Rigorous Schrödinger Quantum Mechanics of Countably Many Degrees of Freedom

Xabier Oianguren-Asua

EBERHARD KARLS UNIVERSITÄT TÜBINGEN

DAAD Deutscher Akademischer Austauschdienst German Academic Exchange Service

"la Caixa" Foundation

CLAIM:

Even for a quantized SCALAR field, a rigorous Schrödinger wavefunction in "field-configuration representation" is a "SPINOR" of UNCOUNTABLY MANY wavefunctionals.

(Alternatively, one needs additional sectors of Fock space allowing infinitely many bosons)

DISCLAIMER for other MATHEMATICIANS:

- Some statements will be DELIBERATELY SLOPPY!
- Check master's thesis for fully precise versions & proofs

Go to: tinyurl.com/XabierOA

1. MOTIVATION: WHY "$\lim_{n\to\infty} L^2(\mathbb{R}^n, d^n x)$"?

2. WHY $\nexists$ LITERAL "$L^2(\mathbb{R}^\infty, d^\infty x)$"?

3. BETTER: TAKE $\lim_{n\to\infty} \underbrace{L^2(\mathbb{R}, dx) \otimes \cdots \otimes L^2(\mathbb{R}, dx)}_{n}$

4. JOINT DIAGONALIZATION of POSITION OPERATORS

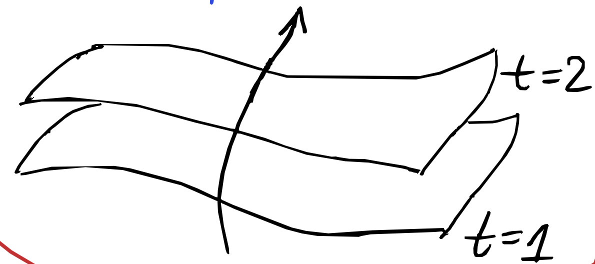
5. TEMPLATE for RIGOROUS PILOT-WAVES on $\mathbb{R}^\infty$

6. RIGOROUS WAVEFUNCTIONALs

7. FOCK vs MYRIOTIC CCR REPRESENTATIONS

1. MOTIVATION

System to be modelled & "Time" parameter



$\Downarrow$

PARAMETRIZE configurations of system per "time" with REAL numbers

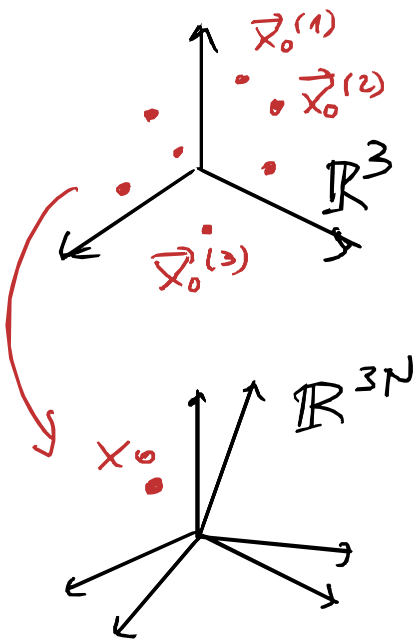
$\Downarrow$

DEGREES of FREEDOM (DOFs)

&

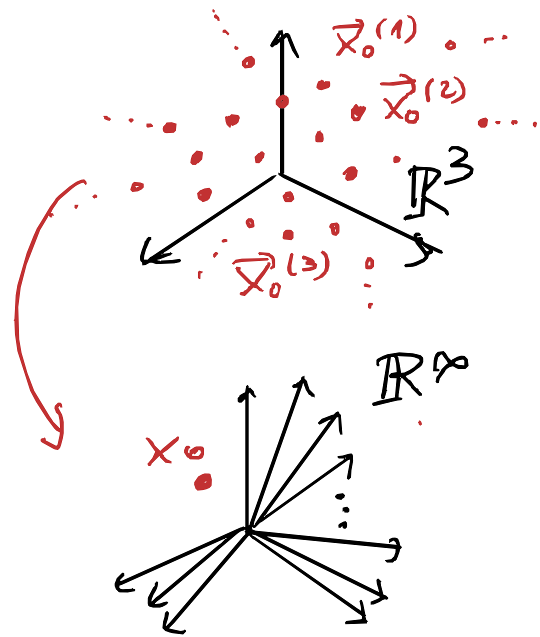
CONFIGURATION SPACE

EXAMPLE 1: N POINT-LIKE BODIES



"PHYSICAL SPACE"

CONFIGURAT. SPACE



$$\mathbb{R}^\infty := \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \ldots$$

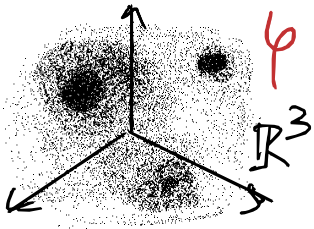
N is FINITE

e.g. Newtonian or Bohmian Mechs.

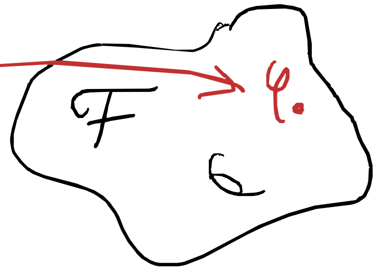
N is COUNTABLY INFINITE

e.g. Thermodynamic lims.

EXAMPLE 2: A SCALAR FIELD



A SCALAR FIELD



FIELD CONFIGURATION SPACE

If separable Hilbert space

$\Downarrow$

$\exists$ COUNTABLE ONB

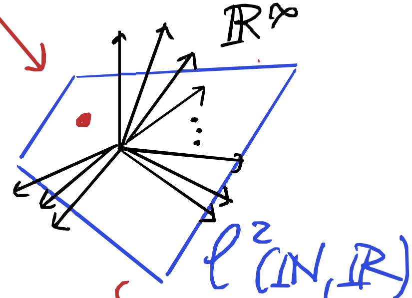
$\{\phi_1, \phi_2, \dots\}$

Each field φ

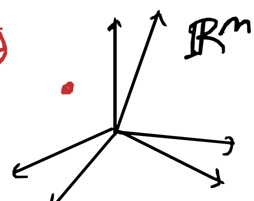
UNIQUE EXPANSION COEFFICIENTS

$(\alpha_1, \alpha_2, \alpha_3, \dots)$:

$$\varphi = \sum_{n=1}^{\infty} \alpha_n \phi_n$$



"UV CUTOFF"



EXPANSION COEFFICIENT SPACE

PILOT-WAVE QM of $m \in \mathbb{N}$ DOFs

- Prescribe (i) Initial pilot-wave $\psi_0 \in L^2(\mathbb{R}^m, d^m x)$

  (ii) Hamiltonian operator $(\hat{H}, D(\hat{H}))$ $\left\{ \hat{U}_t := e^{-\frac{i}{\hbar}\hat{H}t} \right.$ Propagator

- Postulate: dynamical law for pilot-wave: $t \in \mathbb{R}$

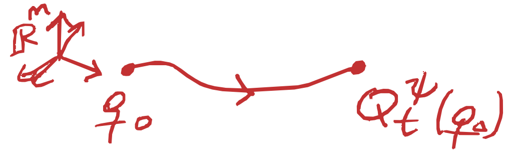
$$\psi_t := \hat{U}_t \psi_0 \iff i\hbar \frac{d\psi_t}{dt} = \hat{H}\psi_t$$

SCHRÖD. EQT.

