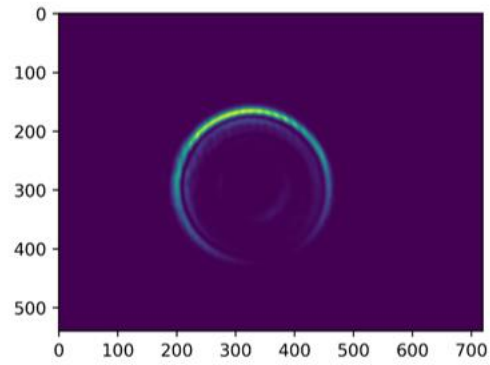


(d) LP $\pi/4$ rad

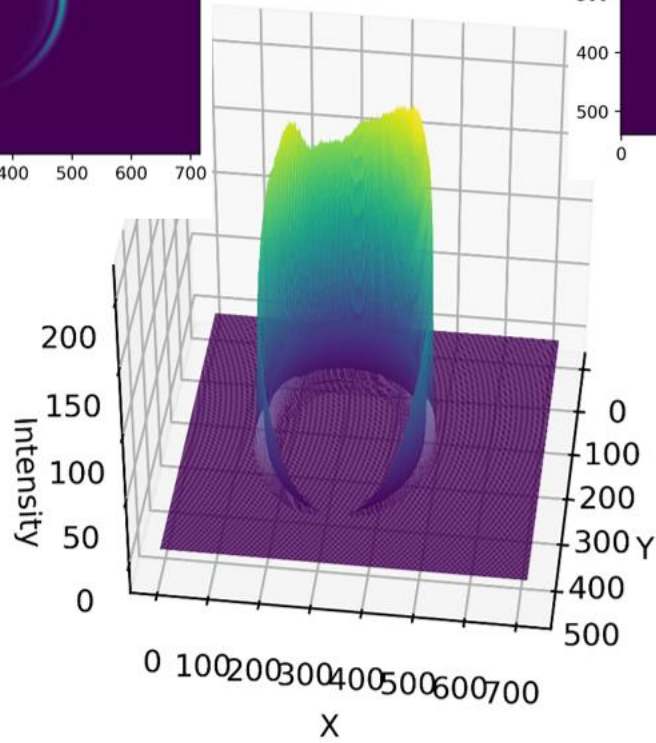
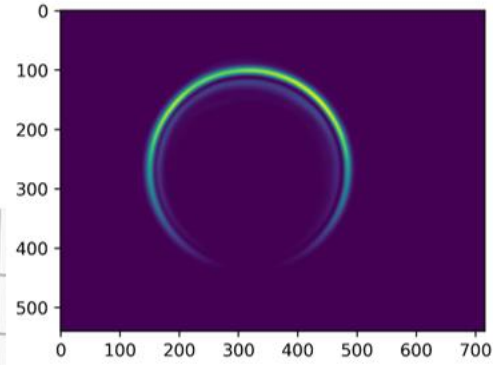
(b) The Experiment



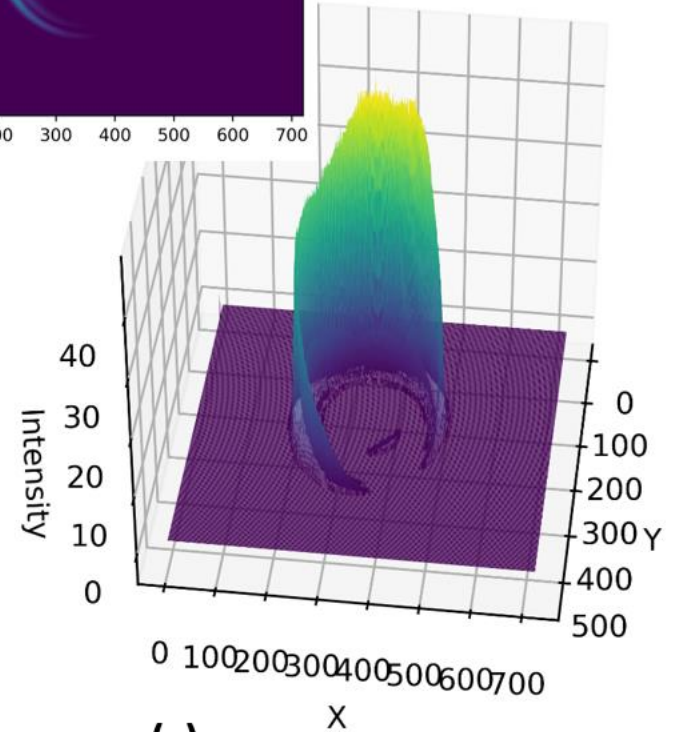
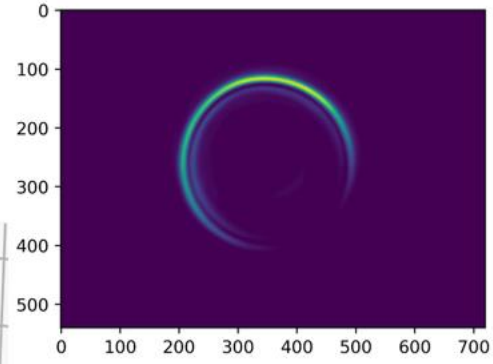
(b) The Experiment



(a)

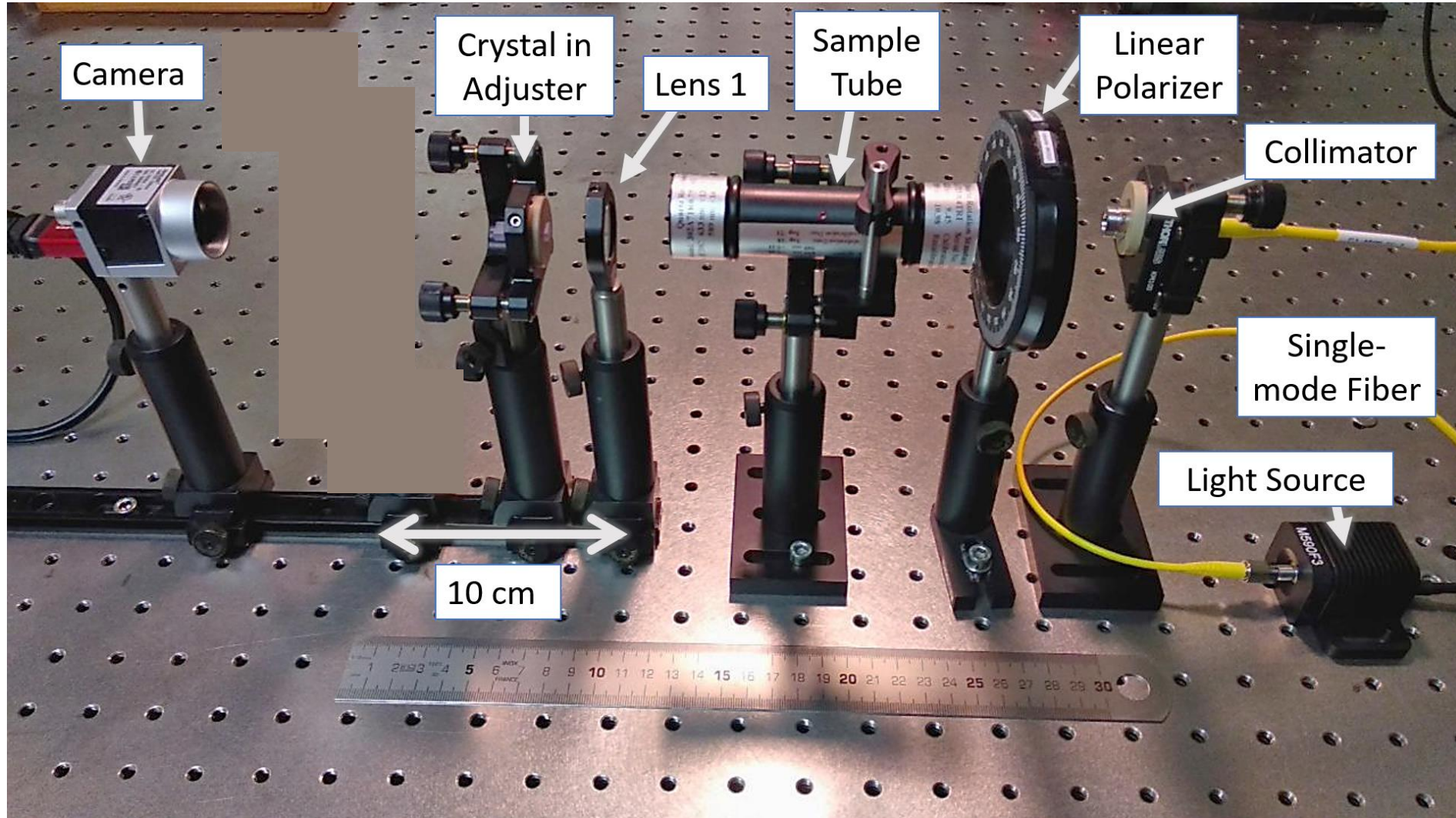


(b)



(c)

(c) A Snapshot Polarimeter



(c) A Snapshot Polarimeter

Lizana, Angel, et al. “*Implementation and performance of an in-line **incomplete Stokes polarimeter** based on a single biaxial crystal.*” *Applied Optics* 54.29 (2015): 8758-8765.

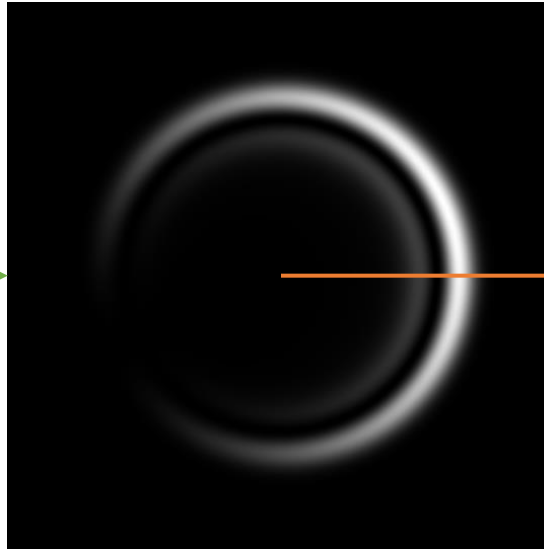
Peinado, Alba, et al. “*Optimization, tolerance analysis and implementation of a **Stokes polarimeter** based on the conical refraction phenomenon.*” *Optics express* 23.5 (2015): 5636-5652.

Peinado, Alba, et al. “***Snapshot polarimeter** based on the conical refraction phenomenon.*” *Modeling Aspects in Optical Metrology V. Vol. 9526. SPIE*, (2015).

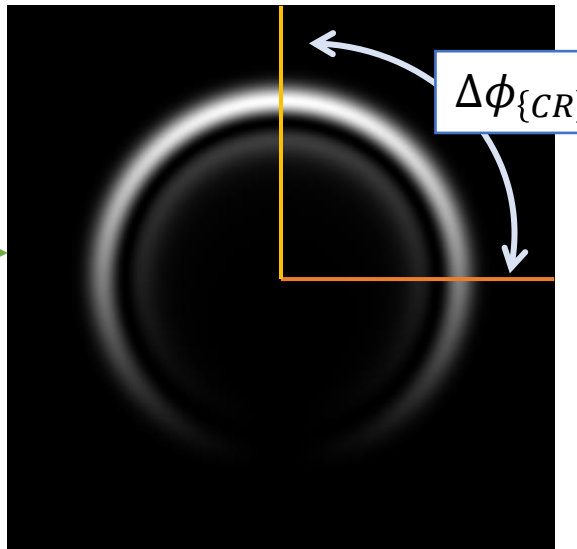
The Algorithms: From Images to Angles

(a) The Computer Vision Task

Reference Sample



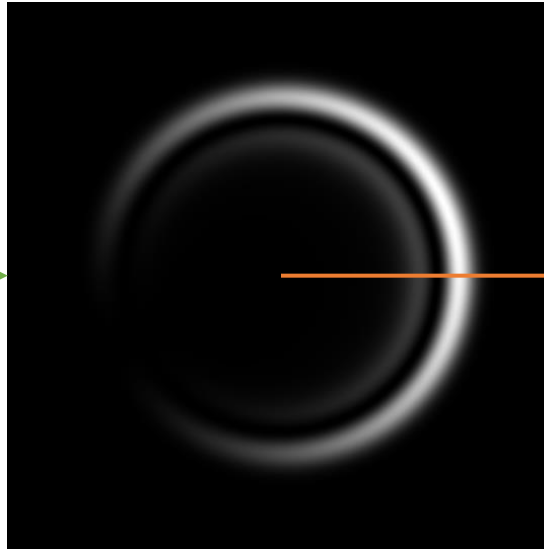
Sample that Rotates Pol. $\pi/4$



$$\Delta\text{Polarization} = \frac{\Delta\phi_{\{CR\}}}{2} = \frac{\pi}{4}$$

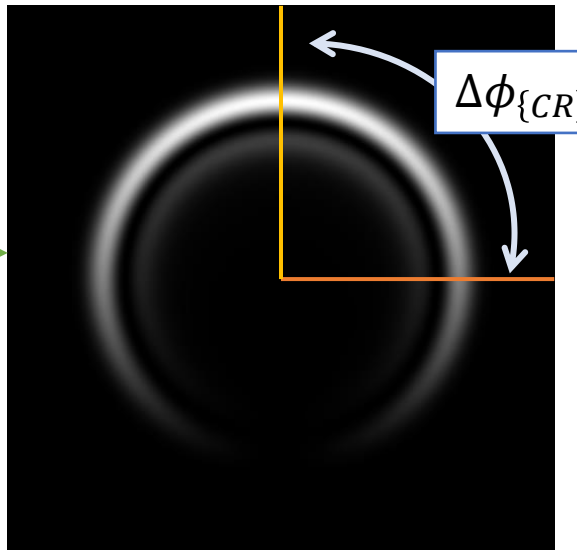
(a) The Computer Vision Task

Reference Sample



Trivial?

Sample that Rotates Pol. $\pi/4$

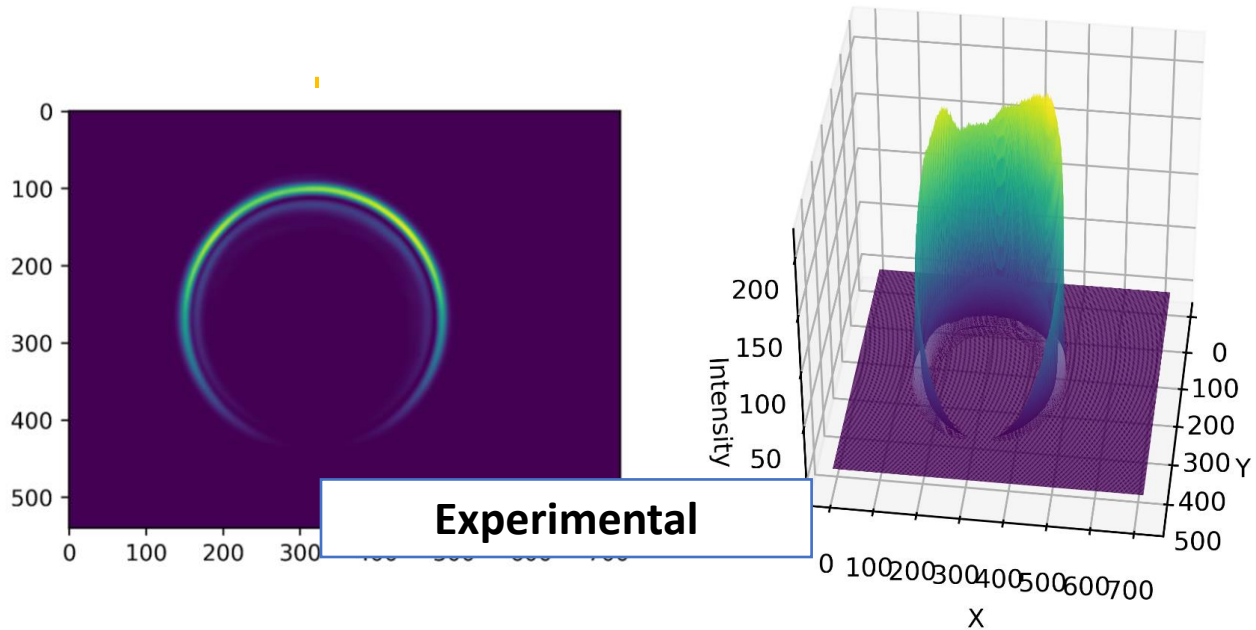
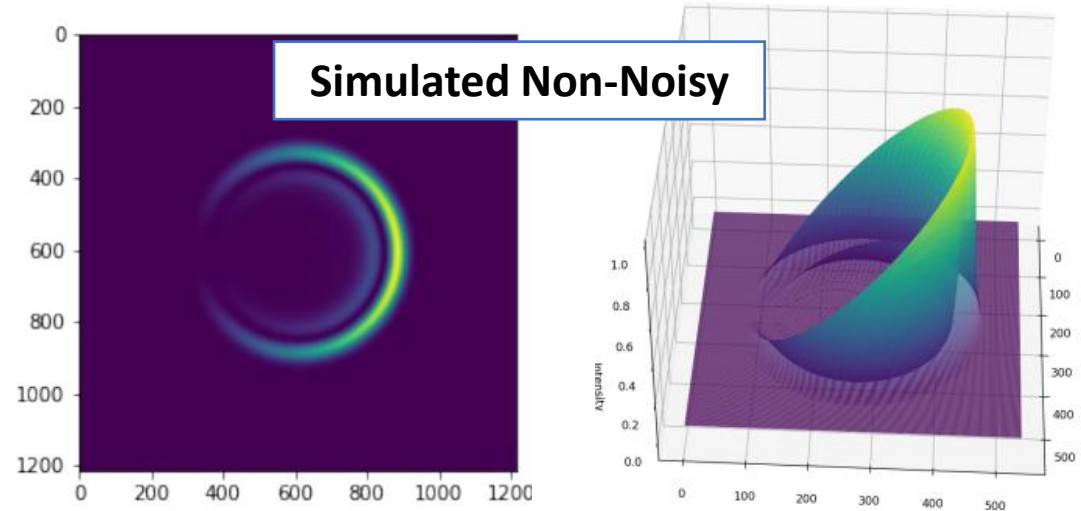


$$\Delta\text{Polarization} = \frac{\Delta\phi_{\{CR\}}}{2} = \frac{\pi}{4}$$

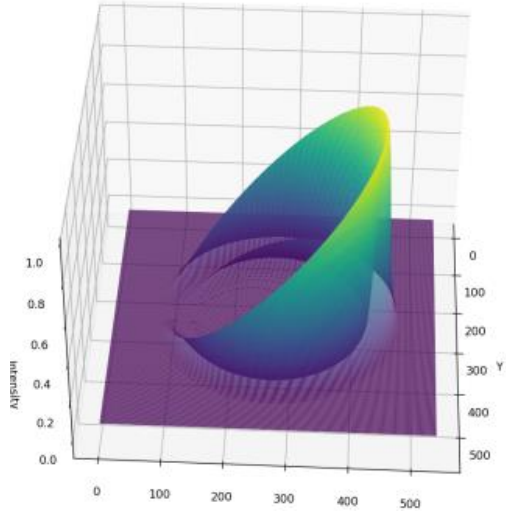
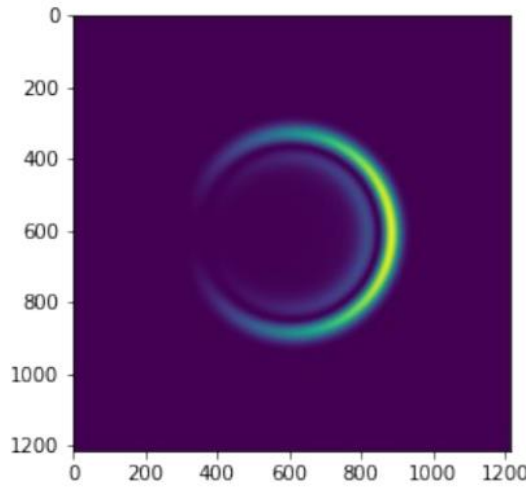
(a) The Computer Vision Task

Trivial?

