# Weak Values: a new paradigm to characterize nanoscale systems

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## 0 – Traditional Theory-Experiment Link

## I – Weak Value Theory-Experiment Link

## II – Unprecedented Characterization

- **(a)** Closed system **Quantum Thermalization**
- **(b) Expected current in THz electronics**
- **(c) The dwell-time** in **nanoscale transistors**
- **(d)** Quantum **Work** and **two-time correlations**

## III – Non-Markovian Stochastic Schrödinger Equations

- **(a) Unravelling Non-Markovian open** quantum systems
- **(b) Quantum electron** transport with **Monte Carlo trajectories**

# 0 - Traditional Theory-Experiment Link



**THEORY**

$$\langle\psi|\hat{A}|\psi\rangle$$

**Bra-ket of observable A** (with operator $\hat{A}$) for $|\psi\rangle$

**EXPERIMENT**

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$$\langle\psi|\hat{A}|\psi\rangle$$

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**EXPERIMENT**

1. Prepare $|\psi(t)\rangle$
2. Measure **A strongly**
3. Get **average of A** measurements

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$$\mathbb{E}[A]$$

**Expected A** for $|\psi\rangle$

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**Uncertainty of A**

$$\langle\psi|\hat{A}^2|\psi\rangle - \langle\psi|\hat{A}|\psi\rangle \quad = \quad (\Delta A)^2$$

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**Cross-correlation** between **A,B**

$$\langle\psi(t_2)|\hat{B}\hat{U}_{t_1}^{t_2}\hat{A}|\psi(t_1)\rangle \;=\; \mathbb{E}[A(t_1)B(t_2)]$$

$$if\ \hat{A},\hat{B}\ commute$$

# I – Weak Value
# Theory-Experiment Link

# I – Weak Value Theory-Experiment Link



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$$\frac{\langle b_k|\hat{A}|\psi(t)\rangle}{\langle b_k|\psi(t)\rangle}$$

**Weak value** of A
on $|\psi\rangle$, if $|b_k\rangle$

**EXPERIMENT**

*Y. Aharonov, D. Z. Albert, and L. Vaidman, Phys. Rev. Lett. 60, 1351 (1988).*

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- Is a **complex** number
- $|b_k\rangle$ **eigenstate** of $\hat{B} = \sum_k b_k |b_k\rangle\langle b_k|$
- **Recover expectation**

$$\mathbb{E}[A] = \langle\psi|\hat{A}|\psi\rangle = \sum_k |\langle b_k|\psi\rangle|^2 \, {}_{\langle b_k|}A_{|\psi\rangle}$$

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$$\mathbb{E}\left[A_{weak}|B = b_k\right]$$

**Conditional expectation**
of A for $|\psi\rangle$

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**THEORY**

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**THEORY** **EXPERIMENT**

$$Re\left\{\frac{\langle b_k|\hat{A}|\psi(t)\rangle}{\langle b_k|\psi(t)\rangle}\right\} = \mathbb{E}[A_{weak}|B = b_k]$$

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*D. Pandey, R. Sampaio, T. Ala-Nissila, G. Albareda, and X. Oriols, Phys. Rev. A 103, 052219 (2021).*

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**THEORY**

**EXPERIMENT**

***Im*** **if** $a_{weak}$ is **weak ancilla's momentum**

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- **Im and Re** parts using **weak post-selected** measurement **(expectations)**

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*D. Pandey et al., Phys. Rev. A 103, 052219 (2021).*

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*E. Cohen and E. Pollak, Phys. Rev. A 98, 042112 (2018).*

*F. De Zela. Phys. Rev. A 105.4, 042202 (2022)*

# I – Weak Value Theory-Experiment Link



Measuring the **wavefunction** $\psi(\vec{x}, t)$

$$c\,\psi(\vec{x}, t) = \frac{\langle \vec{p} = 0 | \vec{x} \rangle \langle \vec{x} | \psi(t) \rangle}{\langle \vec{p} = 0 | \psi(t) \rangle}$$