- Prescribe: (iii) A GUIDANCE LAW:

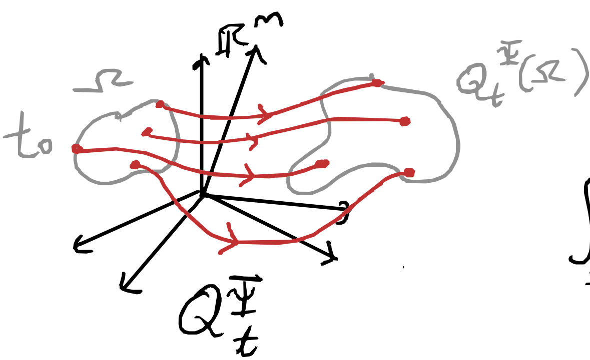
For each $t \in \mathbb{R} \mapsto \psi_t$ a FLOW OF TRAJS.

$Q_t^{\psi}(q_0)$: $q_0 \in \mathbb{R}^m$, $t \in \mathbb{R}$



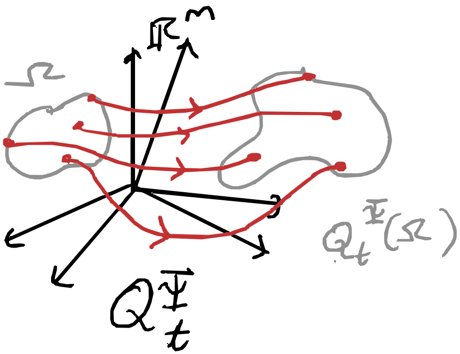
(s.th. $Q_t^{\psi} : \mathbb{R}^m \longrightarrow \mathbb{R}^m$ homeo., $Q_0^{\psi} = Id_{\mathbb{R}^m}$)

EQUIVARIANT with ψ_t's BORN RULE MEAS:



$|\psi_t|^2 d^m x$, i.e.,

$$\int_{q \in \Omega} |\psi_0|^2(q)\, d^m x = \int_{q \in Q_t^{\Psi}(\Omega)} |\psi_t|^2(q)\, d^m x$$



- **POSTULATE:** System follows one of those trajectories & at $t=0$ was $|\psi_0|^2 d^m x$-typical

By Equivar.

Actual conf. of syst. is $|\psi_t|^2 d^m x$-typical $\forall t$

COROLLARY: BORN RULE

OBJECTIVE:

Do the same for COUNTABE ∞ MANY DOFs, $n = |\mathbb{N}|$.

NEED TO CLARIFY:

WHO ARE "$L^2(\mathbb{R}^\infty, d^\infty x)$"

and "$|\psi|^2 d^\infty x$" ?

2. WHY NO TRIVIAL "$L^2(\mathbb{R}^\infty, d^\infty x)$"?

PROP. (Hahn-Kolmog., Oxtoby) (2 ways to understand $d^\infty x$)

(a) $\exists$ multiple candidates of INF. PRODUCT MEASURE

$$d^\infty x\,(E_1 \times \cdots \times E_m \times \mathbb{R} \times \mathbb{R} \times \cdots) = dx(E_1)\cdots dx(E_m)\cdot dx(\mathbb{R})\cdots$$

$\hookrightarrow E_j \subseteq \mathbb{R}$

But all s.th. $d^\infty x\,(\mathcal{U}) = +\infty$ $\forall$ OPEN set $\mathcal{U} \subseteq \mathbb{R}^\infty$.

(b) Any TRANSLATION INVAR. measure $d^\infty\mu$ on $\mathbb{R}^\infty$ is s.th. $d^\infty\mu(\mathcal{U}) = +\infty$ $\forall$ OPEN $\mathcal{U} \subseteq \mathbb{R}^\infty$.

$d^m x$ is a REPLACEABLE MEAS. !

2.

BORN RULE & **EQUIVARIANCE**

(Experimental predictions) ("Ontological" predictions)

$$\underbrace{|\psi|^2 d^m x}_{\text{NOT ALONE!}}$$

- What if there is an equally "flexible" background measure?

PROP. 8: $d^m\mu$ meas. on $\mathbb{R}^m$:

$$\left\{|\psi|^2 d^m\mu\right\}_{\psi\in L^2(\mathbb{R}^m,\, d^m\mu)} = \left\{|\phi|^2 d^m x\right\}_{\phi\in L^2(\mathbb{R}^m,\, d^m x)} \iff d^m\mu \sim d^m x$$

(AGREE on NULL-SETS)

THM. 5: $d^m\mu \sim d^m x$ **iff**

$\exists$ identification $W: L^2(\mathbb{R}^m, d^m x) \longrightarrow L^2(\mathbb{R}^m, d^m\mu)$

s.th. $\psi \longleftrightarrow \tilde{\psi} := W\psi = \psi / \sqrt{RN}$

(i) same empirical predictions (Born rule)

$$\mathbb{P}\left(\begin{smallmatrix} q \in B \\ \text{if } \psi,\, d^m x \end{smallmatrix}\right) = \int_{q \in \Omega} |\psi|^2(q)\, d^m x = \int_{q \in \Omega} |\tilde{\psi}|^2(q)\, d^m\mu = \mathbb{P}\left(\begin{smallmatrix} q \in B \\ \text{if } \tilde{\psi},\, d^m\mu \end{smallmatrix}\right)$$

(ii) same ontological predictions (equivariance constraint)

$$Q^\psi : \quad |\psi|^2 d^m x\text{-EQUIVAR.} \longleftrightarrow |\tilde{\psi}|^2 d^m\mu\text{-EQUIVAR}$$

(iii) Same dynamics for $\psi, \tilde{\psi}$ (a Schrödinger eqt.)

IDEA: Some $d^n\mu$ DO have unique & well-behaved $\lim\limits_{n\to\infty}$

So: $dx \sim d\mu_j \sim d\nu_j \;\; \forall j$. Then,

$$
\begin{tikzcd}
L^2(\mathbb{R}^n, d^n x) \arrow[r, phantom, "\sim"] \arrow[d] \arrow[dr, phantom, "\text{“}n\to\infty\text{”}"] & L^2(\mathbb{R}^n, d^n\mu) \arrow[r, phantom, "\sim"] \arrow[d] \arrow[dr, phantom, "\text{“}n\to\infty\text{”}"] & L^2(\mathbb{R}^n, d^n\nu) \arrow[d] \\
? & L^2(\mathbb{R}^\infty, d^\infty\mu) \arrow[r, phantom, "\overset{?}{\sim}"] & L^2(\mathbb{R}^\infty, d^\infty\nu)
\end{tikzcd}
$$

$\exists$ s.th $d^\infty\mu \perp d^\infty\nu \Longrightarrow$ Literal "$L^2(\mathbb{R}^\infty, d^\infty x)$"

3. INSTEAD: TAKE $\lim_{m\to\infty} \underbrace{L^2(\mathbb{R},dx)\otimes\cdots\otimes L^2(\mathbb{R},dx)}_{m}$

LEM. 11: $L^2(\mathbb{R},dx)\otimes\cdots\otimes L^2(\mathbb{R},dx) \longleftrightarrow L^2(\mathbb{R}^m, d^m x)$

conjugate multilinear forms

$$f_1\otimes\cdots\otimes f_m \longleftrightarrow \left[(x_1,\ldots,x_m)\longmapsto f_1(x_1)\cdots f_m(x_m)\right]$$

classes of functions

CANONICAL IDENTIF.

$$L^2(\mathbb{R},dx)\otimes\cdots\otimes L^2(\mathbb{R},dx) \sim L^2(\mathbb{R}^m, d^m x).$$

"$m\to\infty$"