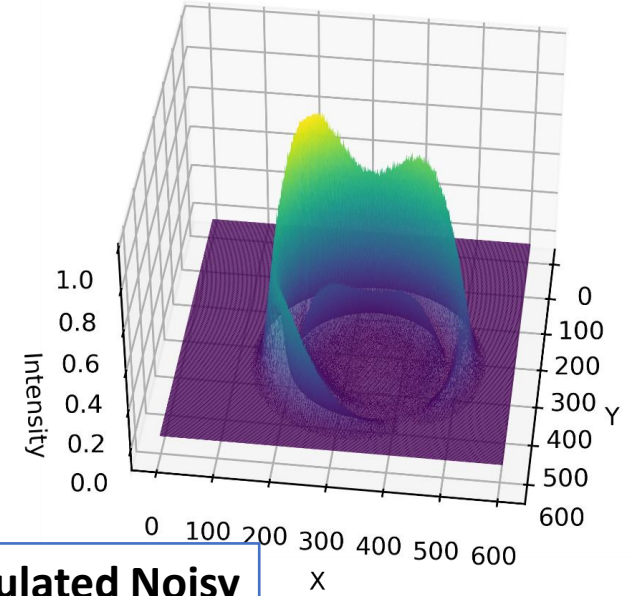
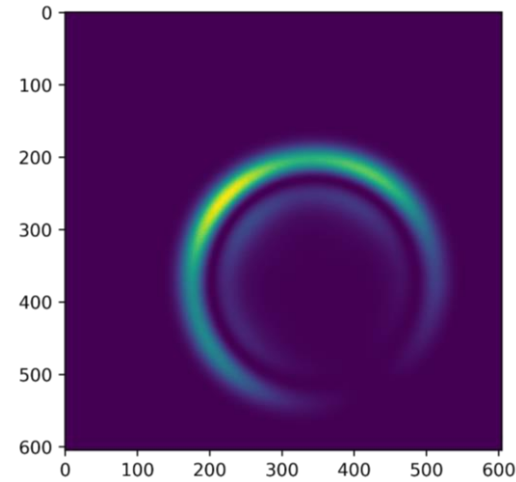
Not for
Quantitative
Results!



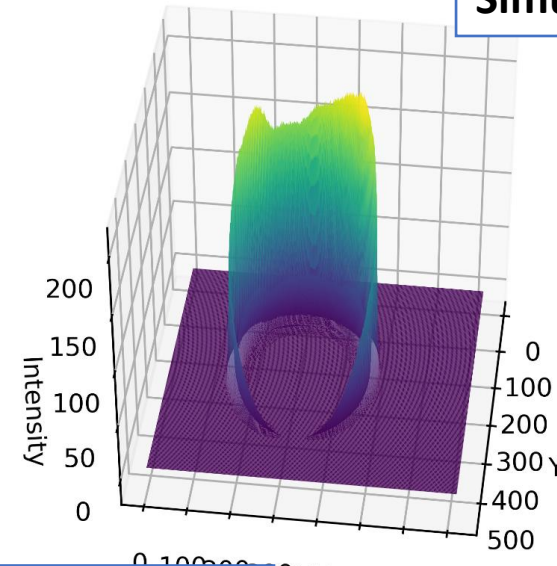
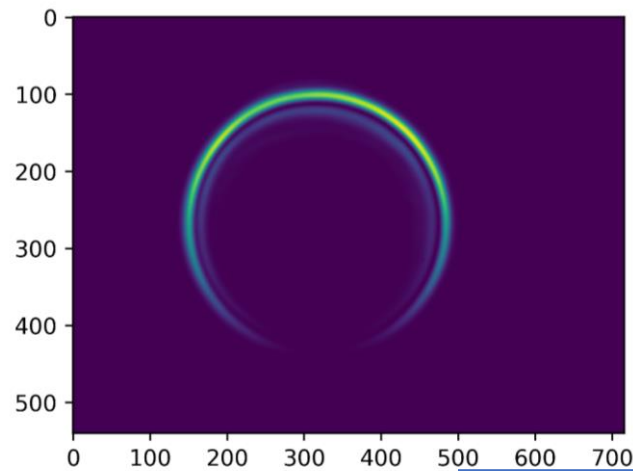
(b) Generated Labelled Datasets



Simulated Non-Noisy



Simulated Noisy



Experimental

(c) Developed Algorithm Types

(i) Geometric Algorithms

(ii) Neural Network Algorithms

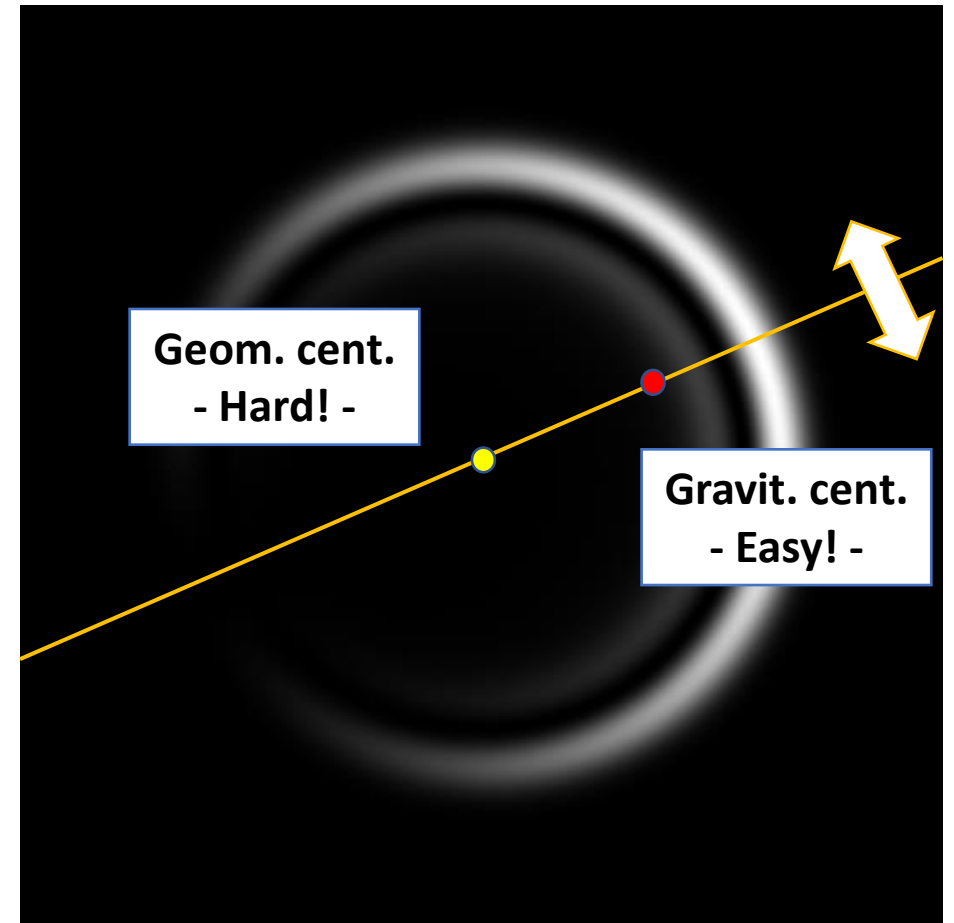
(iii) Embedding Space Algorithms

(iv) Simulation Fitting Algorithms

(c) Developed Algorithm Examples

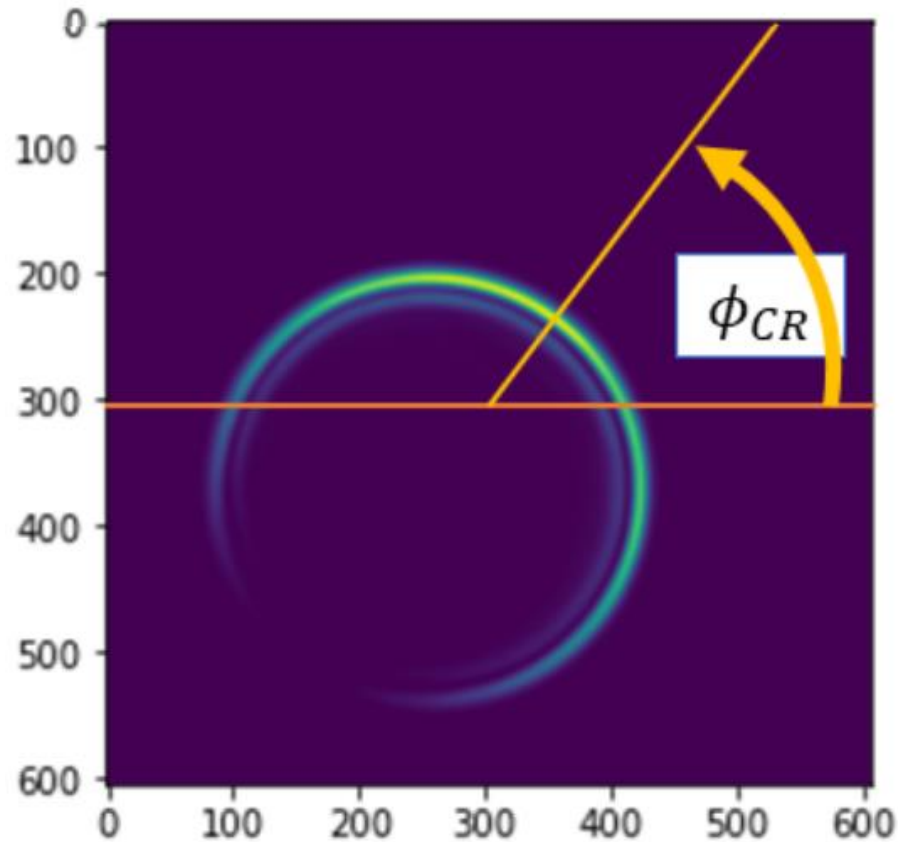
(i) Geometric Algorithms: Theoretical Image-Angle Property 1

- Symmetric about “main axis”
- Gravitat. and Geom. Center aligned in main axis

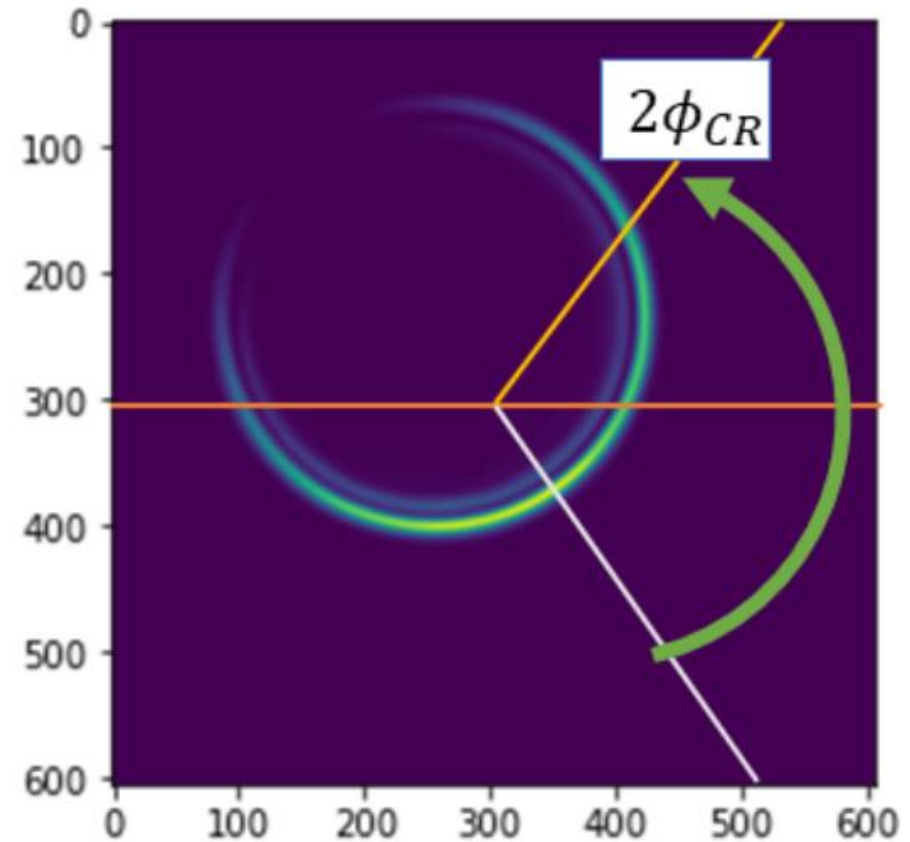


(c) Developed Algorithm Examples

(i) **Geometric** Algorithms:
The **Kalkandjiev-Rotation** Algorithm



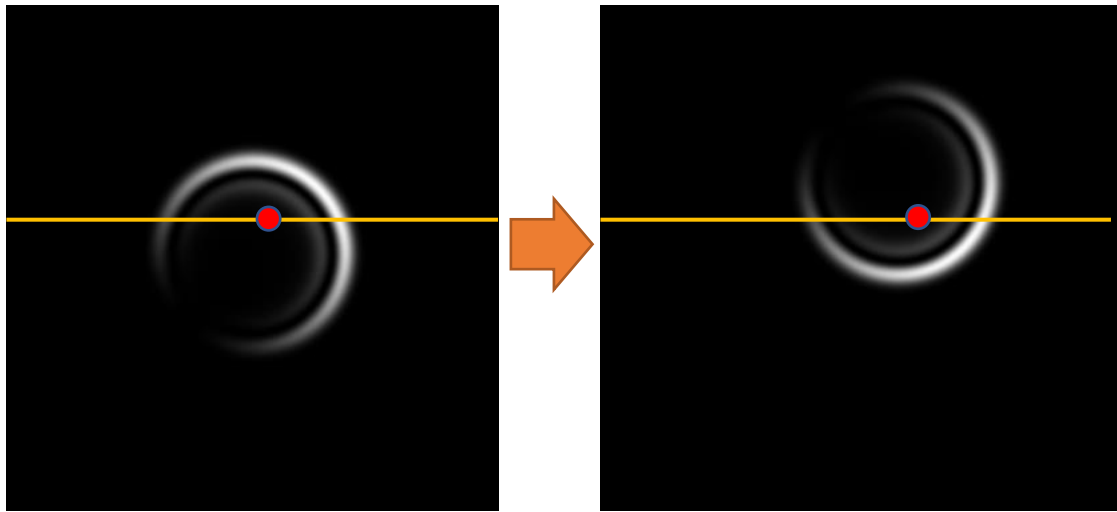
(a)



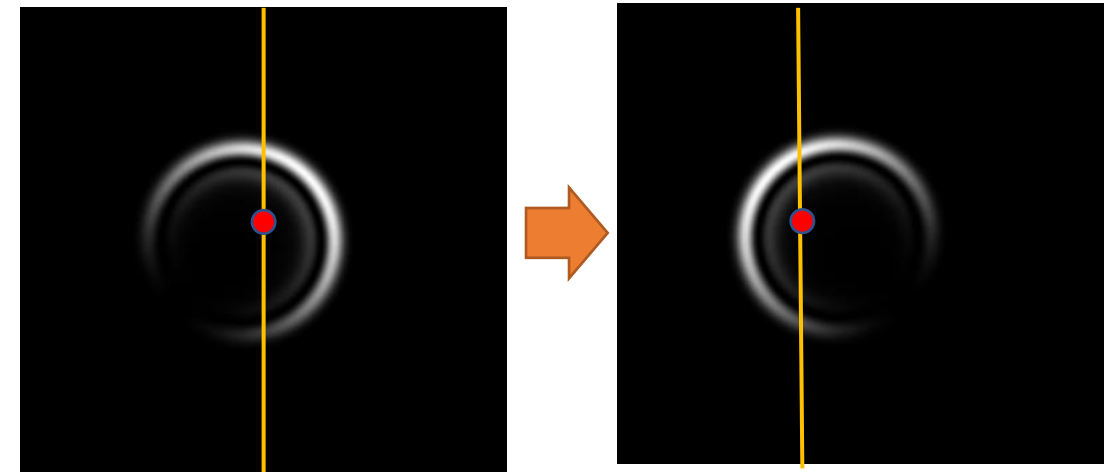
(b)

(c) Developed Algorithm Examples

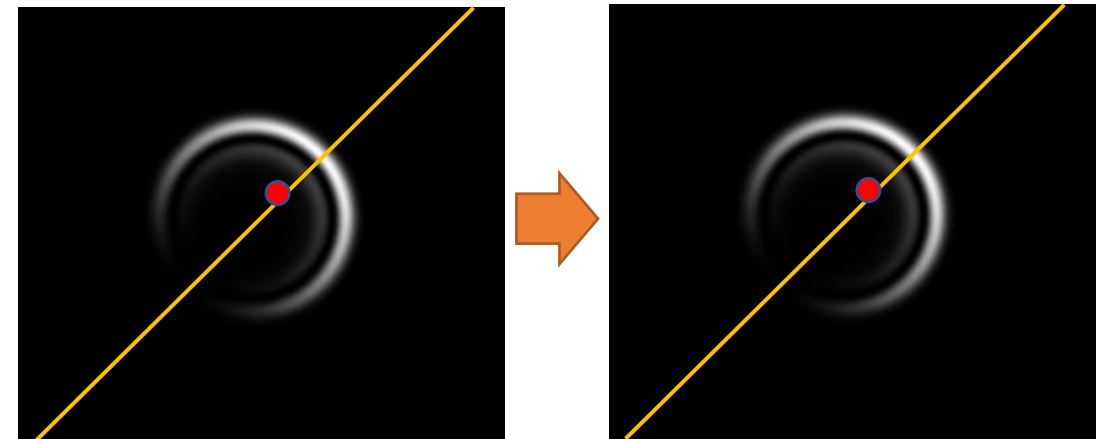
(i) Geometric Algorithms: The **Mirror Flip** Algorithm