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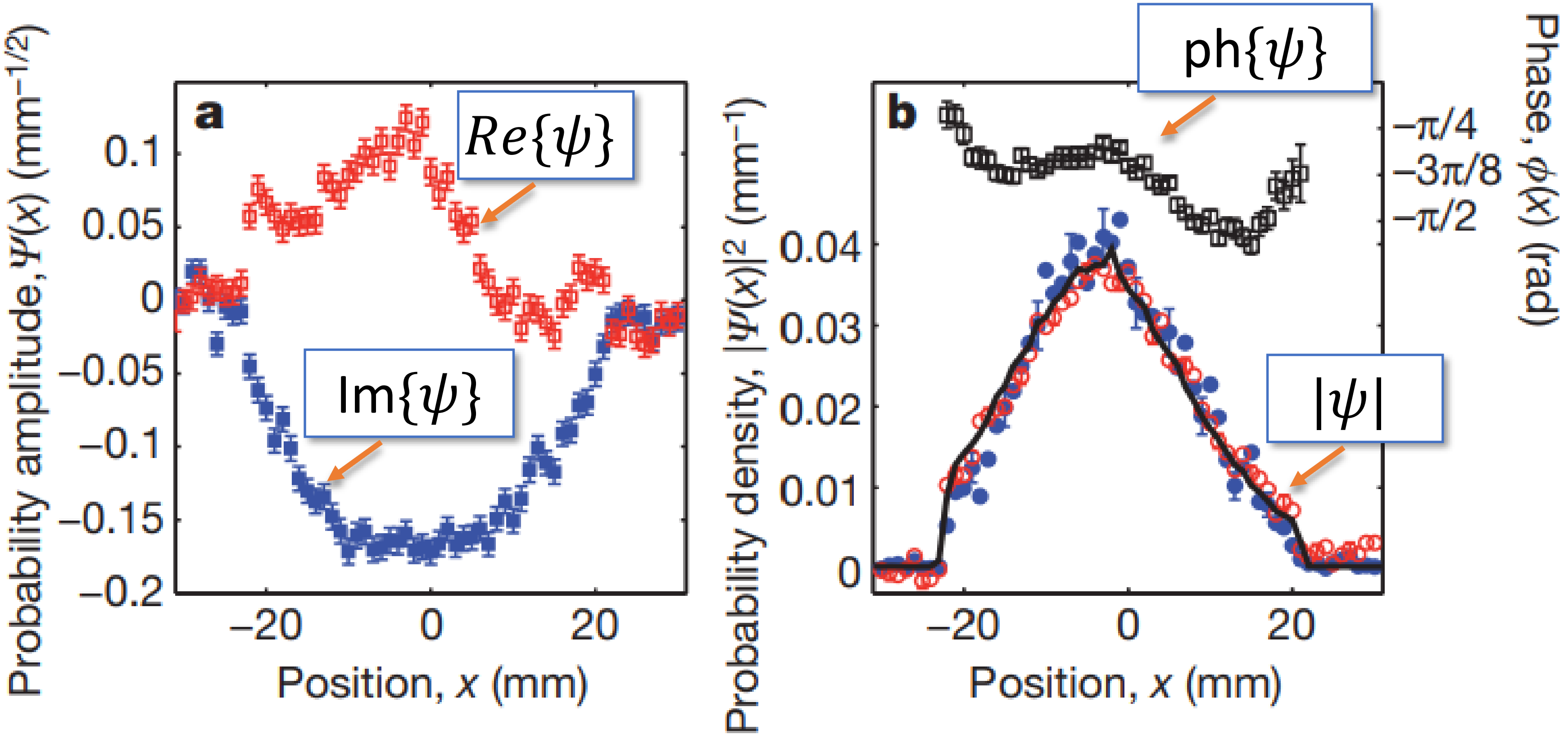
*J. S. Lundeen, B. Sutherland, A. Patel, C. Stewart, and C. Bambe,* ***Nature*** *474, 188 (2011)*



# I – Weak Value Theory-Experiment Link



Quantum Particle **Trajectories**?