(Von Neumann 39)

$\exists!\ \underset{\text{JEDN}}{\otimes}\ L^2(\mathbb{R},dx)$ $\qquad$ $L^2(\mathbb{R}^\infty, d\mu_1)\ \ldots\ L^2(\mathbb{R}^\infty, d\mu_k)$

von Neumann's $\bigotimes_{j\in\mathbb{N}} L^2(\mathbb{R}, dx)$

- $\Psi \in \bigotimes_{j\in\mathbb{N}} L^2(\mathbb{R}, dx)$ is a conjugate multi-linear form: takes $(\phi_1, \phi_2, \dots) \longmapsto \alpha \in \mathbb{C}$

- For each $(\psi_1, \psi_2, \dots) \subset L^2(\mathbb{R}, dx)$ would like to def. $\psi_1 \otimes \psi_2 \otimes \cdots$ as in finite $\otimes$:

$$(\psi_1 \otimes \psi_2 \otimes \cdots)(\phi_1, \phi_2, \dots) := \langle \psi_1, \phi_1 \rangle \langle \psi_2, \phi_2 \rangle \cdots$$

s.th. $\langle \psi_1 \otimes \psi_2 \otimes \cdots, \phi_1 \otimes \phi_2 \otimes \cdots \rangle :=$

INNER PRODUCT

- $\|\psi_1 \otimes \psi_2 \otimes \cdots\| := \|\psi_1\| \cdot \|\psi_2\| \cdots$

only makes sense if $\|\psi_1\| \cdot \|\psi_2\| \cdots < \infty$

$\Rightarrow$ such $(\psi_1, \psi_2, \dots)$ called C-SEQUENCES

- Def. Idea: $\bigotimes_{k \in \mathbb{N}} L^2(\mathbb{R}, dx) = \overline{\left\{ \sum_{\ell=1}^{m} c_\ell \, \psi_1^\ell \otimes \psi_2^\ell \otimes \cdots \right\}}^{\|\cdot\|}$

All linear combs.

$\Downarrow$

NON-SEPARABLE (ONBs are UNCOUNTABLE)

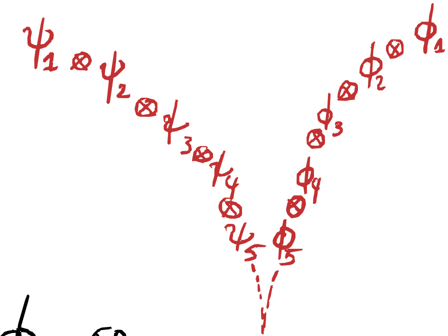
- But von Neumann gave nice "DISSECTION"

$$\psi_1 \otimes \psi_2 \otimes \psi_3 \otimes \psi_4 \otimes \cdots$$

$$\updownarrow \qquad \updownarrow \qquad \updownarrow \qquad \updownarrow$$

$$\phi_1 \otimes \phi_2 \otimes \phi_3 \otimes \phi_4 \otimes \cdots$$

$\underline{\langle \psi_j, \phi_j \rangle} \longrightarrow 1$



- "EQUIVALENT TAILS" RELATION: $\psi_1 \otimes \psi_2 \otimes \cdots \sim \phi_1 \otimes \phi_2 \otimes \cdots$

$$\Updownarrow$$

$$\sum_{j=1}^{\infty} \text{distance}\left(1, \langle \psi_j, \phi_j \rangle\right) \text{ CONVERGES}$$

(von Neumann 39)

- Set of equivalence classes Γ_{tails} ← UNCOUNTABLE!

For each $\mathcal{E} \in \Gamma_{\text{tails}}$:

- $\exists\, \varphi_1^{\mathcal{E}} \otimes \varphi_2^{\mathcal{E}} \otimes \cdots \in \mathcal{E}$: $\|\varphi_j^{\mathcal{E}}\| = 1$ $\Big\}$ call each a GENERATOR of $\mathcal{E}$

- $$\text{closure}\left\{ \sum_{\ell=1}^{m} c_\ell\, \psi_1^\ell \otimes \cdots \otimes \psi_N^\ell \otimes \underline{\varphi_{N+1}^{\mathcal{E}} \otimes \varphi_{N+2}^{\mathcal{E}} \otimes \cdots} \right\}$$

$\|$ FIXED TAIL EQUAL to φ_j's.

$$\text{closure}\left\{ \sum_{\ell=1}^{m} c_\ell\, \psi_1^\ell \otimes \psi_2^\ell \otimes \cdots \;\middle|\; \psi_1^\ell \otimes \psi_2^\ell \otimes \cdots \in \mathcal{E} \right\}$$

Any element in tail class

$\|$

The $\mathcal{E}$-th LAYER of the ITP $\leftarrow$ Why? $\Rightarrow$

3.

ε-th LAYER $:= \left\{ \begin{array}{l} \text{Finite \& } \infty \text{ linear combs.} \\ \text{of } \underline{\text{FIXED TAIL}} \text{ tensors} \end{array} \right\}$

$$\psi_1 \otimes \cdots \otimes \psi_N \otimes \underline{\varphi^{\varepsilon}_{N+1} \otimes \varphi^{\varepsilon}_{N+2} \otimes \cdots}$$

- Because:

(i) Layers <u>PAIRWISE ORTHOGONAL</u> and

(ii) $$\bigotimes_{j \in \mathbb{N}} L^2(\mathbb{R}, dx) = \bigoplus_{\varepsilon \in \Gamma_{\text{tails}}} (\varepsilon\text{-th layer})$$

$$\Psi = (\Psi_\varepsilon)_{\varepsilon \in \Gamma_{\text{tails}}}$$

$\to$ <u>UNCOUNTABLE</u> tuple!

$\hookrightarrow$ Each layer is <u>SEPARABLE</u>! (von Neumann 39)

(COUNTABLE ONB).

- **Issue** of $\bigotimes_{j \in \mathbb{N}} L^2(\mathbb{R}, dx)$

for pilot wave theory:

Elements are conjugate-multi-linear forms

... NOWHERE $\mathbb{R}^\infty$ / config. space!

↳ NO WAVEFUNCTION over $\mathbb{R}^\infty$!

↳ NO BORN RULE MEAS. on $\mathbb{R}^\infty$!

11. Joint Diagonalization of all Position Operators

- State-space $\mathcal{H}$, commuting s.a. ops. $\{\hat{A}_1, \dots, \hat{A}_m\}$

Physicist $\Big\{$ $\exists$ "basis" $|a_1, \dots, a_m\rangle$: $\Psi \in \mathcal{H}$

$$\text{"}\ \Psi = \int_{\mathbb{R}^m} \psi(a_1, \dots, a_m)\, |a_1, \dots, a_m\rangle\, d^m a\ \text{"}$$

$\hookrightarrow$ JOINT $(A_1, \dots, A_m)$–REPRESENTAT. of Ψ

$$\text{"}\ \hat{A}_k = \int_{\mathbb{R}^m} a_k\, |a_1, \dots, a_m\rangle\langle a_1, \dots, a_m|\, d^m a\ \text{"}$$

DIAGONALIZED $\hat{A}_k$!

s.th. $$\text{"}\ \hat{A}_k \Psi = \int_{\mathbb{R}^m} a_k \psi(a_1, \dots, a_m)\, |a_1, \dots, a_m\rangle\, d^m a\ \text{"}$$

- If $m=1$, $\hat{A}$ = Hamitonian

$\Rightarrow$ " $\Psi = \int_{\mathbb{R}} \varphi(E)\,|E\rangle\, dE$ " $\left\{ \varphi(E) \right.$ is ENERGY REP. of Ψ

- If $\hat{A}_j = \hat{p}_j$, $j \in \{1, \dots, m\}$ MOMENTUM OPS.