At 0 rad



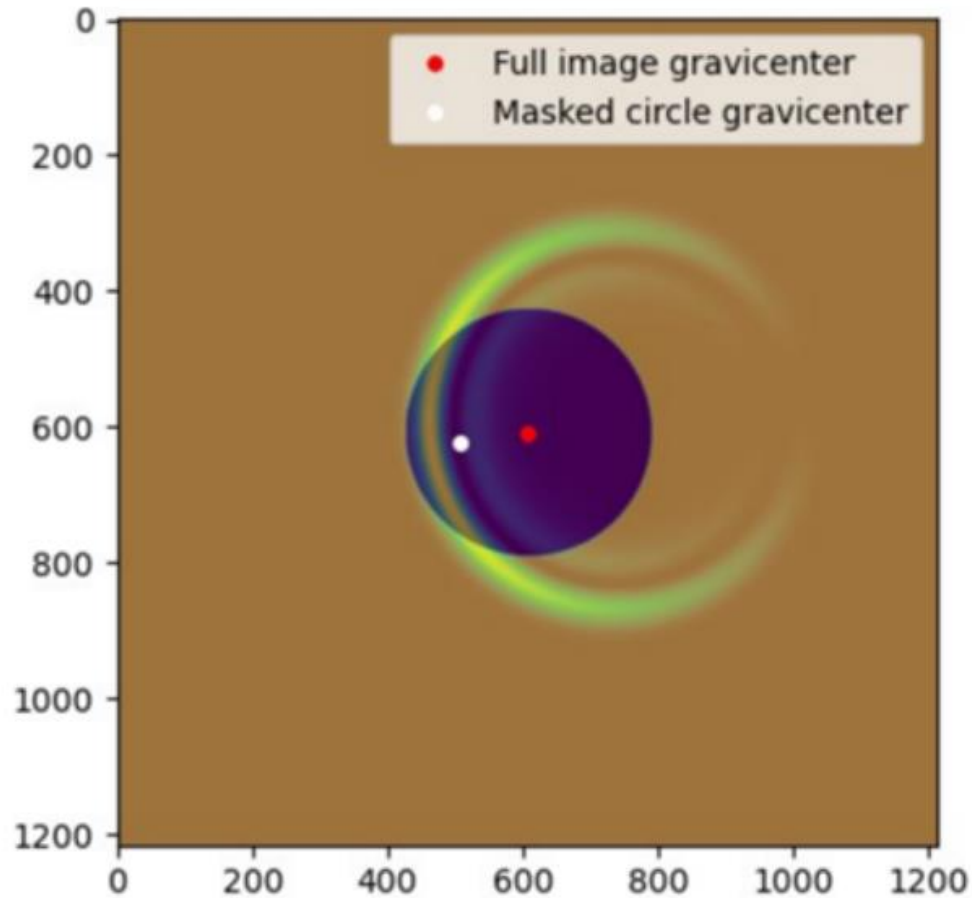
At $\pi/2$ rad



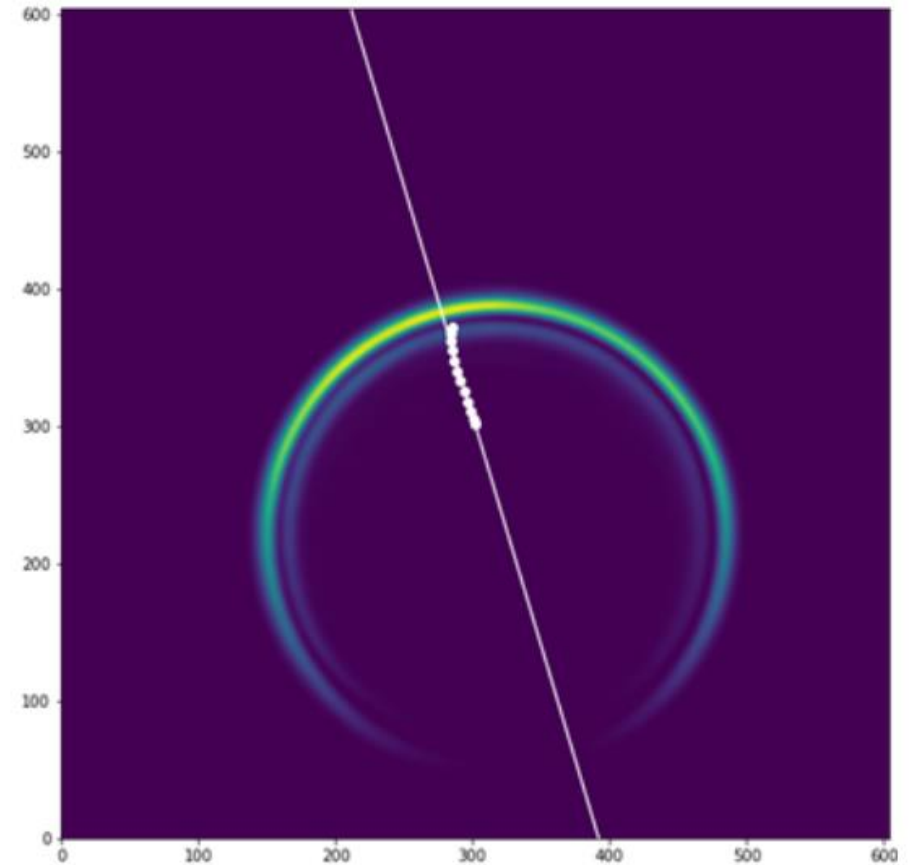
At $\phi_{\{CR\}}$

(d) Developed Algorithm Examples

(i) Geometric Algorithms: The **Gravicenter Regression** Algorithm



(a)

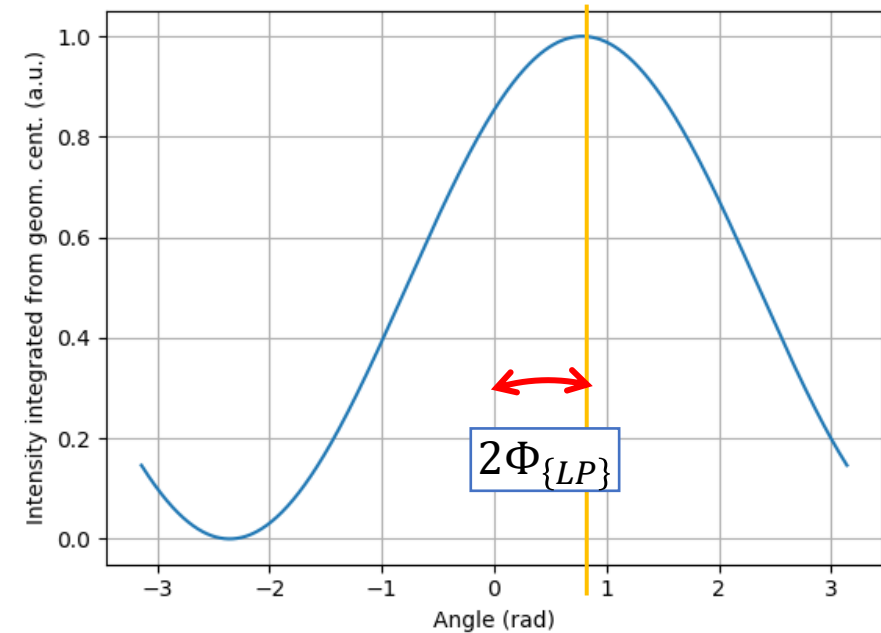
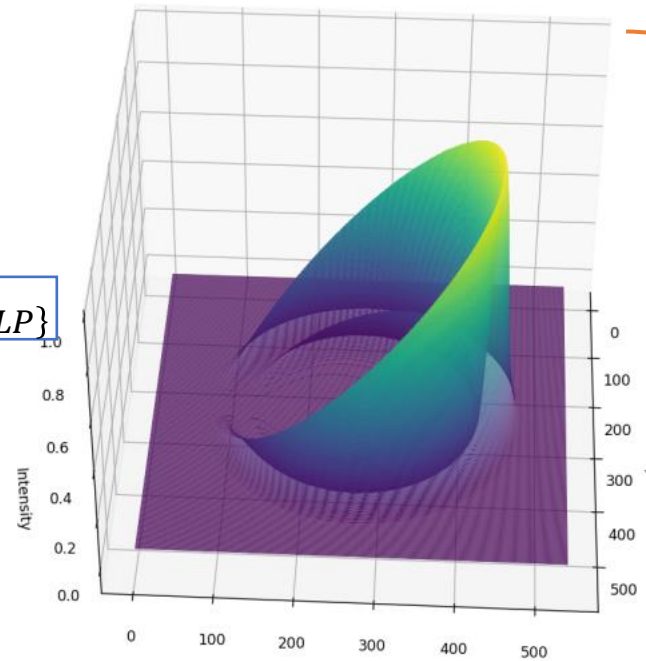
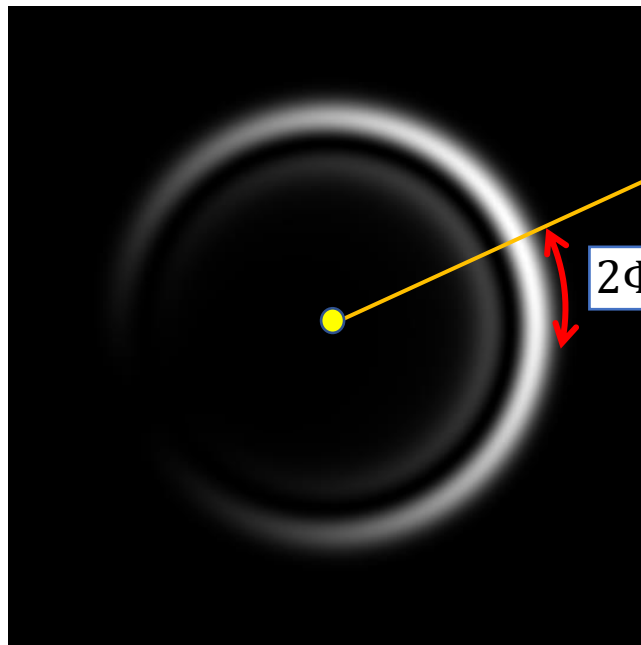


(b)

(c) Developed Algorithm Examples

(i) Geometric Algorithms: Theoretical Image-Angle Property 2

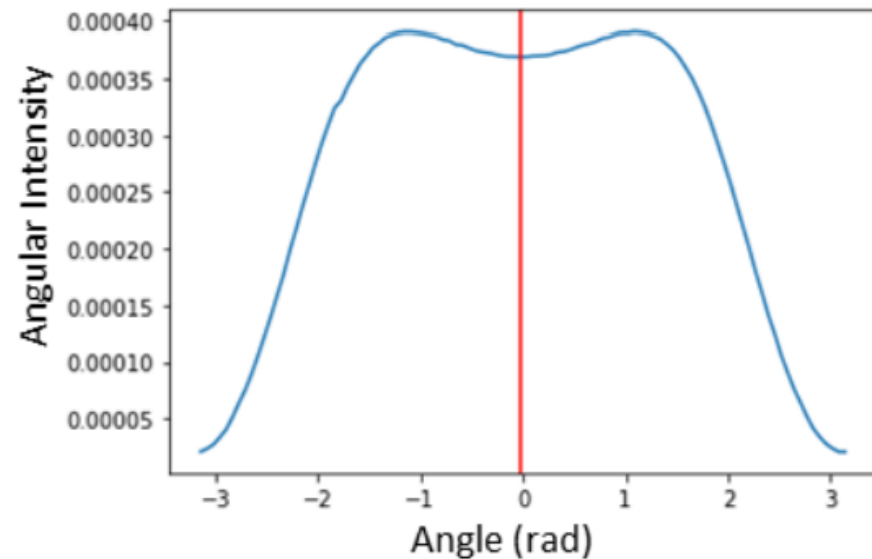
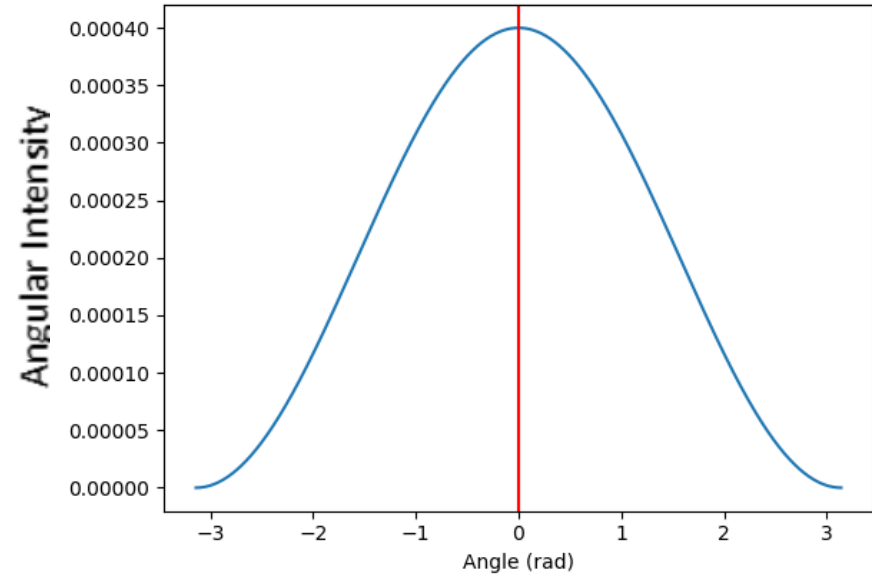
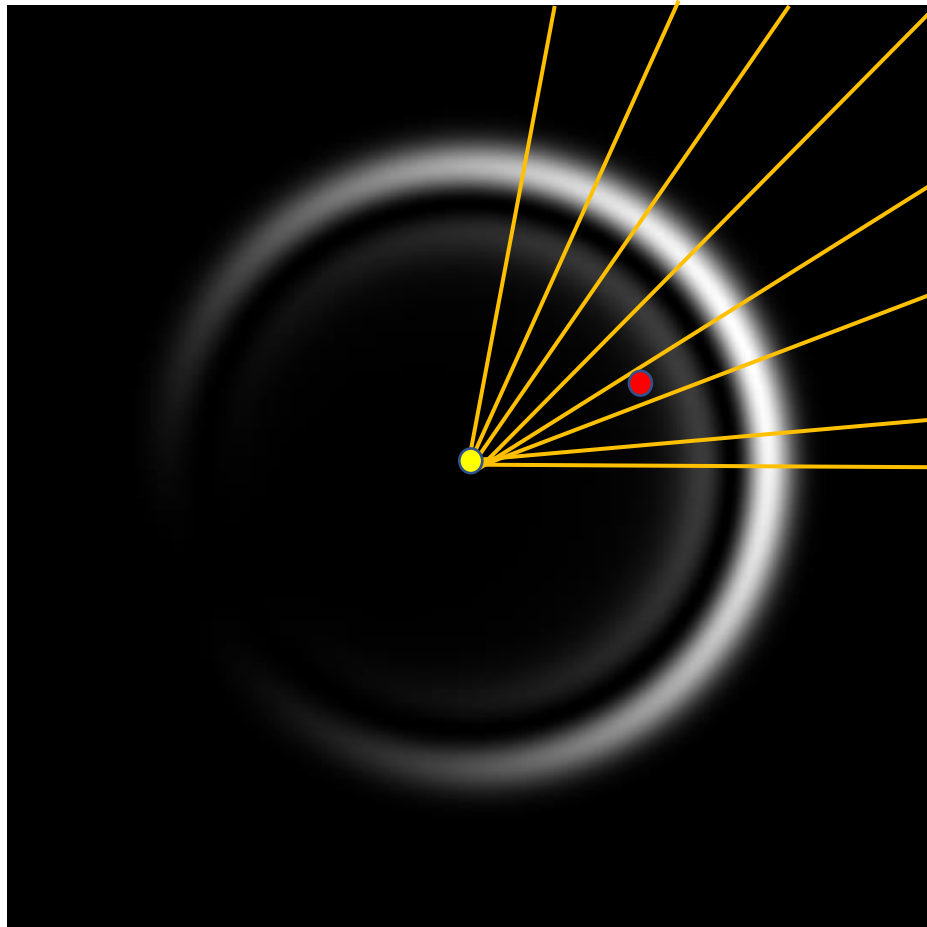
- Angular histogram geom. center, cosine squared displaced by $\phi_{\{CR\}}$



$$I(\theta) = C \cos^2\left(\frac{\theta}{2} - \Phi_{\{LP\}}\right)$$

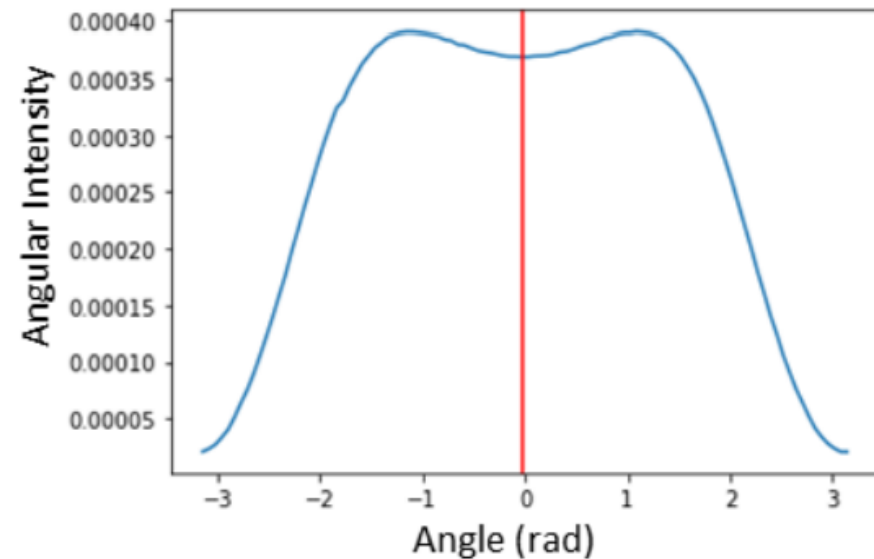
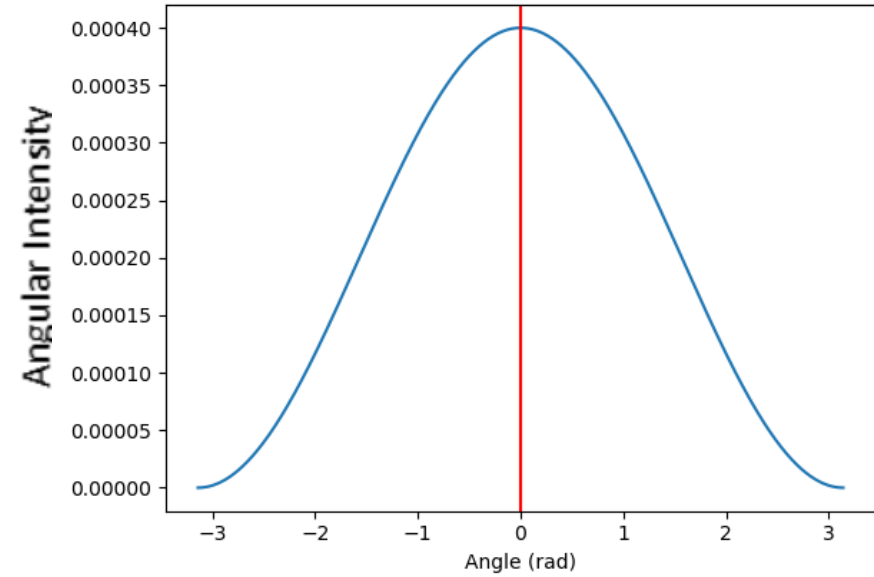
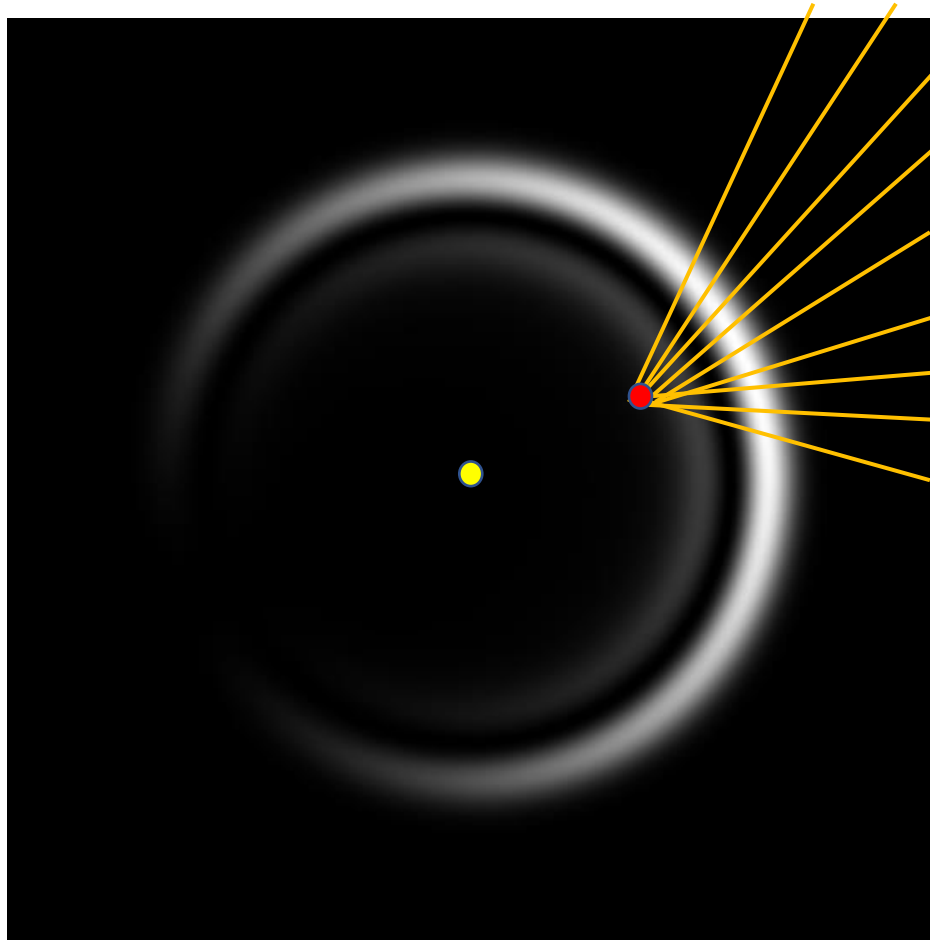
(c) Developed Algorithm Examples