$$\vec{\nabla}S(\vec{x},t) = Re\left\{ \frac{\langle \vec{x} \left| \hat{\vec{p}} \right| \psi(t) \rangle}{\langle \vec{x} | \psi(t) \rangle} \right\}$$

$$\psi(\vec{x},t) = R(\vec{x},t)\, e^{\frac{i}{\hbar}S(\vec{x},t)}$$

**polar form**

*S. Kocsis, B. Braverman, S. Ravets, M. J. Stevens, R. P. Mirin, L. K. Shalm, and A. M. Steinberg.*
***Science** 332, no. 6034 (2011): 1170-1173.*

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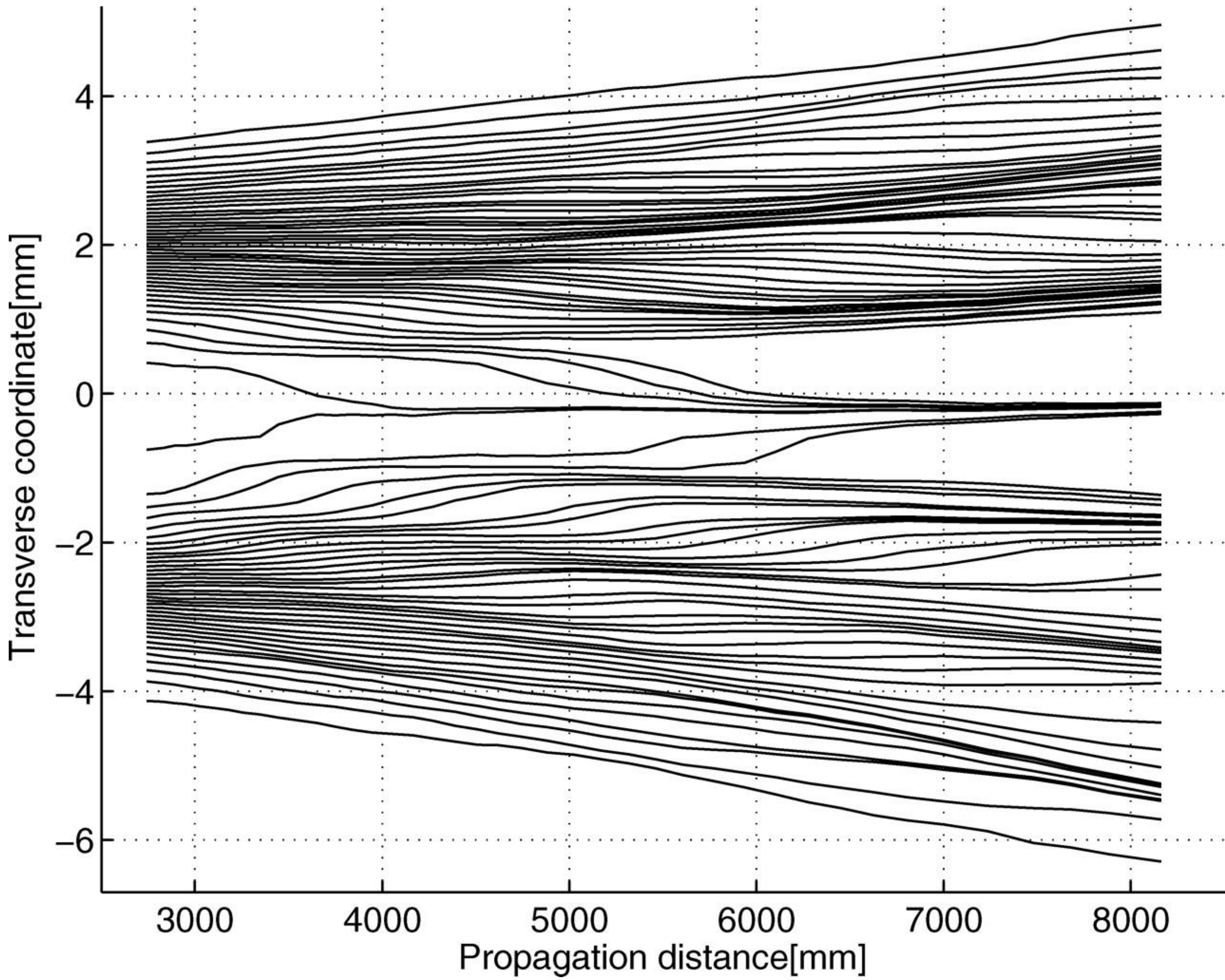