$\Rightarrow$ " $\Psi = \int_{\mathbb{R}^m} \phi(p_1, \dots, p_m)\,|p_1, \dots, p_m\rangle\, d^m p$ ". $\left\{ \phi(p_1, \dots, p_m) \right.$ is MOMENT. REP.

- If $\hat{A}_j = \hat{q}_j$, $j \in \{1, \dots, m\}$ POSITION OPS.

$\Rightarrow$ " $\Psi = \int_{\mathbb{R}^m} \psi(q_1, \dots, q_m)\,|q_1, \dots, q_m\rangle\, d^m q$ "

is the CONFIGURATION REP.

- GENERALIZED BORN RULE:

"joint quantum measurement" of $\hat{A}_1, \dots, \hat{A}_m$ has:

$$\text{“}\mathbb{P}((a_1, \dots, a_m) \in \Omega)\text{”} = \int_\Omega |\psi|^2 (a_1, \dots, a_m)\, d^m a$$

If $\hat{A}_j = \hat{q}_j$ POSITION OPS.

$$\text{“}\mathbb{P}(q \in \Omega) = \int_\Omega |\psi|^2 (q_1, \dots, q_m)\, d^m q\text{”}$$

USUAL BORN RULE in CONFIG. SPACE

IDEA: $\mathcal{H} := \bigotimes_{j\in\mathbb{N}} L^2(\mathbb{R}, dx)$

$\hat{A}_j$ ↷ $\hat{q}_j := \underbrace{\mathrm{Id}\otimes\cdots\otimes\mathrm{Id}\otimes\hat{q}}_{j}\otimes\mathrm{Id}\otimes\cdots\otimes\mathrm{Id}$

j-th POSITION OP.

i.e.,

$$\hat{q}_j\left(\psi_1\otimes\psi_2\otimes\cdots\right) = \psi_1\otimes\cdots\otimes(\hat{q}\psi_j)\otimes\psi_{j+1}\otimes\cdots$$

- $\{\hat{q}_1, \hat{q}_2, \hat{q}_3, \dots\}$ commute $\Rightarrow$ Physicist:

$\exists$ JOINT DIAGONALIZATION: $\forall\, \Psi \in \bigotimes_{j\in\mathbb{N}} L^2(\mathbb{R}, dx)$

$$\text{“}\Psi = \int_{q\in\mathbb{R}^\infty} \psi(q_1, q_2, \dots)\,|q_1, q_2, \dots\rangle\, d^\infty q\text{”}.$$

“$L^2(\mathbb{R}^\infty, d^\infty x)$”

- Mathematically seriously : use

JOINT SPECTRAL THEOREM (multiplic. op. version)

↪ Important details omitted in physics: ⚠

(i) ~~commuting~~ ⟹ STRONGLY commuting

(ii) ∃ Isomorphism $\cancel{\mathcal{H} \longleftrightarrow L^2(\mathbb{R}^m, d^m a)}$

$\cancel{\Psi \longleftrightarrow \psi(a_1, \dots, a_m)}$

$$\mathcal{H} \longleftrightarrow \bigoplus_{k \in \mathcal{K}} L^2(\mathbb{R}^m, da_k)$$

$$\Psi \longleftrightarrow \big(\psi_k(a_1, \dots, a_m)\big)_{k \in \mathcal{K}}$$

(& NON UNIQUE!)

A tuple! Like a SPINOR!

- $|\mathfrak{X}| = 1$ possible only if MULTIPLICITY of JOINT SPECTRUM 1 (simple)

e.g. $\hat{q}_1, \dots, \hat{q}_n$ in $L^2(\mathbb{R}, dx) \otimes \dots \otimes L^2(\mathbb{R}, dx)$

But in general even $|\mathfrak{X}| = \dim(\mathcal{H})$!

- Moreover, joint spectral thm. only well-known for SEPARABLE $\mathcal{H}$

& finite # of ops.

But, $\bigotimes_{j \in \mathbb{N}} L^2(\mathbb{R}, dx)$ NON SEPAR. & $\{\hat{q}_1, \hat{q}_2, \dots\}$ ∞ !

- **THEOREM 19 of Master Thesis:**

  JOINT SPECTRAL THM. for arbitrarily many strongly commuting s.a. ops. on arbitrary non-separable $\mathcal{H}$

- In Prop. 38 explicit spectral PVMs and functional calculi of $\{\hat{q}_j\}_{j\in\mathbb{N}}$

$\Rightarrow$ PVMs commute $\Rightarrow$ can apply Thm. 19

$\Rightarrow$ Explicit joint PVM & functional calc. & associated spectral measures!

Thm. 20: The Configuration Representation of $\bigotimes_{j\in\mathbb{N}} L^2(\mathbb{R}, dx)$

(i) Choose $\mathcal{R} := \{ \varphi_1^{\varepsilon} \otimes \varphi_2^{\varepsilon} \otimes \cdots \}_{\varepsilon \in \Gamma_{\text{tails}}}$:

$\varphi_1^{\varepsilon} \otimes \varphi_2^{\varepsilon} \otimes \cdots$ generator of ε-th layer

s.th. $\varphi_j^{\varepsilon}(x) \neq 0$ a.e. $x \in \mathbb{R}$

call it a Configuration Representat. basis.

(ii) Def. associated BACKGROUND MEASURES on $\mathbb{R}^{\infty}$

$$d\mu_{\varepsilon} := |\varphi_1^{\varepsilon}|^2 \cdot dx_1 \odot |\varphi_2^{\varepsilon}|^2 dx_2 \odot \cdots$$

(iii) For each $\mathcal{E}$-layer, $\exists!$ isomorph.: 4.

$$\left(\mathcal{E}\text{-th LAYER}\right) \xleftrightarrow{W_{\mathcal{E}}^{R}} L^2(\mathbb{R}^\infty, d\mu_{\mathcal{E}})$$

$$\psi_1 \otimes \cdots \otimes \psi_m \otimes \underline{\varphi_{m+1}^{\mathcal{E}} \otimes \varphi_{m+2}^{\mathcal{E}} \otimes \cdots} \longleftrightarrow \frac{\psi_1(q_1)}{\varphi_1^{\mathcal{E}}(q_1)} \cdots \frac{\psi_m(q_m)}{\varphi_m^{\mathcal{E}}(q_m)}$$

and thus $\exists$ isom.

$$\bigotimes_{j\in\mathbb{N}} L^2(\mathbb{R}, dx) = \bigoplus_{\mathcal{E}\in\Gamma} \left(\begin{matrix}\mathcal{E}\text{-th}\\ \text{layer}\end{matrix}\right) \xleftrightarrow{W_R} \bigoplus_{\mathcal{E}\in\Gamma} L^2(\mathbb{R}^\infty, d\mu_{\mathcal{E}})$$

$$\Psi = (\Psi_{\mathcal{E}})_{\mathcal{E}\in\Gamma_{\text{tails}}} \longleftrightarrow \left(\psi_{\mathcal{E}}(q_1, q_2, \ldots)\right)_{\mathcal{E}\in\Gamma_{\text{tails}}}$$

A tuple of wavefuncts. on $\mathbb{R}^\infty$!