(i) Geometric Algorithms: The **Cosine Squared Fit** Algorithm



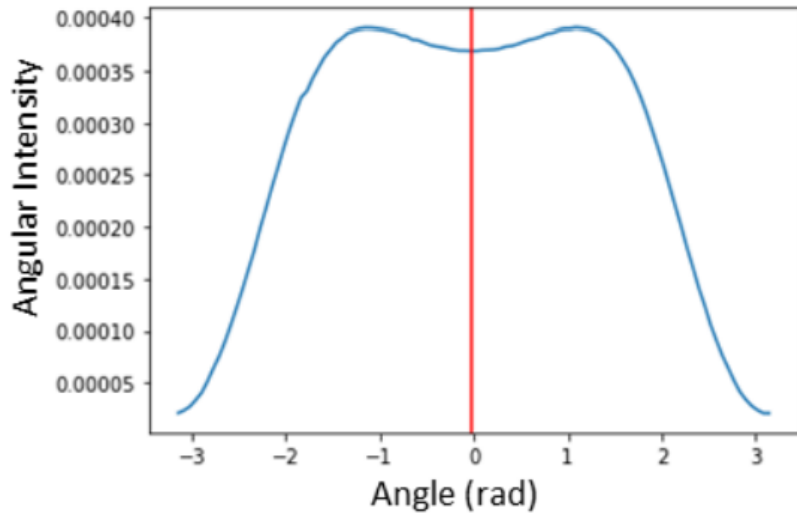
(c) Developed Algorithm Examples

(i) Geometric Algorithms: The **Cosine Squared Fit** Algorithm

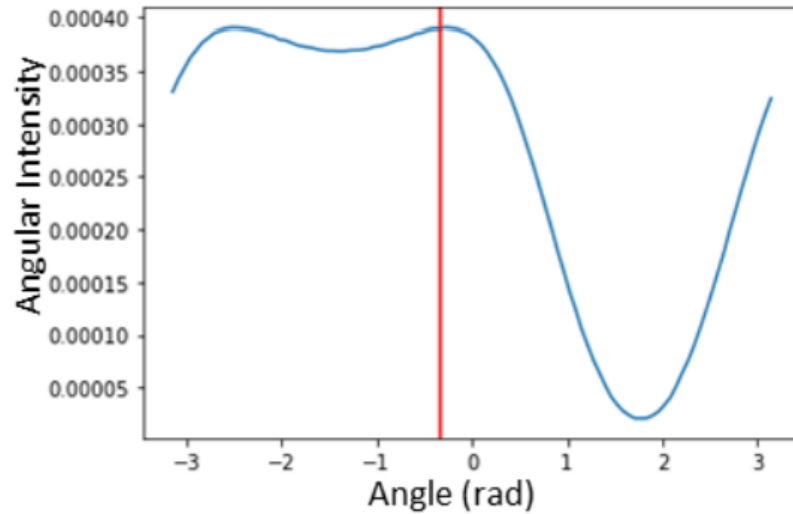


(d) Developed Algorithm Examples

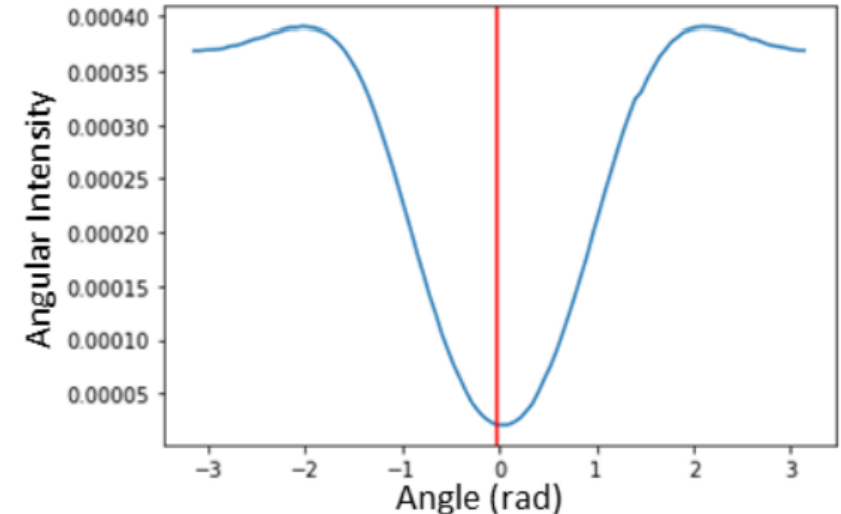
(i) Geometric Algorithms: The **Bimodal Histogram** Algorithm



(a)



(b)

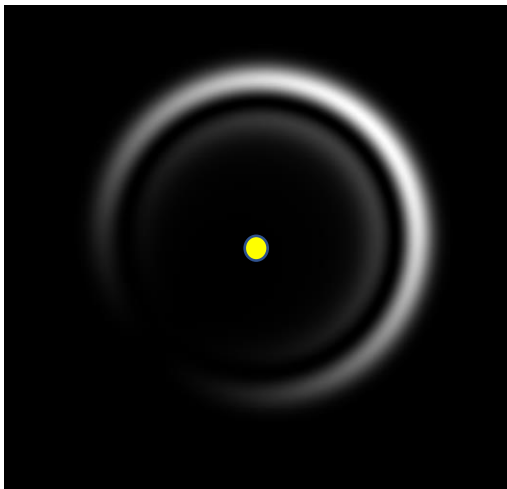


(c)

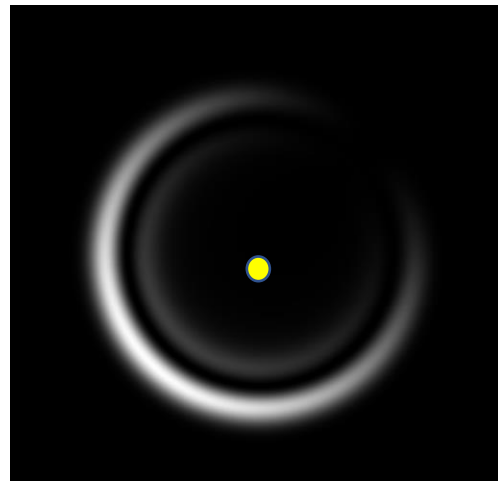
(c) Developed Algorithm Examples

(i) Geometric Algorithms: Theoretical Image-Angle Property 3

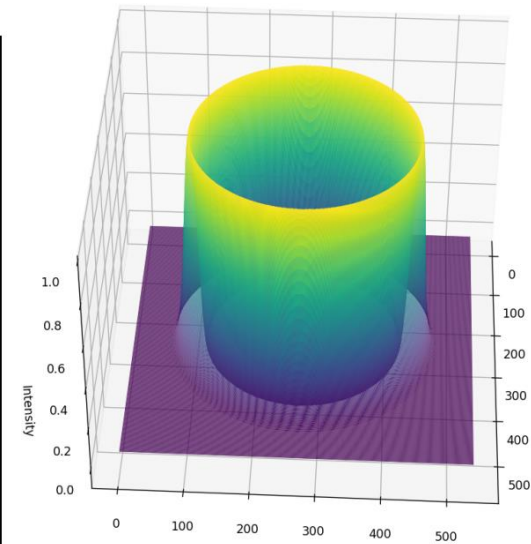
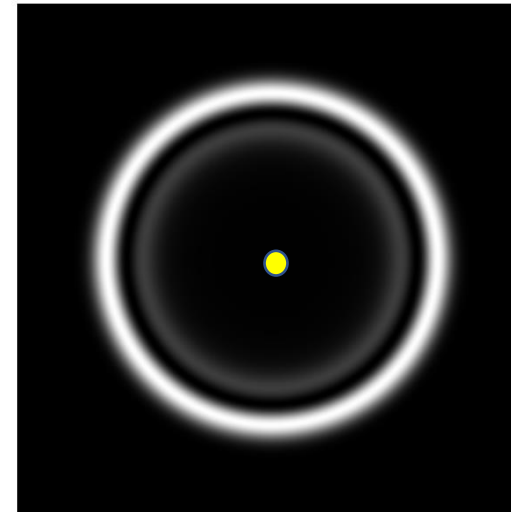
- **Crescent ring + π rotated (abt. Geom. Center) = full ring as CP**



Image

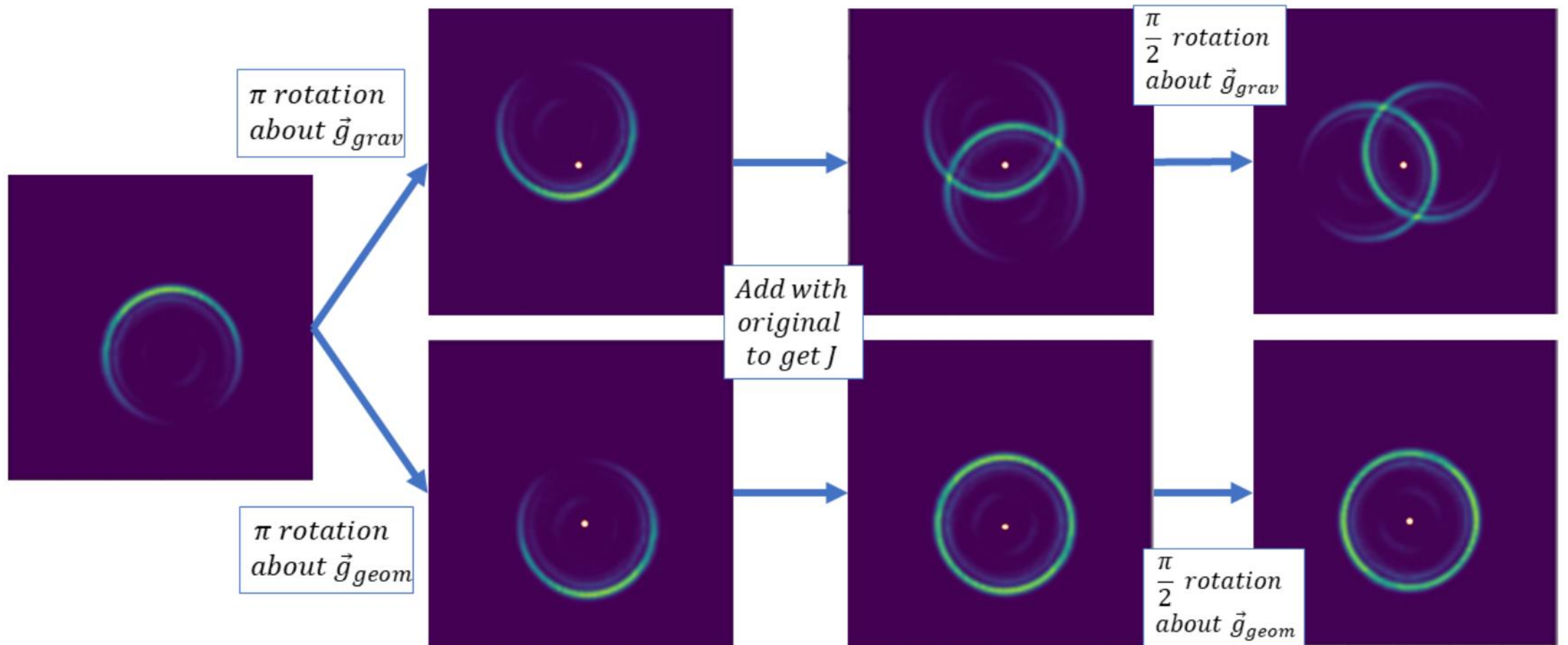


Rot π (Image)



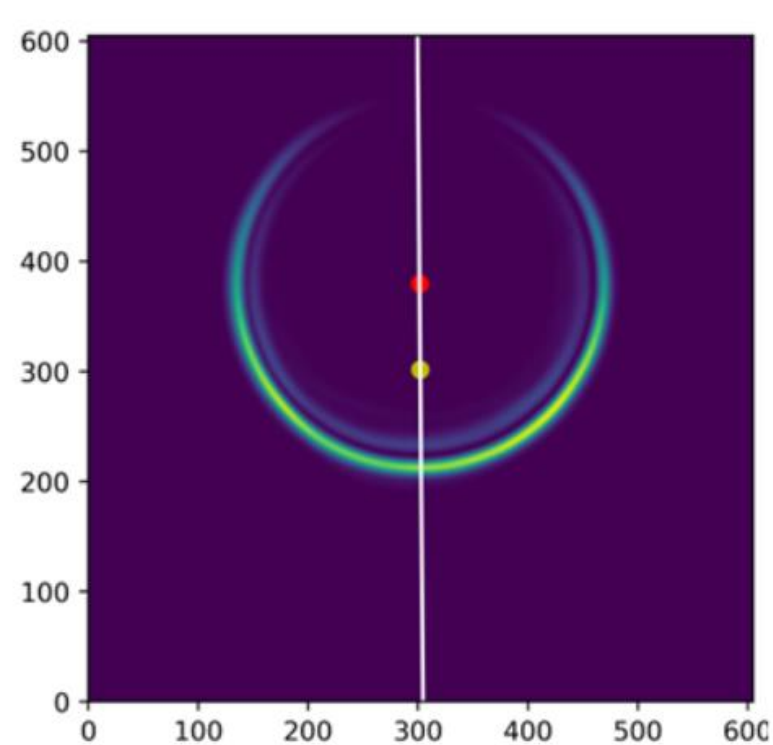
(c) Developed Algorithm Examples

(i) Geometric Algorithms: The **Blazquez** Algorithm

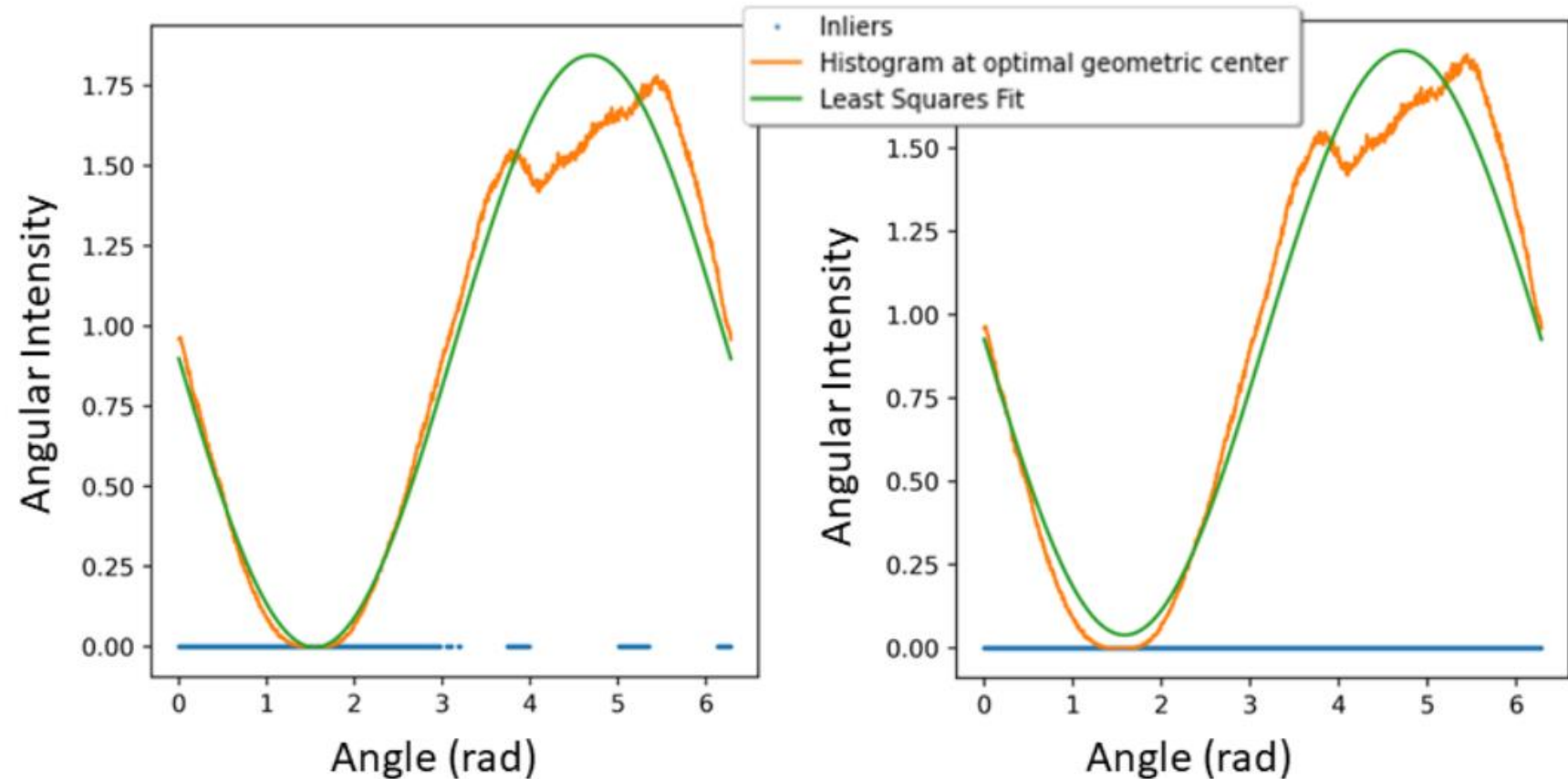


(c) Developed Algorithm Examples

(i) **Geometric** Algorithms:
The **Blazquez + RANSAC Cosine Fit** Algorithm



(a)

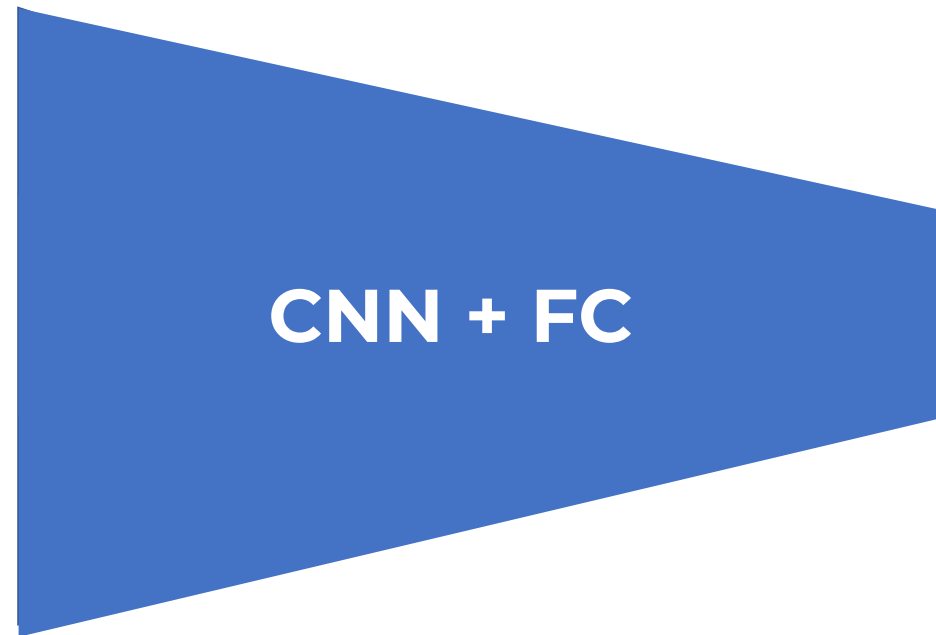


(b)

(c)

(c) Developed Algorithm Examples

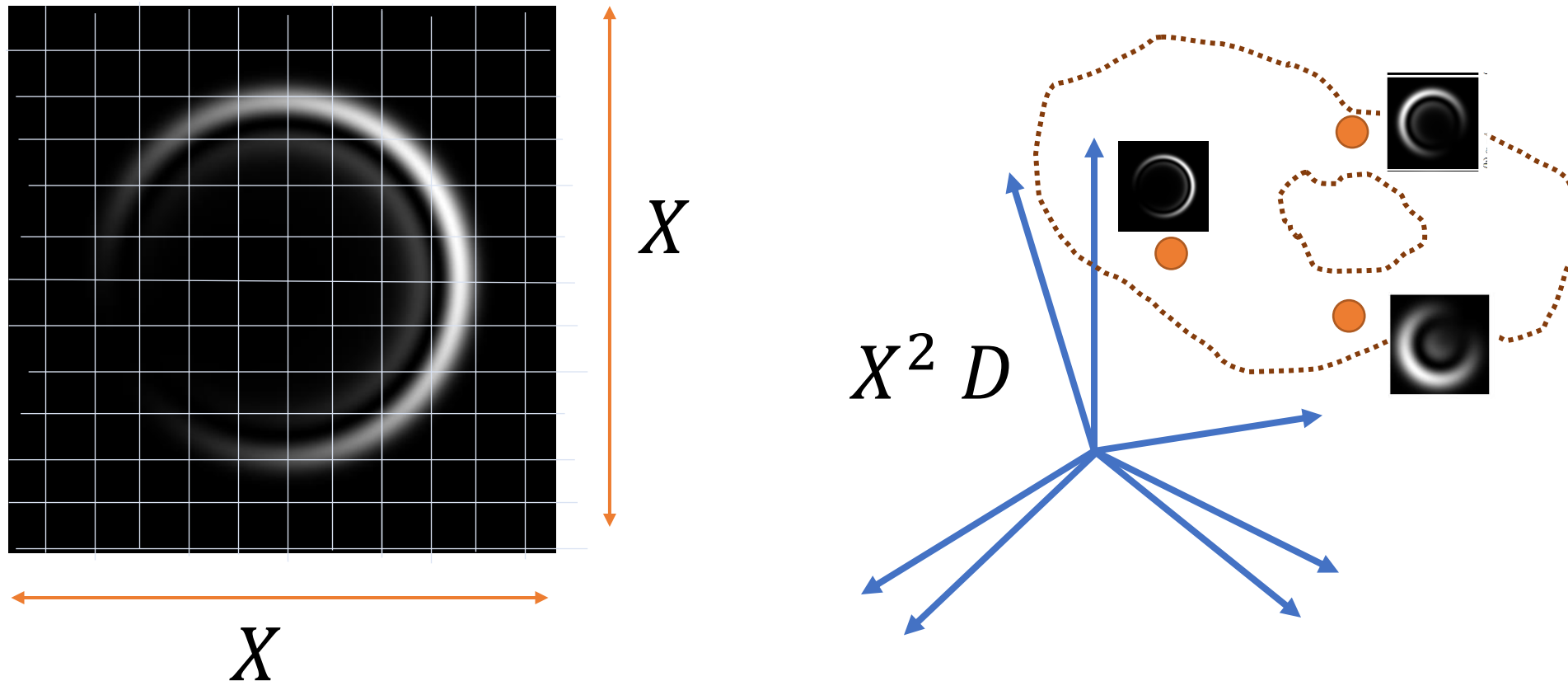
(ii) Neural Network Algorithms



$$\phi_{\{CR\}}$$

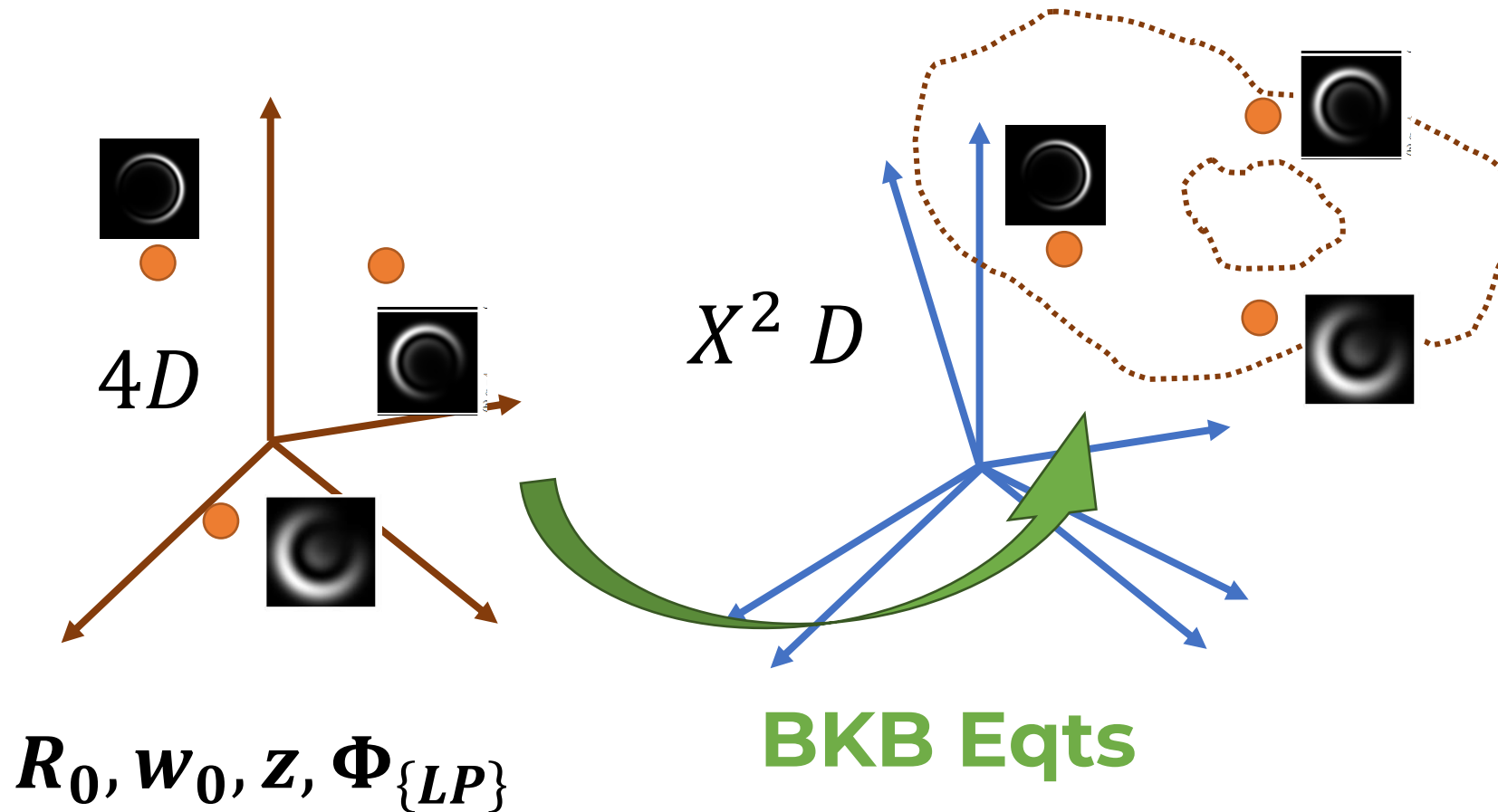
(c) Developed Algorithm Examples

(iii) Embedding Space Algorithms: Dimensionality of the Data Manifold



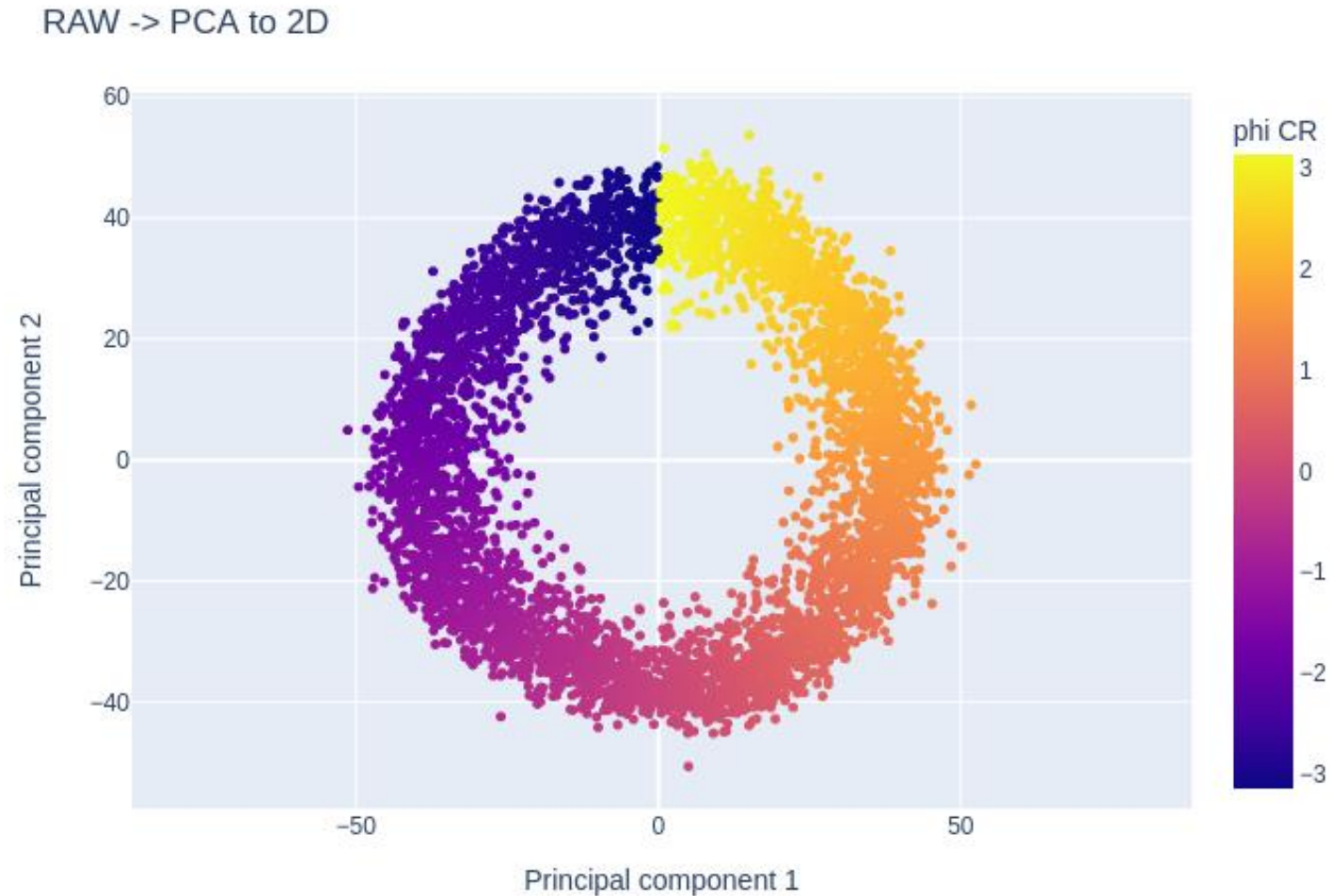
(c) Developed Algorithm Examples

(iii) **Embedding Space** Algorithms:
Observation 1



(c) Developed Algorithm Examples

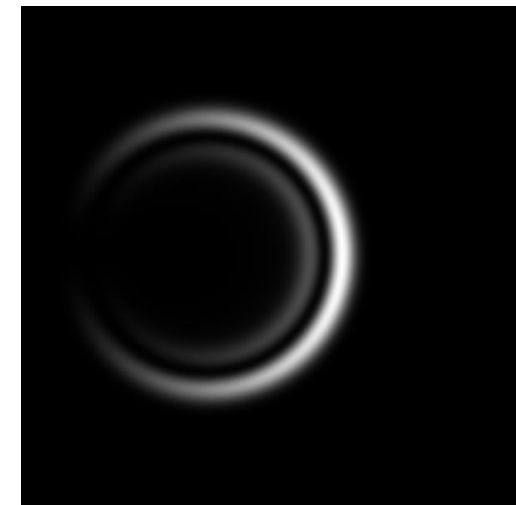
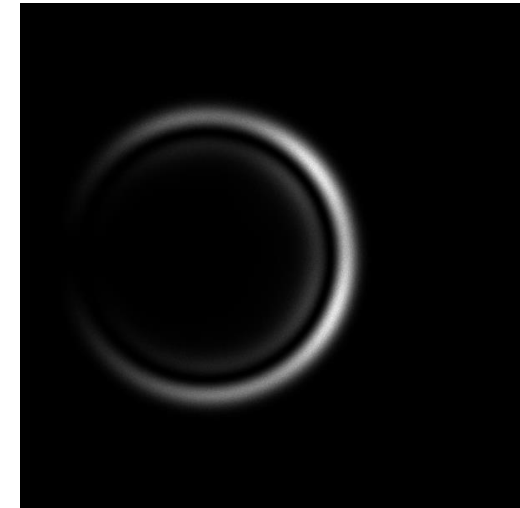
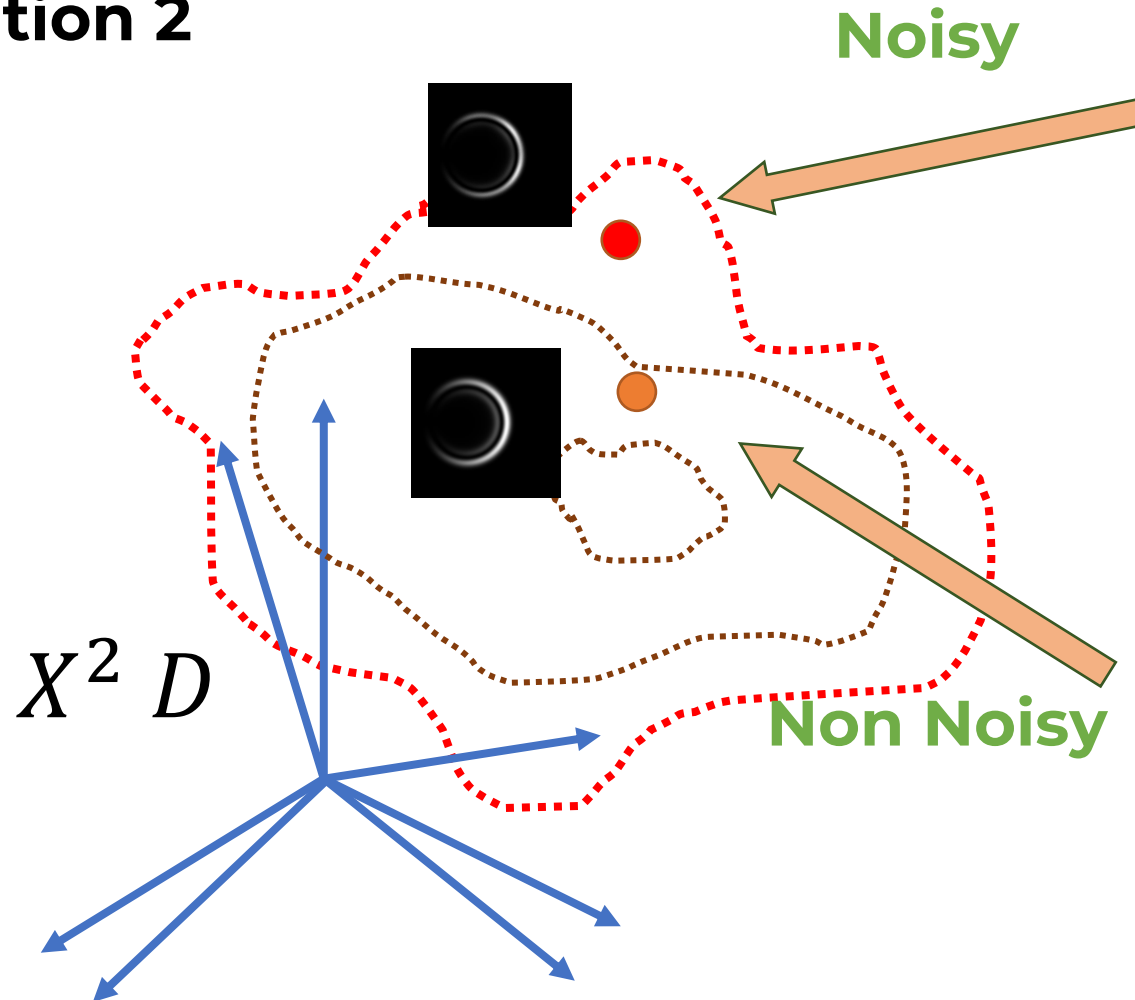
(iii) Embedding Space Algorithms: Observation 1