(iv) $\mathcal{W}_R$ gives the JOINT DIAGONALIZ. of all POSITION OPS. $\{\hat{q}_1, \hat{q}_2, \dots\}$:

So, this is "$L^2(\mathbb{R}^\infty, d^\infty x)$"!

$$\hat{q}_k \Psi \longleftrightarrow \left(q_k \cdot \psi_c(q_1, q_2, \dots) \right)_{c \in \Gamma_{tails}}$$

$\Rightarrow$ the BORN RULE measure of Ψ: $\Omega \subseteq \mathbb{R}^\infty$

$$\mathbb{P}^\Psi(q \in \Omega) = \sum_{c \in \Gamma_{tails}} \int_{q \in \Omega} |\psi_c|^2(q_1, q_2, \dots)\, d\mu_c$$

which is independ. of chosen R.

- It is VERY ANALOGOUS to SPINORS: choose spinor basis $\mathcal{B}$:

$$\begin{array}{ccc} \text{ABSTRACT SPINOR} & & \text{POSITION REP.} \\ \Psi & \xleftrightarrow{\ \mathcal{B}\ } & \begin{pmatrix}\psi_1(q_1,q_2,q_3)\\ \psi_2(q_1,q_2,q_3)\end{pmatrix} \end{array}$$

BORN RULE: $$\mathbb{P}^{\Psi}(\Omega) = \int_{q\in\Omega} |\psi_1|^2(q)\, d^3q + \int_{q\in\Omega} |\psi_2|^2(q)\, d^3q$$

$\Omega \subseteq \mathbb{R}^3$

- Hence, $\Psi \in \bigotimes_{j\in\mathbb{N}} L^2(\mathbb{R}, dx)$ is like a SPINOR of UNCOUNTABLY many wavefunctions on $\mathbb{R}^\infty$.

- Recall: diagonalizat. space is NOT unique!

$\Rightarrow$ COULD $\exists$ another spectral diagonalization with finitely or countably many wavefunctions?

- No... THM. 20.(iv): The SPECTRAL MULT. of JOINT PVM is UNCOUNTABLE!

5. TEMPLATE for RIGOROUS PILOT-WAVE THEORIES on $\mathbb{R}^\infty$

- Prescribe

(i) $\Psi_0 \in \bigotimes_{j\in\mathbb{N}} L^2(\mathbb{R}, dx) \longleftrightarrow \left(\Psi_0^{\mathcal{C}}(q_1, q_2, \dots)\right)_{\mathcal{C}\in\Gamma_{\text{tails}}}$

Initial pilot-wave

(ii) Hamiltonian operator $(\hat{H}, D(\hat{H}))$ $\left\{ \hat{U}_t := e^{-\frac{i}{\hbar}\hat{H}t} \right.$ Propagator

- Postulate: dynamical law for pilot-wave: $t \in \mathbb{R}$

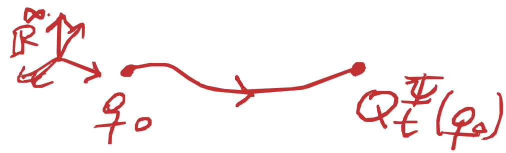
$$\Psi_t := \hat{U}_t \Psi_0 \iff i\hbar \frac{d\Psi_t}{dt} = \hat{H}\Psi_t$$

SCHRÖD. EQT.

- Prescribe: (iii) A GUIDANCE LAW:

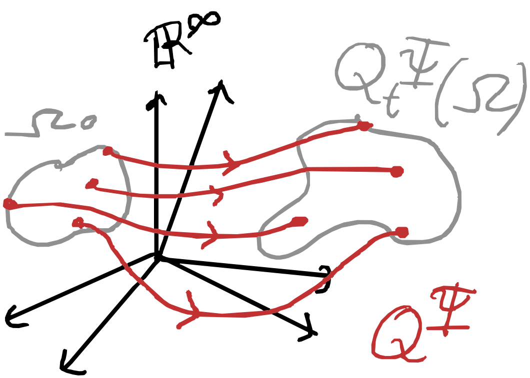
For each $t \in \mathbb{R} \mapsto \Psi_t$ a FLOW OF TRAJS.

$$Q_t^\Psi(q_0) : \quad q_0 \in \mathbb{R}^\infty, \; t \in \mathbb{R}$$



(s.th. $Q_t^\Psi : \mathbb{R}^\infty \to \mathbb{R}^\infty$ homeo., $Q_0^\Psi = Id_{\mathbb{R}^\infty}$)

EQUIVARIANT with Ψ_t's BORN Rule meas.:



$$\sum_{\mathcal{E} \in \Gamma_{tails}} \int_{q \in \Omega} |\psi_0^{\mathcal{E}}|^2(q) \, d\mu_{\mathcal{E}} =$$

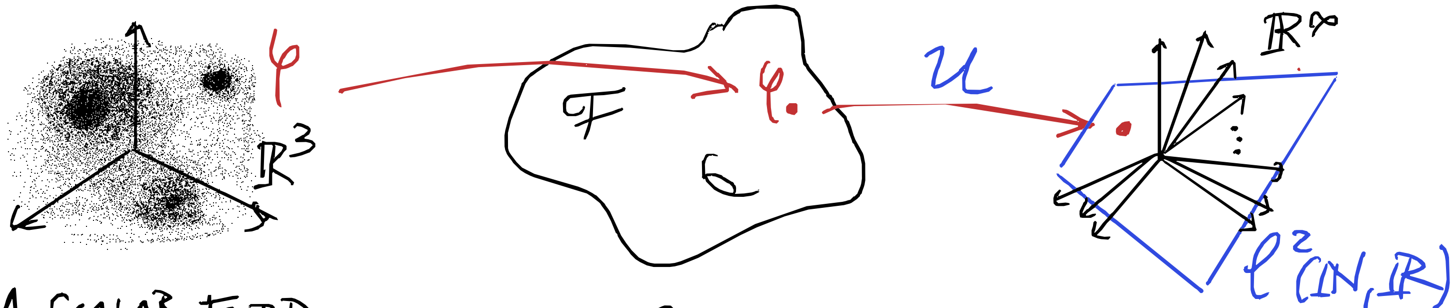
$$= \sum_{\mathcal{E} \in \Gamma_{tails}} \int_{q \in Q_t^\Psi(\Omega)} |\psi_t^{\mathcal{E}}|^2(q) \, d\mu_{\mathcal{E}}$$

- **Postulate:** system follows a **DETERMINISTIC trajectory** $t \in \mathbb{R} \longmapsto Q_t^{\Psi}(q_0)$ with **initial config.** $q_0 \in \mathbb{R}^{\infty}$ unknown to us but **"$|\Psi_0|^2 d^{\infty}x$"-typical**

COROLLARY: **BORN RULE**, $\Omega \subseteq \mathbb{R}^{\infty}$,

$$\mathbb{P}(q \in \Omega \text{ if } \Psi_t) = \sum_{e \in \Gamma_{\text{tails}}} \int_{q \in \Omega} |\psi_t^e|^2(q)\, d\mu_e$$

6. Rigorous Wavefunctionals



A Scalar Field

Field Configuration Space

Expansion Coeff. Space

- ONB

$$(\varphi_n)_{n\in\mathbb{N}} \subset \mathcal{F}$$

$$\mathcal{U} : \mathcal{F} \longrightarrow \ell^2(\mathbb{N},\mathbb{R})$$

$$\varphi = \sum_{n=1}^{\infty} \alpha_n \varphi_n \longmapsto (\alpha_1, \alpha_2, \dots)$$

- Can we do: "$L^2(\cancel{\mathbb{R}^\infty}, d^\infty x)$" ?