PCA to 2D
Non-Noisy
Data

(c) Developed Algorithm Examples

(iii) **Embedding Space** Algorithms:
Observation 2

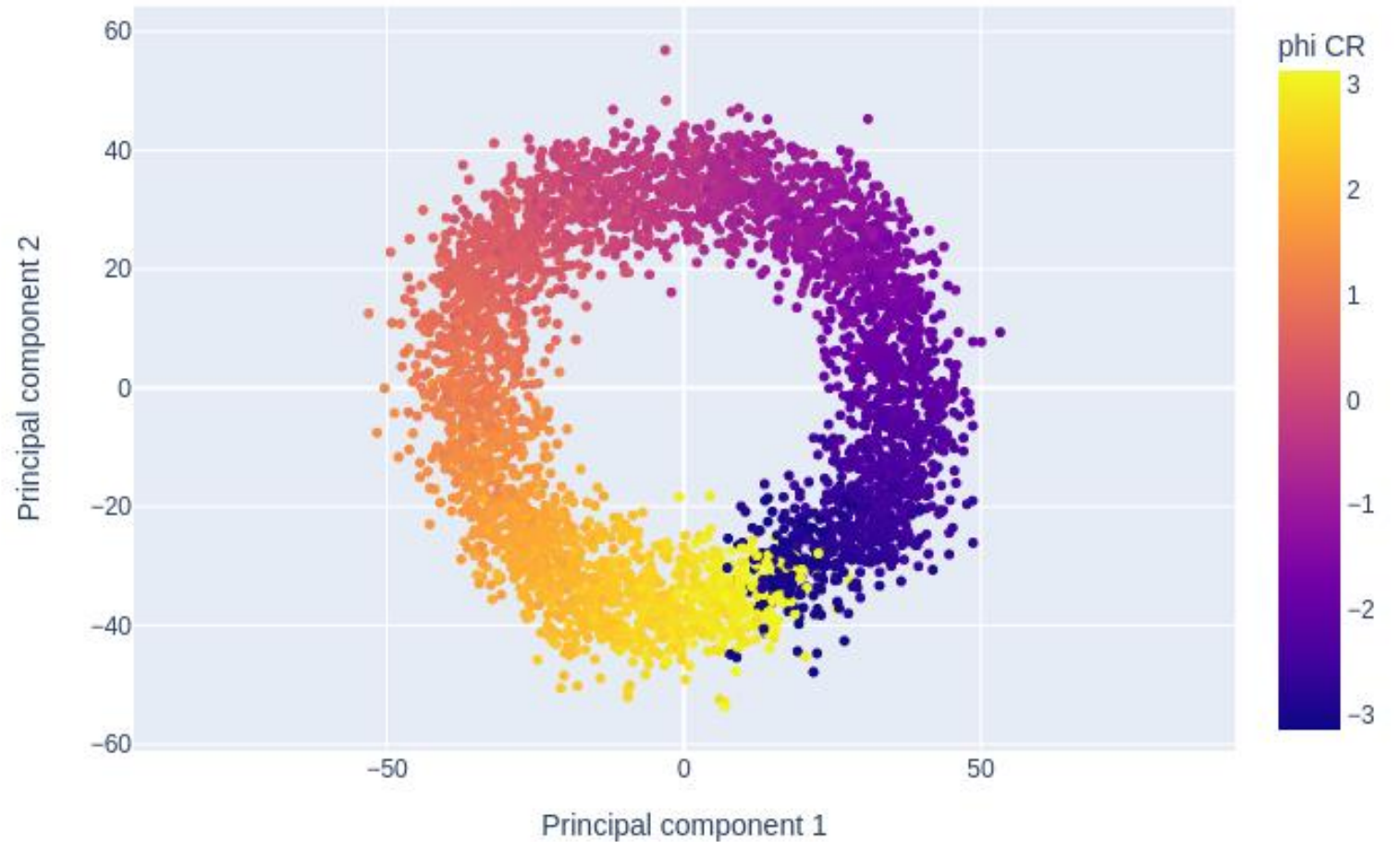


(c) Developed Algorithm Examples

(iii) Embedding Space Algorithms: Observation 2

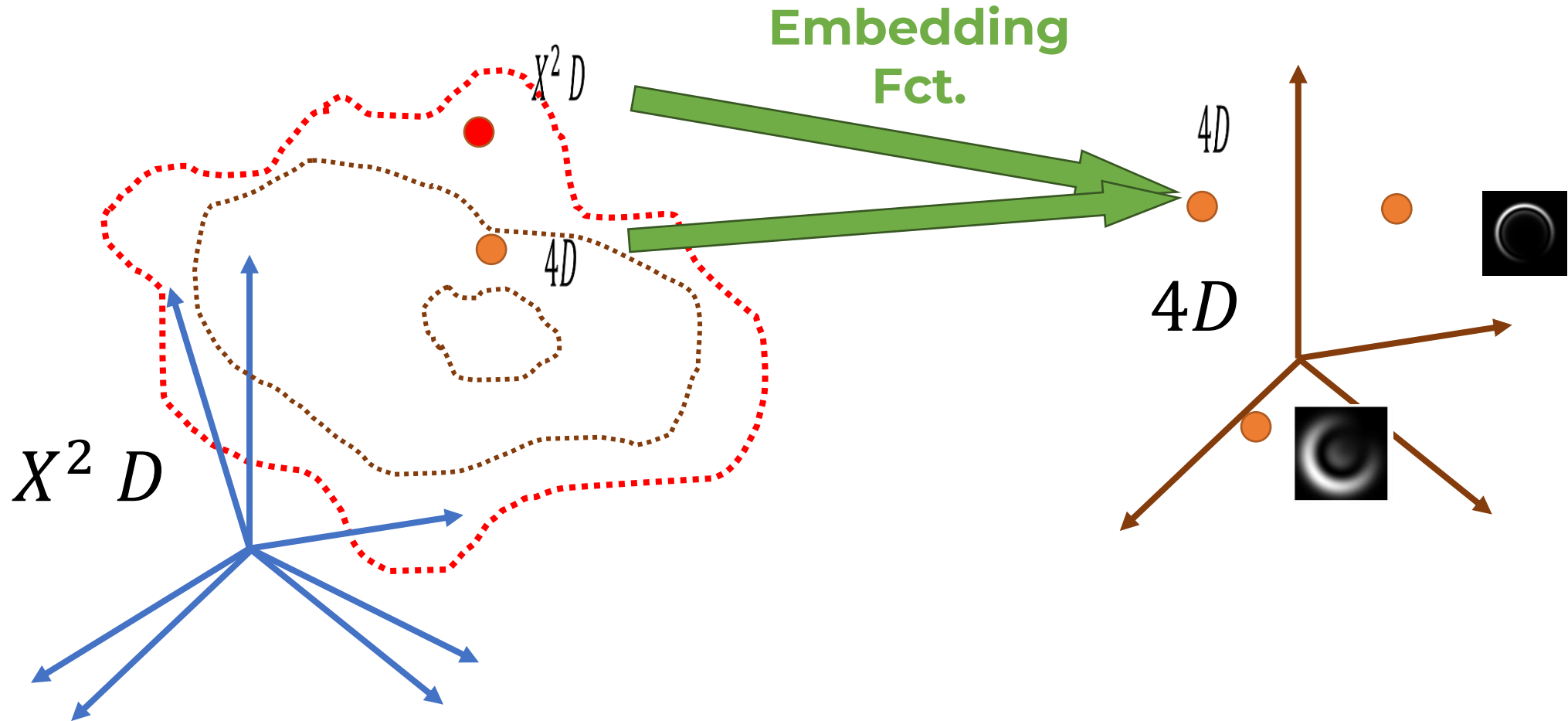
PCA to 2D
Noisy
Data

RAW -> PCA to 2D



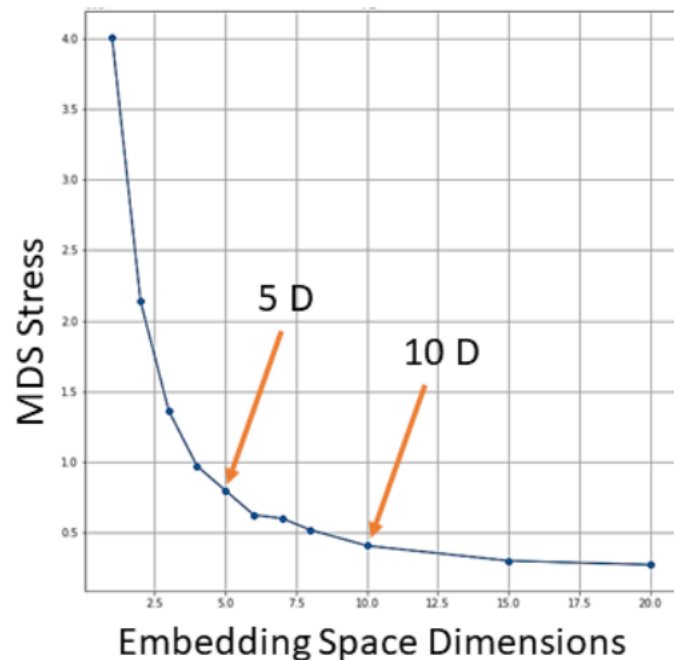
(c) Developed Algorithm Examples

(iii) **Embedding Space** Algorithms:
Happy Idea

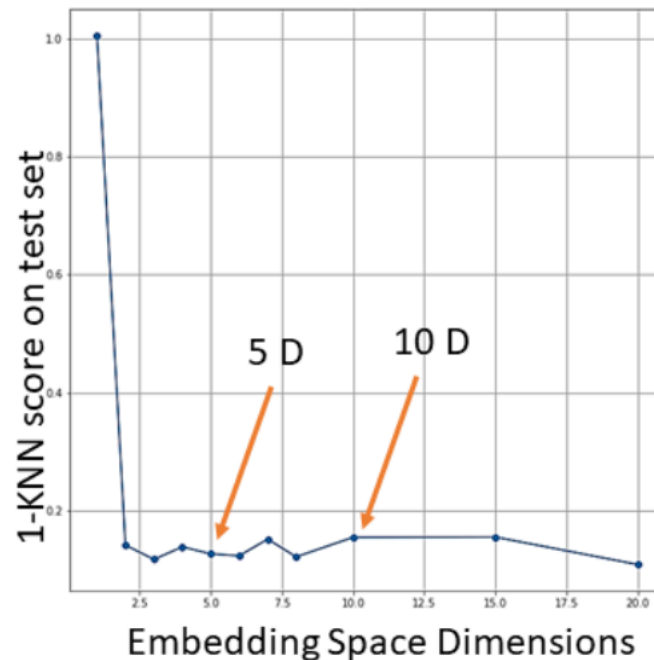


(c) Developed Algorithm Examples

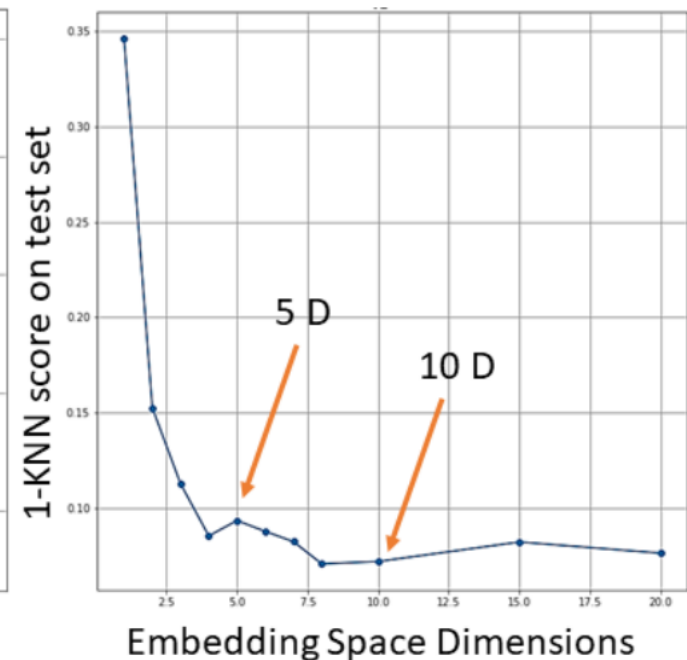
(iii) Embedding Space Algorithms: Dimensionality of the Data Manifold



(a)



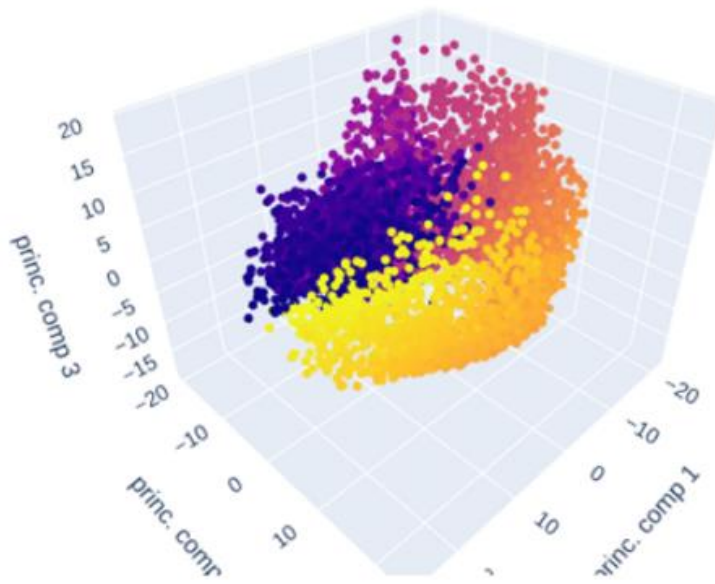
(b)



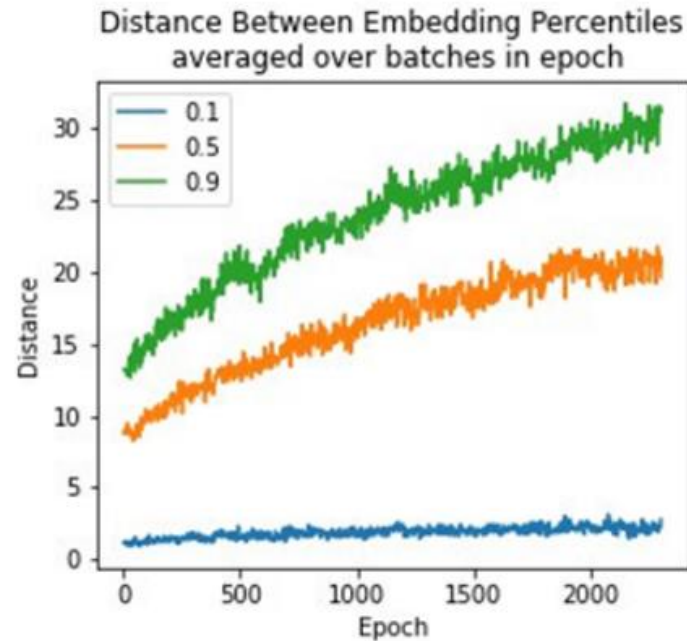
(c)

(c) Developed Algorithm Examples

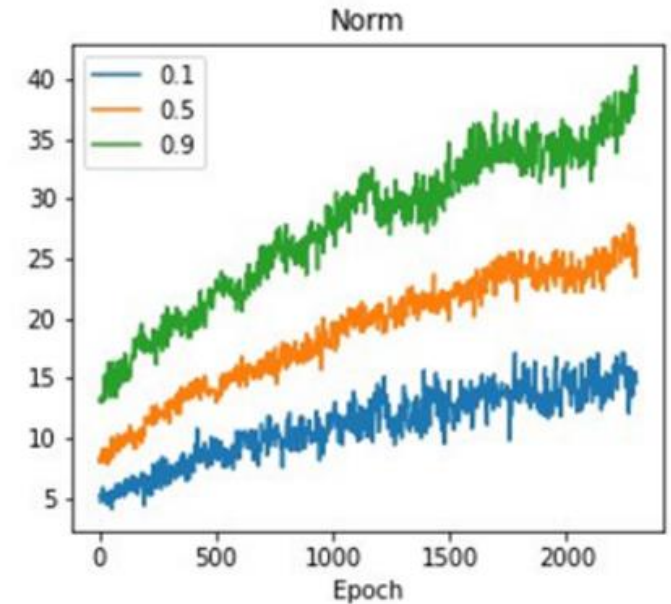
(iii) Embedding Space Algorithms: Triplet Loss Neural Network + KNN



(a)



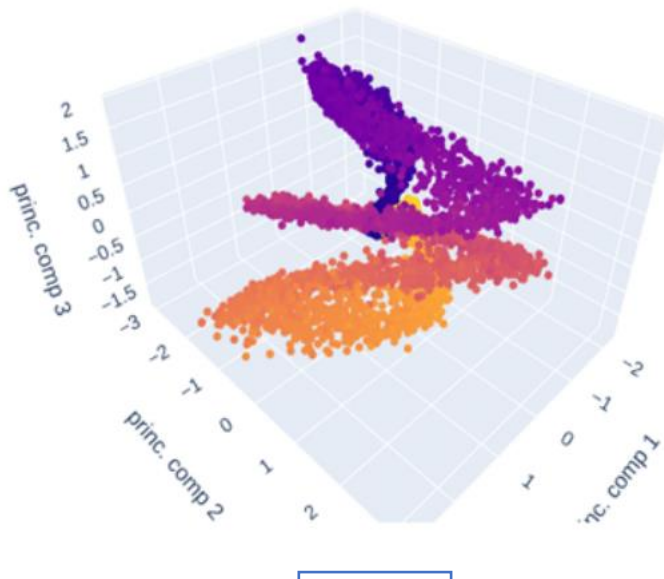
(b)



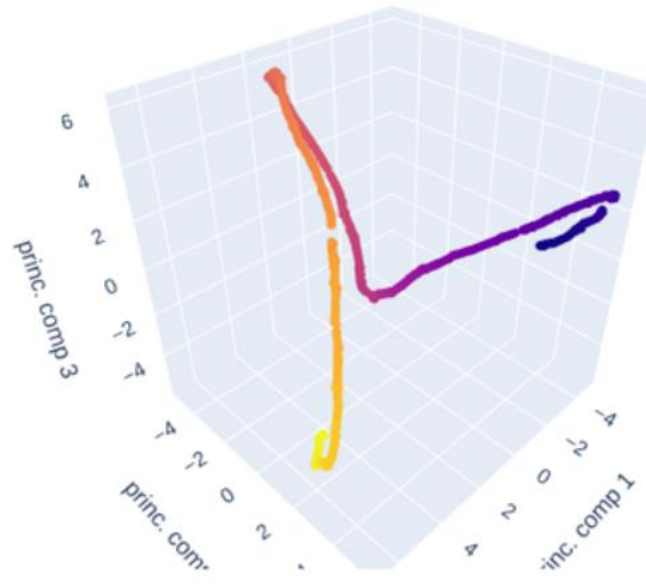
(c)

(c) Developed Algorithm Examples

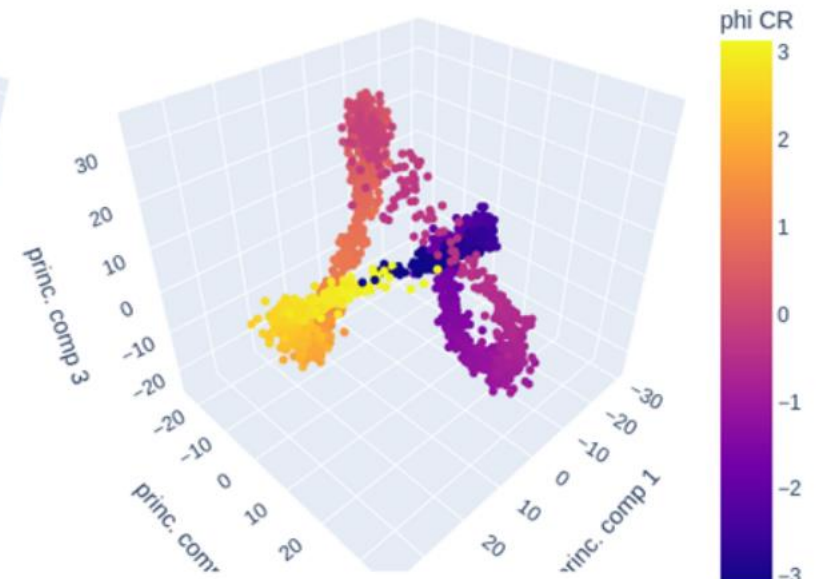
(iii) **Embedding Space** Algorithms:
PCA, UMAP, NCA + KNN



PCA



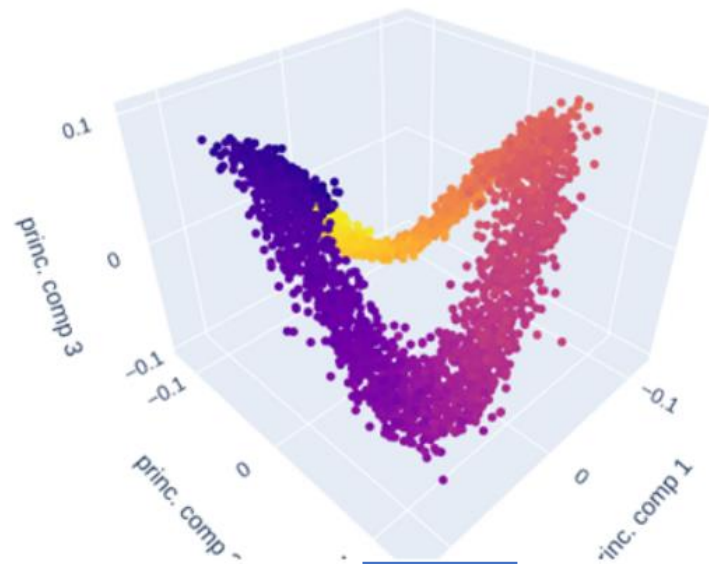
UMAP



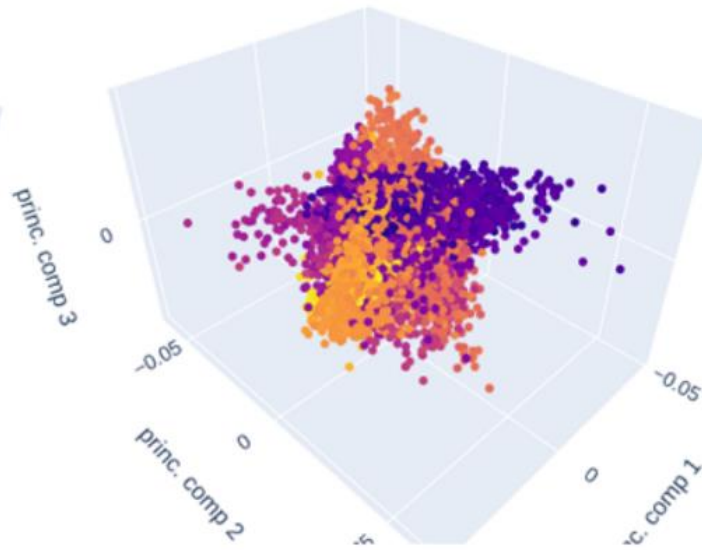
NCA

(c) Developed Algorithm Examples

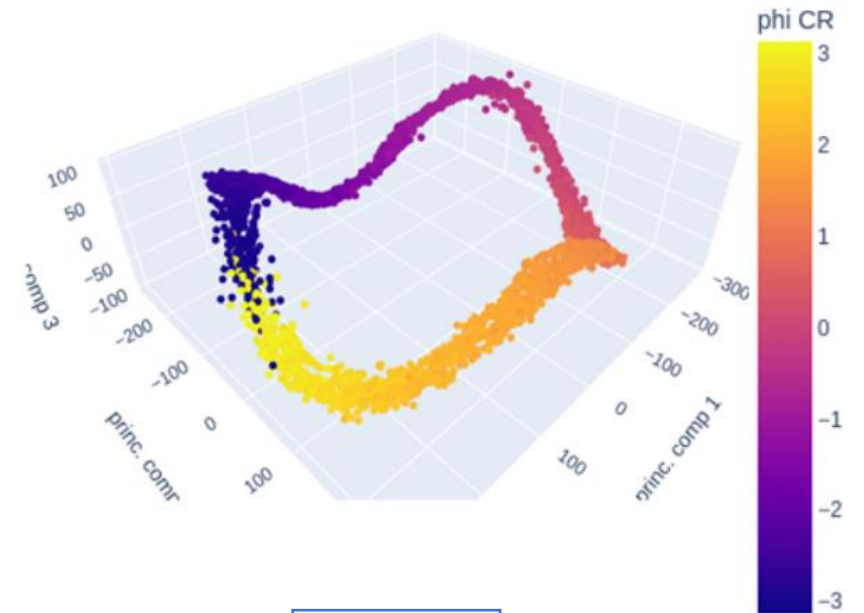
(iii) **Embedding Space** Algorithms:
KPCA, LLE, ISOMAP + KNN



KPCA



LLE



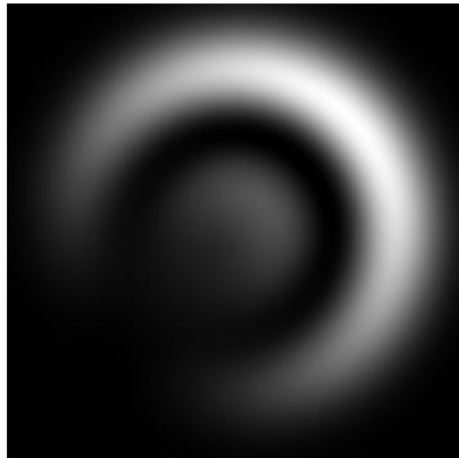
ISOMAP

(c) Developed Algorithm Examples

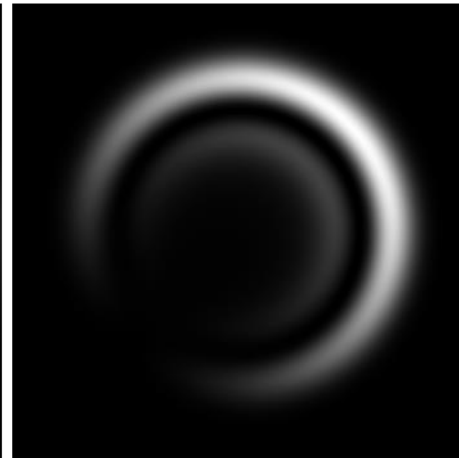
(iv) Simulation Fitting Algorithms:

$R_0, w_0, z, \Phi_{\{LP\}}$

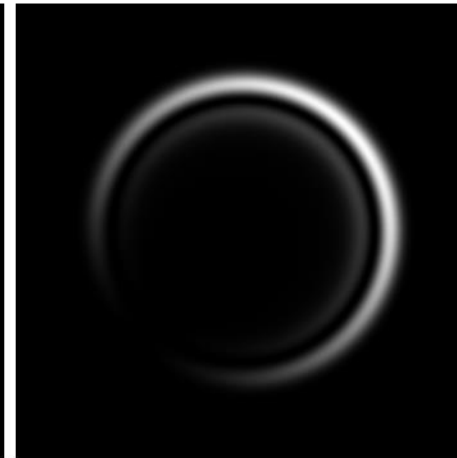
$z = 0,$
 $LP = \pi/8$



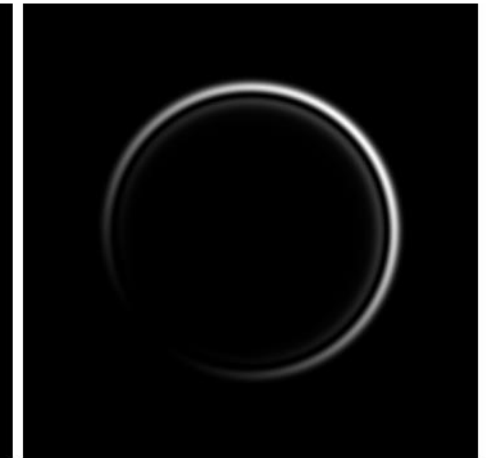
(a) $\rho_0 = 2$
 $w_0 = 83.5 R_0 = 167$



(b) $\rho_0 = 4$
 $w_0 = 41.75 R_0 = 167$

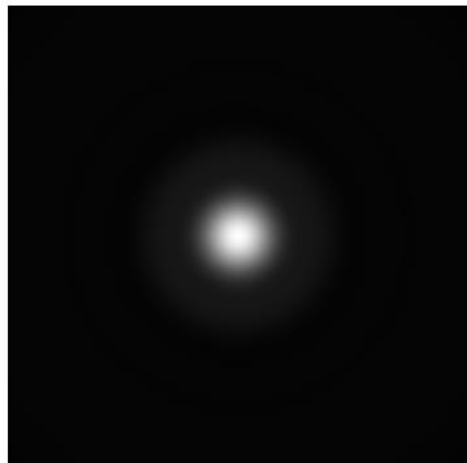


(c) $\rho_0 = 8$
 $w_0 = 20.87 R_0 = 167$

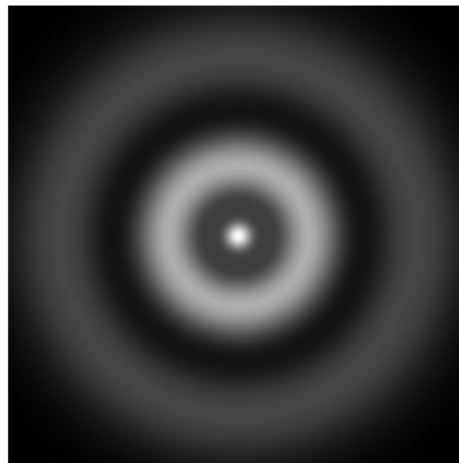


(d) $\rho_0 = 16$
 $w_0 = 10.44 R_0 = 167$

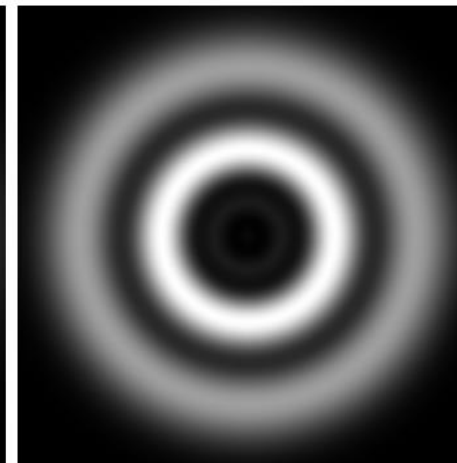
$R_0 = 167,$
 $w_0 = 24,$
 CP



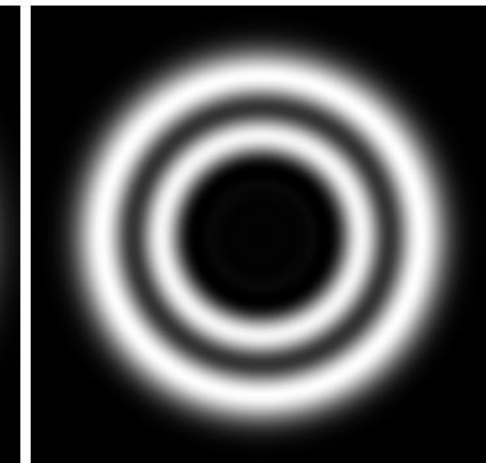
(a) $z = \pm 3$



(b) $z = \pm 1.8$



(c) $z = \pm 1.5$



(d) $z = \pm 1.2$

(d) Algorithm Benchmark

(i) Simulated Non-Noisy Images

Best-case
Scenario
for
540 x 540
camera

Algorithm		Absolute Error (deg)			Times (s)	
Name	Winner Variant	Mean	90% CI		90% CI	
Mirror Flip	Quadratic Opt., Lanczos Interpol., Preproc. Norm. to Max	0.00106	0.00102	0.00109	0.218	0.220
Gravicenter Regression	Preproc. Norm. to Max	0.00196	0.00189	0.00203	0.453	0.454
Kalkandjiev-Rotation	Quadratic Opt., Lanczos Interpol., Preproc. Norm. to Max	0.00205	0.00198	0.00213	0.221	0.222
Blazquez + Cosine Fit	w=0.999, Nelder-Mead Opt., Preproc. Norm. to Max	0.00455	0.00437	0.00475	3.38	3.39
Cosine Square Fit	Preproc. Norm. to Max	0.00466	0.00452	0.00480	2.820	2.830
Blazquez	Preproc. Norm. to Max	0.124	0.0517	0.274	0.454	0.456
Simulation Fit	Opt. Nelder-Mead, About Grav. Preproc. Norm. to Max	0.140	0.123	0.171	1.283	1.316
KNN+Embedder	NCA Embedder, Trained with Non-Noisy Data	0.280	0.140	2.32	0.130	0.160
KNN	Trained with Non-Noisy Data	0.820	0.484	1.40	4.960	5.610
Bimodal Histogram	Preproc. Norm. to Max	1.66	1.58	1.76	0.131	0.132
Relative Rotation	Quadratic Opt., Bicubic Interpol., Preproc. Norm. to Max	1.73	1.61	1.83	0.068	0.069
Polar Registration	Preproc. Norm. to Max	2.83	2.64	3.04	0.245	0.245
CNN+fc	Encoder A	3.41	2.94	4.06	1.093	1.094

(d) Algorithm Benchmark

(ii) Simulated Noisy Images

Worst-case Scenario for 540 x 540 camera

Algorithm		Absolute Error (deg)			Times (s)	
Name	Winner Variant	Mean	90% CI		90% CI	
Simulation Fit	Opt. Powell, About Gravicenter, Preproc. Sigmoid k=35 c=0.1	1.86	1.75	1.98	1.24	1.25
Kalkandjiev-Rotation	Opt. Fibo., Bicubic Interpol., Preproc. Sigmoid k=35, c=0.1	1.93	1.85	2.01	0.218	0.219
Mirror Flip	Opt. Quad., Lanczos Interpol., Preproc. Sigmoid k=35, c=0.1	2.01	1.92	2.12	0.220	0.222
Simulation Fit + Embedder	Opt. Nelder-Mead, About Gravicenter, Preproc. NCA trained w. Noisy Data	2.02	1.88	2.21	2.68	2.73
Blazquez + Cosine Fit	w=0.9999, Opt. Nelder-Mead	2.29	2.18	2.40	3.46	3.47
Blazquez	Opt. Nelder-Mead, final estimation using RANSAC+cos fit	2.32	2.20	2.46	0.510	0.512
KNN + Embedder	Embedder NCA, Trained w. Non-Noisy Data	2.32	2.01	2.94	0.04	0.05
Cosine Square Fit	Opt. Nelder-Mead	2.83	2.73	2.93	2.865	2.866
Gravicenter Regression	Preproc. Sigmoid k=35 c=0.1	2.88	2.77	2.98	0.45	0.45
Polar Registration	Preproc. Sigmoid k=35 c=0.3	2.88	2.74	3.10	0.246	0.247
Relative Rotation	Opt. Quad., Bicubic Interpol., Preproc. Opening s=11	2.94	2.82	3.06	0.067	0.068
Bimodal Histogram	Preproc. Sigmoid k=35, c=0.1	3.41	3.28	3.54	0.131	0.132
KNN	Trained w. Non-Noisy Data	3.69	3.19	4.46	0.886	0.893
CNN+fc	Encoder A	4.09	3.69	4.65	1.092	1.093

(d) Algorithm Benchmark

(iii) Experimental Images

Lab
Scenario
for
540 x 540
camera

Algorithm		Abs. Error (deg)		Times (s)	
Name	Winner Variant	Mean	Std.	Mean	Std.
Cosine squared fit	Opt. Nelder-Mead, final estimation using RANSAC+cos fit	0.50(5)	0.29	2.88	0.012
Blaquez	Opt. Nelder-Mead, final estimation using RANSAC+cos fit	0.51(7)	0.43	0.52	0.020
Blazquez+Cosine Fit	w=0.999, Opt. Nelder-Mead	0.52(4)	0.32	3.43	0.041
Polar Registration	Preproc. Sigmoid k=35, c=0.3	0.54	0.54	0.26	0.011
Mirror flip	Opt. Quad., Bicubic Interpol., Preproc. Sigmoid k=35, c=0.3	0.56	0.41	0.14	0.0080
Kalkandjiev-Rotation	Opt. Fibo., Bicubic Interpol., Preproc. Sigmoid k=35, c=0.3	0.57	0.42	0.22	0.020
Simulation Fit	About Gravic., Opt. Nelder-Mead, Preproc. Norm. to Max	0.58	0.39	1.57	0.042
Relative Rotation	Opt. Quad., Bicubic Interpol., Preproc. Sigmoid k=35, c=0.3	0.61	0.45	0.069	0.0080
Simulation Fit + Embedder	Opt. Quad., Bicubic Interpol., Preproc. Sigmoid k=35, c=0.3	0.88	0.52	2.17	0.033
Bimodal Histogram	Preproc. Sigmoid k=35, c=0.5	1.48	1.59	0.14	0.0025
Gravicenter Regression	Preproc. Sigmoid k=35, c=0.3	2.20	1.96	0.48	0.017
KNN+Embedder	UMAP Embedder, Trained on Non-Noisy Data	2.76	2.05	3.33	0.0
KNN	Trained w. (simulated) Noisy Data	4.23	2.64	1.80	0.0