$\ell^2(\mathbb{N},\mathbb{R})$

$$\Psi(\alpha_1, \alpha_2, \dots)$$

$$\rightsquigarrow$$

$$\Psi\Big(\sum_{n=1}^{\infty} \alpha_n \varphi_n\Big)$$

Cor. 17: For each $\mathfrak{C} \in \Gamma_{\text{tails}}$

- either $d\mu_{\mathfrak{C}}(\ell^2) = d\mu_{\mathfrak{C}}(\mathbb{R}^\infty)$ | FULL MEASURE

  $\Longrightarrow$ trivially $L^2(\mathbb{R}^\infty, d\mu_{\mathfrak{C}}) \cong L^2(\ell^2, d\mu_{\mathfrak{C}})$

- or $d\mu_{\mathfrak{C}}(\ell^2) = 0 \Longrightarrow L^2(\ell^2, d\mu_{\mathfrak{C}}) = \{0\}$

  $\Longrightarrow$ can ignore the layer

independently of basis $R = \left(\bigotimes_{j \in \mathbb{N}} \Upsilon_j^{\mathfrak{C}} \right)_{\mathfrak{C} \in \Gamma_{\text{tails}}}$.

- Layers to Keep:

$$\Gamma_{\ell^2} := \left\{ \mathcal{E} \in \Gamma_{tails} \;\middle|\; d\mu_{\mathcal{E}}(\ell^2) = d\mu_{\mathcal{E}}(\mathbb{R}^\infty) \right\}$$

- Then,

$$``L^2(\ell^2, d^\infty x)" := \bigoplus_{\mathcal{E}\in\Gamma_{\ell^2}} \begin{pmatrix} \mathcal{E}\text{-th layer} \\ \text{of } \bigotimes_{j\in\mathbb{N}} L^2(\mathbb{R}, dx) \end{pmatrix} =$$

$$\overset{R}{=} \bigoplus_{\mathcal{E}\in\Gamma_{\ell^2}} L^2(\ell^2, d\mu_{\mathcal{E}})$$

- So,

$$\Psi \in ``L^2(\ell^2, d^\infty x)" \longleftrightarrow \left(\Psi^{\mathcal{E}}(\alpha_1, \alpha_2, \dots)\right)_{\mathcal{E}\in\Gamma_{\ell^2}}$$

$$\Psi_\varepsilon \in L^2(\ell^2, d\mu_\varepsilon)$$

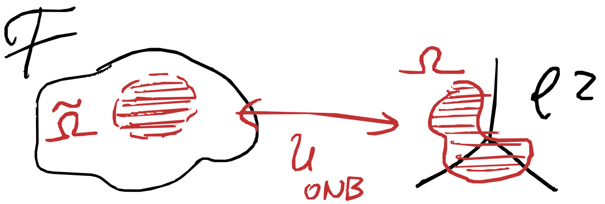
$$(\alpha_1, \alpha_2, \dots) \in \ell^2 \longmapsto \Psi_\varepsilon(\alpha_1, \alpha_2, \dots)$$

$\mathcal{U} \in$ UNITARY!

Pullback arguments to $\mathcal{F}$

$$\varphi \in \mathcal{F} \longmapsto \Phi_\varepsilon(\varphi) := \Psi_\varepsilon(\langle \varphi_1, \varphi \rangle, \langle \varphi_2, \varphi \rangle, \dots)$$

$$\Phi_\varepsilon \in L^2(\mathcal{F}, d\tilde{\mu}_\varepsilon)$$



$$d\tilde{\mu}_\varepsilon(\tilde{\Omega}) := d\mu_\varepsilon(\Omega)$$

- ONB $(\varphi_n)_{n \in \mathbb{N}} \subset \mathcal{F}$

$$\mathcal{U} : \mathcal{F} \longrightarrow \ell^2(\mathbb{N}, \mathbb{R})$$

$$\varphi = \sum_{n=1}^{\infty} \alpha_n \varphi_n \longmapsto (\alpha_1, \alpha_2, \dots)$$

WHY SEEMS CORRECT PATH?

(i) Position Ops. become Literal Field Ops.

$$\hat{q}_\varphi := \sum_{m=1}^{\infty} \langle \varphi_m, \varphi \rangle \, \hat{q}_m \quad \xleftrightarrow{\;\mathcal{U}_\varepsilon\;} \quad \hat{\Phi}_\varphi$$

of $L^2(\mathbb{R}^\infty, d\mu_\varepsilon)$!

$$\left(\hat{q}_\varphi \Psi\right)(\alpha) = \sum_{m=1}^{\infty} \langle \varphi_m, \varphi \rangle \, \alpha_m \, \Psi(\alpha)$$

Prop. 41

$\mathcal{U}_\varepsilon$

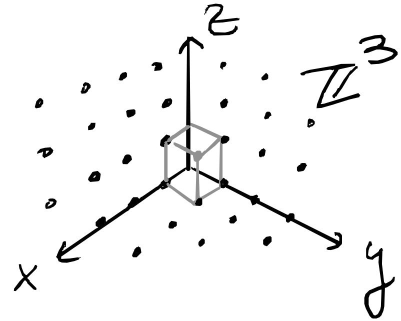
$$\left(\hat{\Phi}_\varphi \Phi\right)(\phi) = \langle \varphi, \phi \rangle \cdot \Phi(\phi)$$

- $\{\hat{\Phi}_\varphi\}_{\varphi \in \mathcal{F}}$ are self-adjoint: the Field OP.s.

EXAMPLE 1: If SPACE DISCRETE

$$\mathcal{F} = L^2(\mathbb{Z}^3, \mathbb{R}, d\nu)$$

SCALAR FIELDS



$\Rightarrow \quad \forall x \in \mathbb{Z}^3 \quad \delta_x \in \mathcal{F}$ (DIRAC DELTA is "legal")

$$\left(\hat{\Phi}_{\delta_x} \Phi\right)(\phi) = \phi(x) \cdot \Phi(\phi)$$

just like: $(\hat{q}_j \psi)(q) = q_j \psi(q)$

EXAMPLE 2: If SPACE CONTINOUS

$$\mathcal{F} = L^2(\mathbb{R}^3, \mathbb{R}, d^3x)$$ SCALAR FIELDS $\forall \varphi \in \mathcal{F}$,

$$\left(\hat{\Phi}_\varphi \, \Phi\right)(\phi) = \int_{x\in\mathbb{R}^3} \varphi(x)\cdot \underbrace{\phi(x)\, \Phi(\phi)}_{=:\left(\hat{\Phi}_x \Phi\right)(\phi)} \, d^3x =$$

$$= \left(\int_{x\in\mathbb{R}^3} \varphi(x)\, \hat{\Phi}_x \, d^3x\right) \Phi(\phi)$$

LITERAL SMEARED FIELD OP.

or OP.-VALUED DISTRIBUTION

(ii) MOMENTUM OPs. BECOME LITERAL FUNCTIONAL DERIVATIVES. 6.[redacted]

$$\hat{P}_\varphi := \sum_{m=1}^{\infty} \langle \varphi_m, \varphi \rangle \hat{P}_m \xleftrightarrow{\mathcal{U}_\varepsilon} \hat{\pi}_\varphi$$

of $L^2(\mathbb{R}^\infty, d\mu_\varepsilon)$

$$(\hat{P}_\varphi \Psi)(\alpha) = \sum_{m=1}^{\infty} \langle \varphi_m, \varphi \rangle (-i\hbar \partial_m) \Psi(\alpha) + G^\varepsilon \Psi(\alpha)$$

$G^\varepsilon :=$ "Gauge multiplier"

$\mathcal{U}_\varepsilon$

$$(\hat{\pi}_\varphi \Phi)(\phi) = -i\hbar \frac{d}{ds} \Phi(\phi + s\varphi)\Big|_{s=0} + \tilde{G}^\varepsilon \Phi(\phi)$$

DIRECTIONAL DER. in φ's DIRECTION

$$=: -i\hbar \left\langle \frac{\delta \Phi}{\delta \psi}\Big|_\phi, \varphi \right\rangle$$

41

$$\hat{P}_\varphi := \sum_{k=1}^{\infty} \beta_k \hat{P}_k$$

$D f|_\varphi \eta = \frac{d}{dt} f(\varphi + t\eta)_{t=0} = \langle \frac{\delta f}{\delta \varphi}|\varphi, \eta \rangle$

$\mathcal{F}\phi \quad \mathcal{F}_n(\phi \ldots) \quad (\mathcal{F}_1 \otimes \mathcal{F}_2 \otimes \cdots) q_n$

$\mathcal{F}_\mu P_\mu = q_\mu \mathcal{F}_\mu$

$\langle \frac{\delta \Phi}{\delta \phi}\phi, \varphi \rangle$

$\hat{P} \mathcal{F}$.