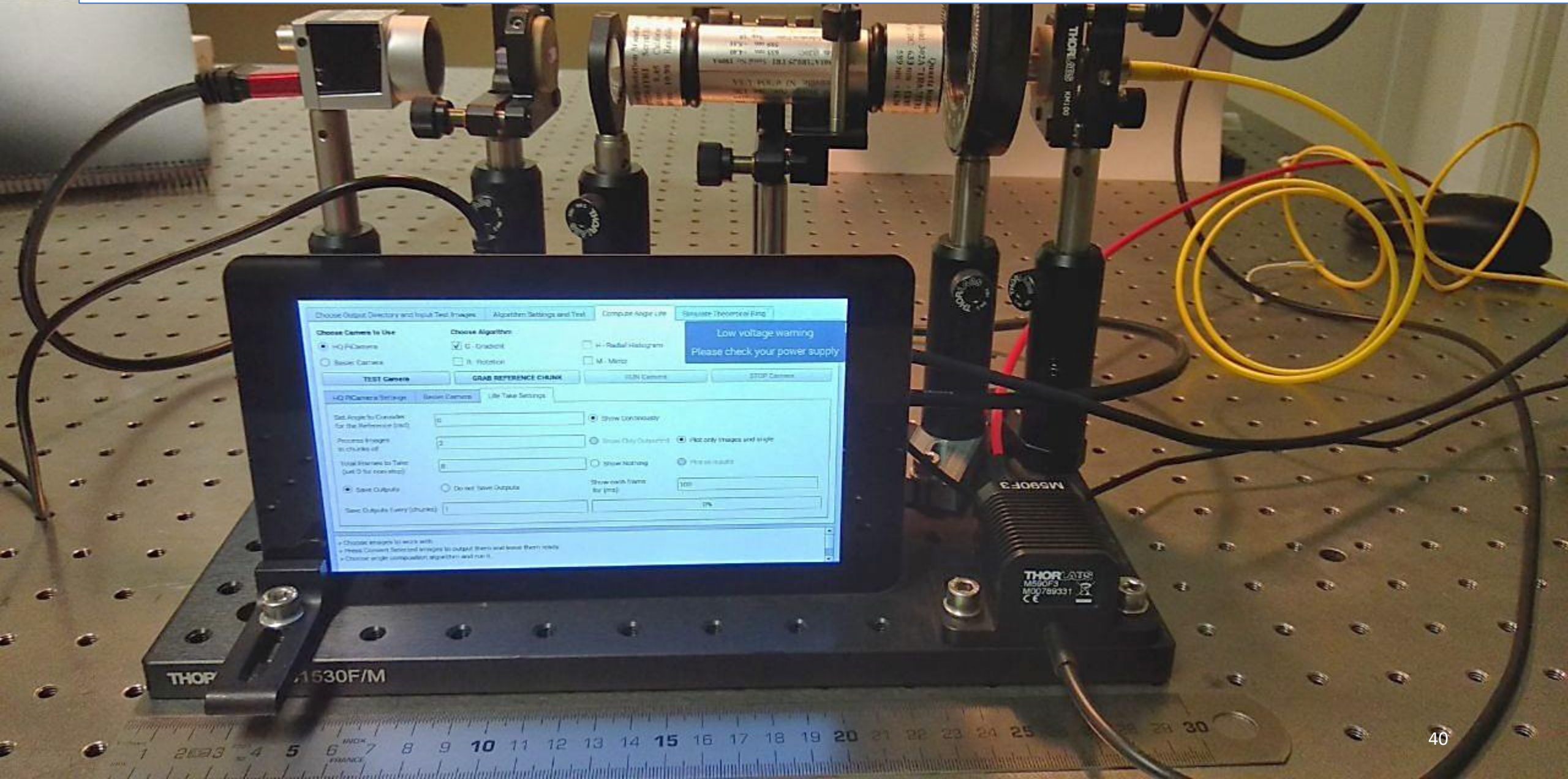
(c) Algorithm Benchmark

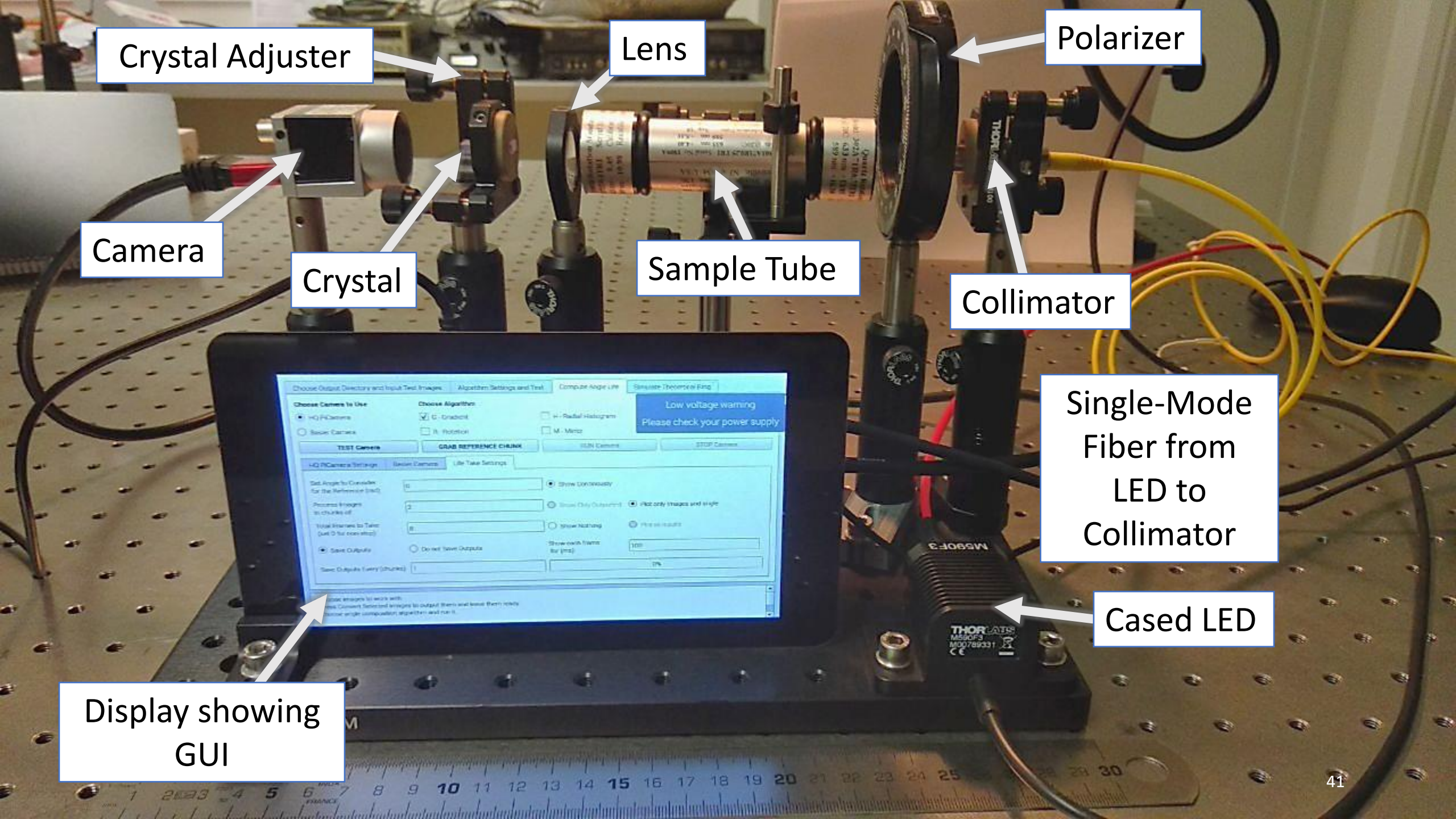
540 x 540 camera	Best-Case	Worst-Case	Exp-Case (mag. Lens)
Best Mean Abs. Error (deg)	0.001	1.93	0.51
Best Alg. Time/meas. (s)	0.22	0.22	0.52

- **Best Algorithm depends on noise level!**

The Device

(a) A Proof-of-Concept





Crystal Adjuster

Lens

Polarizer

Camera

Crystal

Sample Tube

Collimator

Single-Mode
Fiber from
LED to
Collimator

Cased LED

Display showing
GUI

Choose Output Directory and Input Test Images Algorithm Settings and Test Compute Angle List Simulate Theoretical Ring

Choose Camera to Use
 HQ Camera
 Basic Camera

Choose Algorithm:
 G - Gradient
 R - Radial Histogram
 M - Mirror
 H - Histogram
 P - Polaroid

TEST Camera GRAB REFERENCE CHUNK GRAB Camera STOP Camera

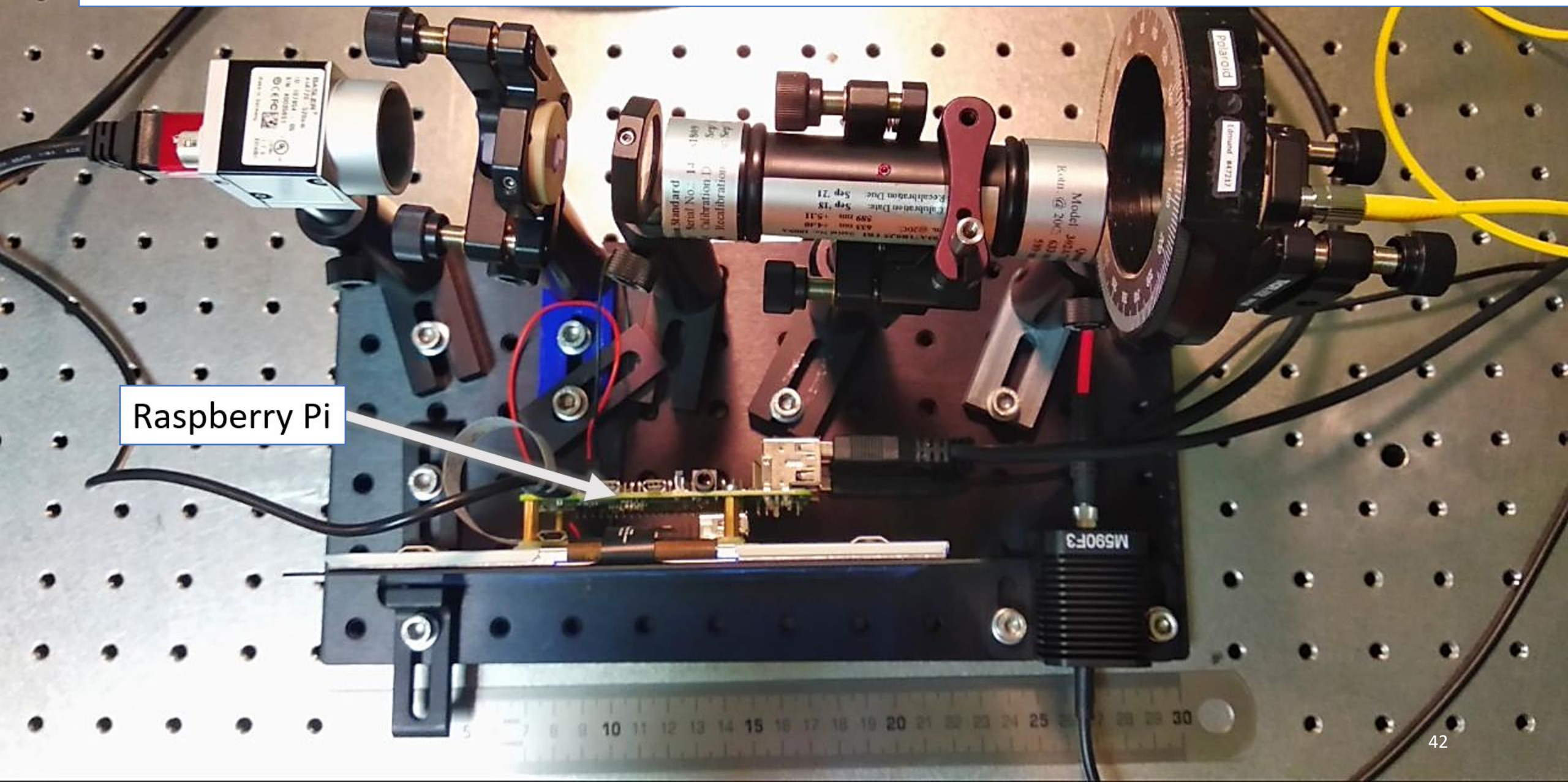
HQ Camera Settings Basic Camera Live Take Settings

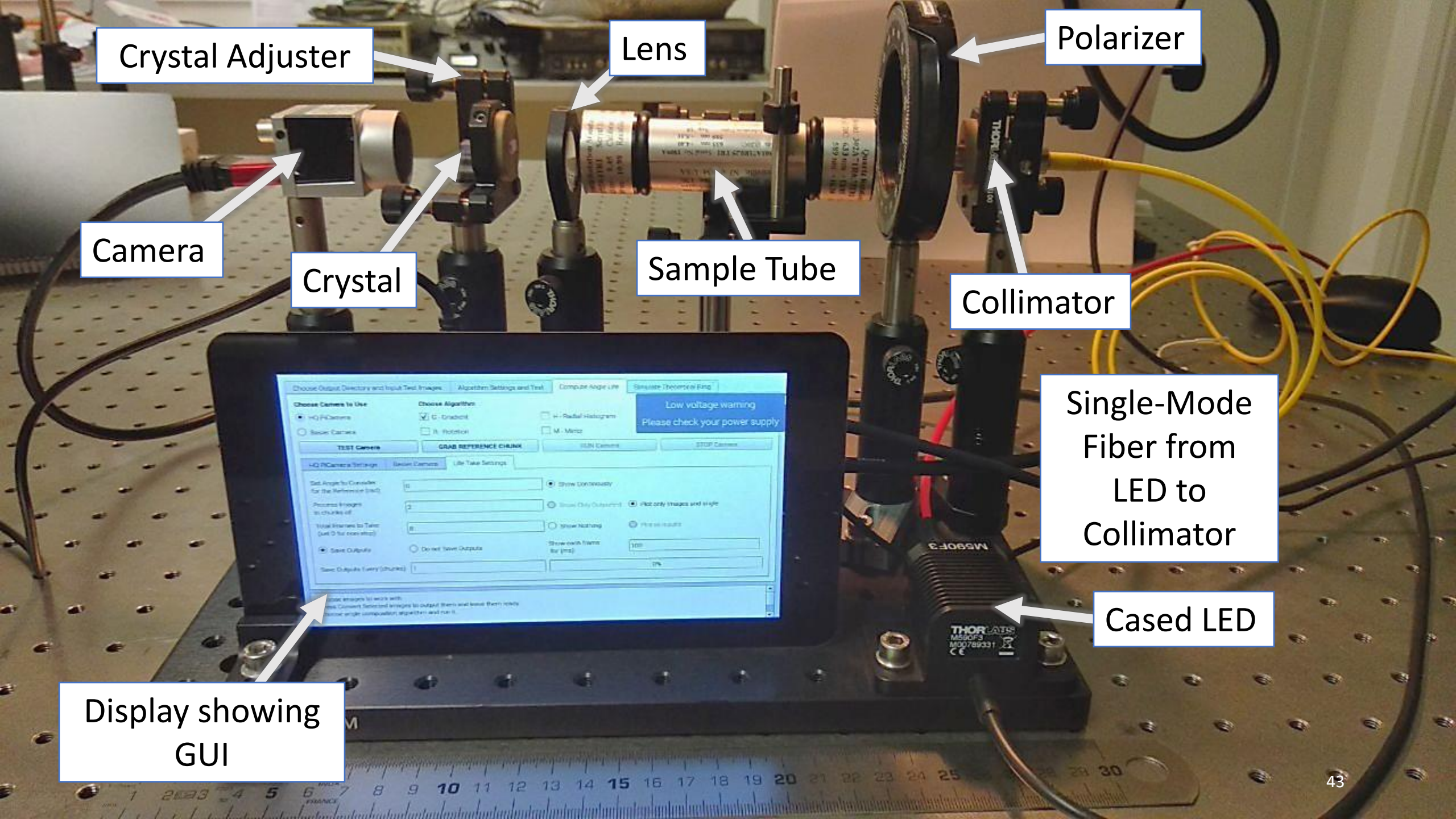
Set Angle for Camera for the Reference (rad) Show Contour
Process Images in chunks of Show Only Outlines Plot only Images and Angle
Total Images to Take (set 0 for once stop) Show Nothing Plot in Image
 Save Outputs Do not Save Outputs Show each frame for (ms)
Save Outputs Every (chunks)

Load images to work with
Click Command Selected images to output them and leave them ready for single comparison algorithm and run it.

Low voltage warning
Please check your power supply

(a) A Proof-of-Concept





Crystal Adjuster

Lens

Polarizer

Camera

Crystal

Sample Tube

Collimator

Single-Mode
Fiber from
LED to
Collimator

Cased LED

Display showing
GUI

Choose Output Directory and Input Test Images Algorithm Settings and Test Compute Angle List Simulate Theoretical Ring

Choose Camera to Use
 HQ Camera Basic Camera

Choose Algorithm:
 G - Gradient Radial Histogram
 H - Hough M - Morph

TEST Camera GRAB REFERENCE CHUNK GRAB Camera STOP Camera

HQ Camera Settings Basic Camera Live Take Settings

Set Angle for Convolver for the Reference (rad): Show Contour Show Only Outlines Plot only images and angle

Process Images in chunks of: Show Nothing Plot in outline

Total Images to Take (set 0 for once stop): Show with frame for (ms): Show with frame for (ms): Show with frame for (ms):

Save Outputs Do not Save Outputs

Save Outputs Every (chunks):

Load images to work with:
Click on a Selected images to output them and leave them ready for single comparison algorithm and run it.

Low voltage warning
Please check your power supply

(b) Commercial Viability

Component	Vendor	Price (euro)
Polarizer film	Thorlabs	9
3V 1A Power Supply for light source	Amazon	10
Lens 100 mm	Thorlabs	19
Casing and element support 3D printer material	Estimated	50
Raspberry Pi 4	Amazon	40
Raspberry Touchscreen 7"	Farnell	50
Single-mode fiber apt for 590 nm	Thorlabs	72
Biaxial crystal cut in the right crystallographic plane	Altechna (priv. comm.)	300
LED M590F3 - 590 nm 3.3 mW	Thorlabs	430
Basler acA720-520um camera	Basler	330
Total	-	1310

540 x 540	Best-Case	Exp-Case (mag. Lens)	Proof-of- Concept (no mag.)
Abs. Error (deg)	0.001	0.51	0.72
Time/meas. (s)	0.22	0.52	0.36

(b) Commercial Viability

Our CR
Polarimeter

				Price (euro)
Total		-		1310
540 x 540	Best-Case	Exp-Case (mag. Lens)	Proof-of- Concept (no mag.)	
Abs. Error (deg)	0.001	0.51	0.72	
Time/meas. (s)	0.22	0.52	0.36	

3 kg!

Polarimeter Brand and model	Price (euro)	Accuracy (deg)	Source	Features
Krüß Polarimeter P3000 Cat. N. 10 61 13000	8,150 + VAT	±0.01	LED 589 nm	- Fast Meas. time ~ 1s - Thermometer - 28 kg; 645 x 200 x 360 mm



(b) Commercial Viability

Our CR
Polarimeter

				Price (euro)
Total		-		1310
540 x 540	Best-Case	Exp-Case (mag. Lens)	Proof-of- Concept (no mag.)	
Abs. Error (deg)	0.001	0.51	0.72	
Time/meas. (s)	0.22	0.52	0.36	

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(b) Commercial Viability

Our CR Polarimeter

Total		-	Price (euro)
			1310
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(b) Commercial Viability

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Future Work

- Find **limit precision** for **increasing pixel density**
- **Improve Proof-of-concept** alignment/casing
- **Deep Learning** with **experimental** labelled dataset

Questions