$\left(\sum_{k=1}^{\infty} \beta_k \hat{q}_k\right) \mathcal{F}$

$$\hat{P}_{L^2}(\varphi)(\cdot)(x) = \sum_{m=1}^{N} \beta_m (-i\hbar)\left(\partial_m - \frac{\partial_m \rho_m^\varepsilon(x_m)}{\rho_m^\varepsilon(x_m)}\right)$$

$\mathcal{M}$

En $\psi_1 \cdots \psi_N$ ke sou dense

$$\partial_m \text{Apoly} - \frac{\partial_m \rho_m}{\rho_m} \text{Apoly}$$

Dense / denda

$$\sum^N \beta_m \partial_m \psi(x_1 x_2 \ldots) - \frac{\partial_m \rho_m}{\rho_m} \psi(x_1, x_2 \ldots)$$

$$\sum^N \beta_m \lim_{h \to 0} \frac{\psi(x_m + h) - \psi(x_m)}{h} - \frac{\partial_m \rho_m}{\rho_m} \psi(x)$$

$$\lim_{h \to 0} \sum^N \beta_m \frac{\psi(x_m + h) - \psi(x_m)}{h} - \frac{\partial_m \rho_m}{\rho_m} \psi(x)$$

$\|$

$$\lim_{h \to 0} \sum^N \beta_m \frac{\Psi(\Sigma(\alpha_m + h_m)\phi_m) - \Psi(\Sigma \alpha_m \phi_m)}{h}$$

$$\phi + h_m \varphi_m \quad \frac{\Psi(\phi + h \varphi_i) - \Psi(\phi)}{h}$$

$$\sum_{k=1}^{N} \beta_m$$

$$\sum_{n=1}^{N} \beta_m \begin{smallmatrix}\text{dir}\\\text{der. in}\\\text{dir } \phi_m\end{smallmatrix} \Psi(\phi) = \text{dir. der in dir}$$

Linty of dir deriut. $\Sigma \beta_m \varphi_m$

φ

$$\langle \varphi, \varphi_m \rangle = \int_{\phi \in \text{Tal}} \overline{\varphi(x)} \varphi_m(x) d^3x$$

$= \text{Dir der in } \varphi_m \Psi(\phi)$

$\|$

$\partial_m \psi(x)$

$$\langle \nabla f, v \rangle = D_v f = df(v)$$

$$\psi = \sum_{n=1}^{\infty} \partial_m \psi \cdot \varphi_m$$

$\int \nabla f v$

$\|$

$\langle \nabla f, v \rangle$

$\|$

$D f v$

$\|$

lin. Tal

$$\langle \psi, \Psi \rangle$$

(iii) They satisfy the CCR

6. [redacted]

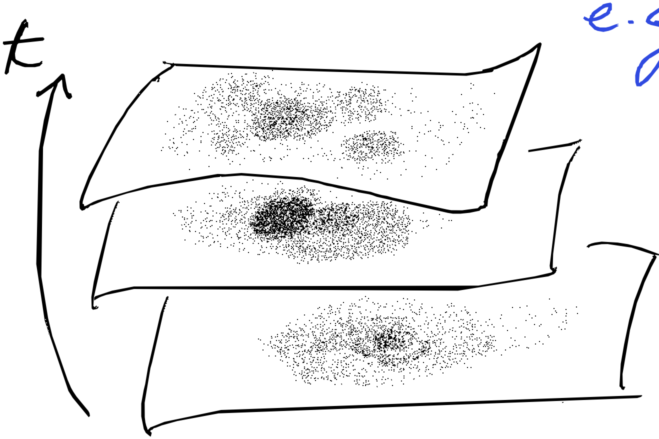
$$[\hat{\Phi}_\varphi, \hat{\pi}_\psi] = i\hbar \langle \varphi, \psi \rangle$$

$$[\hat{\Phi}_\varphi, \hat{\Phi}_\psi] = 0 = [\hat{\pi}_\varphi, \hat{\pi}_\psi]$$

(In Preparation ...)

OUTLOOK: $\mathcal{F}$ can be tensor-valued fields

e.g., asymptotically decaying perturbations of background spacetime metric in each leaf



$\Rightarrow$ wavefunctionals over asympt. Euclidean metrics

7. ITP, Fock Space & Haag

THMS. 29, 30: $\{\varphi_m\}_{m\in\mathbb{N}} \subseteq \mathcal{F}$ ONB, $E := \operatorname{span}\{\varphi_m\}_{m\in\mathbb{N}}$

def. now on full $\bigotimes_{d\in\mathbb{N}} L^2(\mathbb{R}, dx)$:

$$\hat{q}_\varphi := \text{"}\sum_{m=1}^{\infty}\text{"} \langle \varphi_m, \varphi \rangle \hat{q}_m \,, \quad \hat{p}_\varphi := \text{"}\sum_{m=1}^{\infty}\text{"} \langle \varphi_m, \varphi \rangle \hat{p}_m$$

Then, $\{\hat{q}_\varphi, \hat{p}_\varphi\}_{\varphi\in E}$ is Heisenberg CCR rep.

$\{e^{i\hat{q}_\varphi}, e^{i\hat{p}_\varphi}\}_{\varphi\in E}$ is Weyl CCR rep.

- Def. in $L^2(\mathbb{R}, dx)$:

$$\phi_m(x) := c_m\, h_m(x)\, e^{-x^2/2}$$

Hermite funct. ONB = Eigenfuncts of Harmonic oscillator

- Ladder ops. $\hat{a}, \hat{a}^+$, lifted:

$$\hat{a}_j := \underbrace{Id \otimes \cdots \otimes Id \otimes \hat{a}}_{j} \otimes Id \otimes \cdots$$

THMS. 31, PROP. 78:

$$\hat{a}_\varphi := \text{``}\sum_{n=1}^{\infty}\text{''} \langle \varphi_m, \varphi \rangle\, \hat{a}_m\,, \quad \varphi \in E_{\mathbb{C}}$$

Then,

$\{\hat{a}_\varphi, \hat{a}_\varphi^*\}_{\varphi \in E_{\mathbb{C}}}$ is CREAT-ANNIH. CCR rep.

THM. 35: All reps. are REDUCED by every layer $\mathcal{C} \in \Gamma$ & reduced parts are IRREDUCIBLE representations!

- Moreover:

$$\left(\begin{array}{c} \mathcal{C}, \mathcal{D} \in \Gamma \\ \text{IRREPS are} \\ \text{EQUIVALENT} \end{array} \right) \Leftrightarrow \left(\begin{array}{l} \mathcal{C} \overset{q}{\sim} \mathcal{D}, \text{ i.e. } \exists (\theta_j)_{j \in \mathbb{N}} \subset [-\pi, \pi): \\ f_1 \otimes f_2 \otimes \cdots \in \mathcal{C},\ g_1 \otimes g_2 \otimes \cdots \in \mathcal{D} \\ f_1 \otimes f_2 \otimes \cdots \sim (e^{i\theta_1} g_1) \otimes (e^{i\theta_2} g_2) \otimes \cdots \end{array} \right)$$

(Klauder et al. 66)

$\rightsquigarrow$ i.e., layers that differ in factor-wise global phases e.g. $f_1 \otimes f_2 \otimes \cdots$ & $(-f_1) \otimes (-f_2) \otimes \cdots$

DEF59: $(\mathcal{H}, \mathcal{D}, \{\hat{A}(f)\}_{f\in\mathcal{V}})$ arb. creat-annih. CCR rep., def. **QUANTUM VACUUM:**

unit $\Omega \in \mathcal{H}$, cyclic for $\{\hat{A}(f), \hat{A}(f)^*\}_{f\in\mathcal{V}}$

($\mathcal{H}_\Omega := \mathrm{span}\{\Omega, \hat{c}(f_1)^{\#_1} \cdots \hat{c}(f_m)^{\#_m}\Omega\}$ DENSE

— states finite bosons away from Ω —)

(a) If $\hat{c}(f)\Omega = 0 \ \forall f \in \mathcal{V} \Rightarrow$ EMPTY Q. VACUUM

(b) If $\exists \tilde{\Omega} \in \mathcal{H}_\Omega$ empty $\Rightarrow \Omega$ ALMOST EMPTY QV

(c) Else Ω is MYRIOTIC Q. VACUUM

THM. 40: $\forall$ LAYER $\mathcal{E}$, every generator 7.

$\bigotimes_{j\in\mathbb{N}} \mathcal{J}_j^{\mathcal{E}}$ is a Quantum VACUUM of irrep

$$K(\mathcal{E}) := \{\hat{a}_\varphi|_\mathcal{E}, \hat{a}_\varphi|_\mathcal{E}^*\}_{\mathcal{E}\in\Gamma_{tails}}.$$

Def.: $\phi_0(x) := \frac{1}{\sqrt[4]{\pi}} e^{-x^2/2}$ & $\Psi_G := \phi_0 \otimes \phi_0 \otimes \cdots \in \mathcal{E}_G$

$\mathcal{E}_G$ (layer of harm. osc. GROUND STATE

THM. 40: (Using Arai 2019)

$\Big($ $K(\mathcal{E})$ is EQUIVALT. to (bosonic) Fock CCR irrep $\Big)$ $\Leftrightarrow$ $\Big($ $\mathcal{E}$-th LAYER has a genertor EMPTY QV $\Big)$ $\Leftrightarrow$ $\Big($ $\mathcal{E} \overset{q}{\sim} \mathcal{E}_G$ up to global phase it's Gaussian lay. $\Big)$

7.

Cor. 33:

$$\left(\begin{array}{c}K(\mathcal{E})\text{ is INEQUIVALENT}\\ \text{to Fock CCR}\\ \text{irrep}\end{array}\right) \Leftrightarrow \left(\begin{array}{c}\text{EVERY quantum}\\ \text{VACUUM of }\mathcal{E}\\ \text{is MYRIOTIC}\end{array}\right) \Leftrightarrow \left(\mathcal{E} \overset{\varphi}{\not\simeq} \mathcal{E}_G\right)$$

REP. EQUIVALENCE: $\mathcal{E}_G$ layer $\sim$ Fock rep

$$\phi_0 \otimes \phi_0 \otimes \cdots \longleftrightarrow \Omega_{Fock} := (1, 0, 0, \ldots)$$

$$\phi_1 \otimes \phi_0 \otimes \phi_0 \otimes \cdots \longleftrightarrow \hat{A}(\tilde{e}_1)^\dagger \Omega_{Fock} \propto (0, \tilde{e}_1, 0, \ldots)$$

$$\phi_0 \otimes \phi_3 \otimes \phi_0 \otimes \cdots \longleftrightarrow \hat{A}(\tilde{e}_2)^\dagger \hat{A}(\tilde{e}_2)^\dagger \hat{A}(\tilde{e}_2)^\dagger \Omega_{Fock}$$

$$\propto (0, 0, 0, \underbrace{\tilde{e}_2 \otimes \tilde{e}_2 \otimes \tilde{e}_2}, 0, \ldots)$$

↳ 3 excitats. in 2nd mode

↳ 3 bosons of mode 2

BOSONS BEYOND FOCK SP.

- $\phi_{m_1} \otimes \phi_{m_2} \otimes \phi_{m_3} \otimes \cdots$ describes:

m_1 bosons in mode 1, m_2 bosons in mode 2, etc.

$$\begin{cases} \phi_0 \otimes \phi_0 \otimes \cdots \rightsquigarrow 0 \text{ bosons.} \rightsquigarrow \underline{\text{EMPTY}}\ \underline{\text{VACUUM}} \\ \phi_1 \otimes \phi_1 \otimes \cdots \rightsquigarrow 1 \text{ boson / mode} = \infty \text{ bosons} \end{cases}$$

- If $\sum_{j=1}^{\infty} m_j = +\infty$ <u>cannot be annihilated by finite</u> $\hat{a}(\beta) \rightsquigarrow$ <u>MYRIADS of BOSONS</u>

  ↳ <u>Myriotic vacuum.</u>

- If $\left.\begin{matrix} m_j \neq m_j' \text{ for} \\ \infty\text{ly many } j \end{matrix}\right\}$ $\phi_{m_1} \otimes \phi_{m_2} \otimes \cdots$'s layer $\neq$ $\phi_{m_1'} \otimes \phi_{m_2'} \otimes \cdots$'s layer

- From field ontology's perspective:

(i) Bosons literally virtual particles!

↳ Literally "excitations of wavefunct."

(ii) No problem with interactions that "generate ∞ly many bosons"

↳ just transition of wavefct. betw. layers

BUT PRIMITIVE ONTOL. well-def'd $\forall t$

HAAG'S THEOREM'S ORIGIN

- Original announcm. by van Hove (1952) due to priv. comm. Segal in 1950 ← BOTH using ITP!

- $\hat{H}_{free} = \sum_{m=1}^{\infty} \omega_m (\hat{p}_m^2 + \hat{q}_m^2)$ rigorous in $\mathcal{E}_G$ (proves Reed 69), but if interact:

$$\hat{H}_{interact} = \hat{H}_{free} + \sum_{m=1}^{\infty} c_m \hat{q}_m^4 + \sum_{k,l,m,n=1}^{\infty} d_{klmn} \hat{q}_k \hat{q}_l \hat{q}_m \hat{q}_n$$

cannot be "implemented" in $\mathcal{E}_G$

need other CCR irreps → Non-Gauss. layers

⇒ HAAG's THM. is NO MYSTERY

If $t \to -\infty$ free finitely many or 0 bosons,

⇒ $\phi_0 \otimes \phi_0 \otimes \cdots$'s layer (or equiv. Fock sp.) describes state. But if interaction generates ∞ly many bosons by $t = 0$

↳ Need other layers

↳ i.e. irreps. inequivalent to Fock

Solut.:

↳ Either change state-space dynamically...

↳ or... use full ITP as state-space!!!

Thank you
for your
Attention!!!