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# II – Unprecedented Characterization

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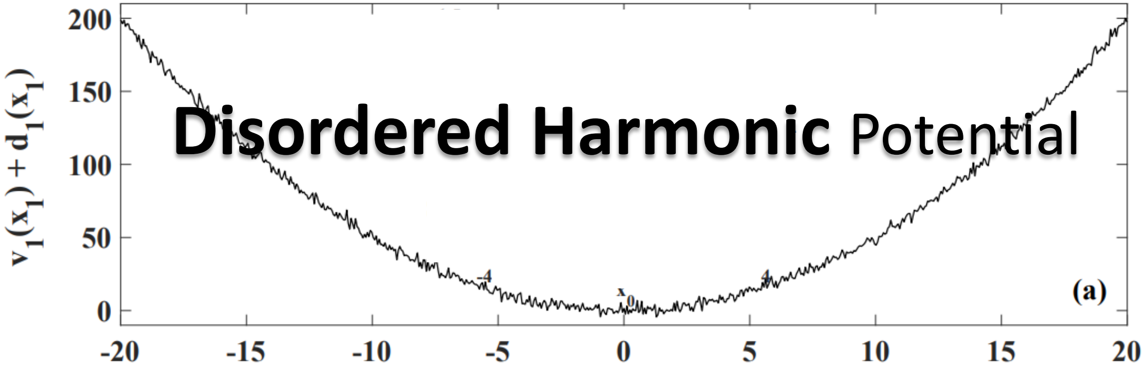
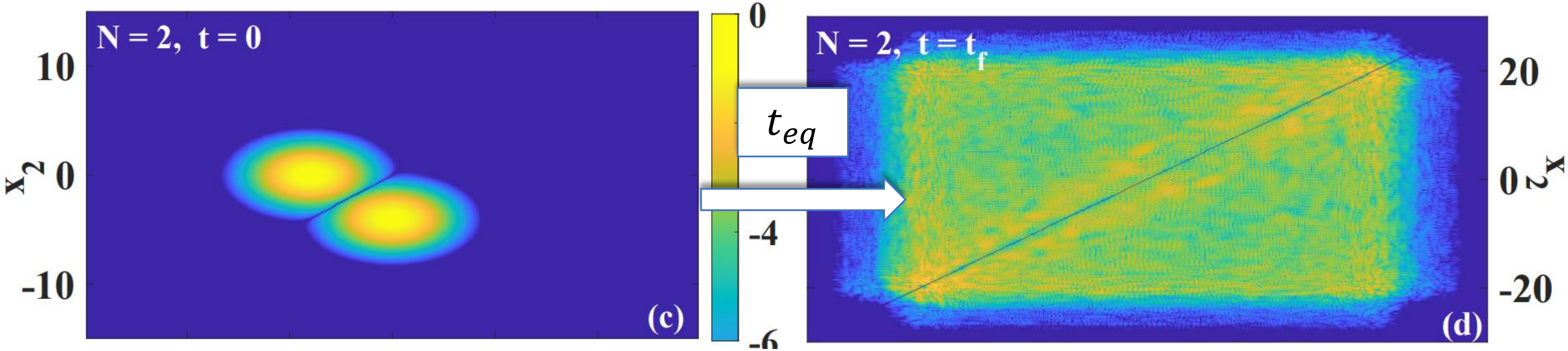
## (a) Closed system Quantum Thermalization




*C. F. Destefani and X. Oriols, Phys. Rev. A 107, 012213 (2023).*

*Destefani, Carlos F., and Xavier Oriols. arXiv preprint arXiv:2305.02026 (2023).*

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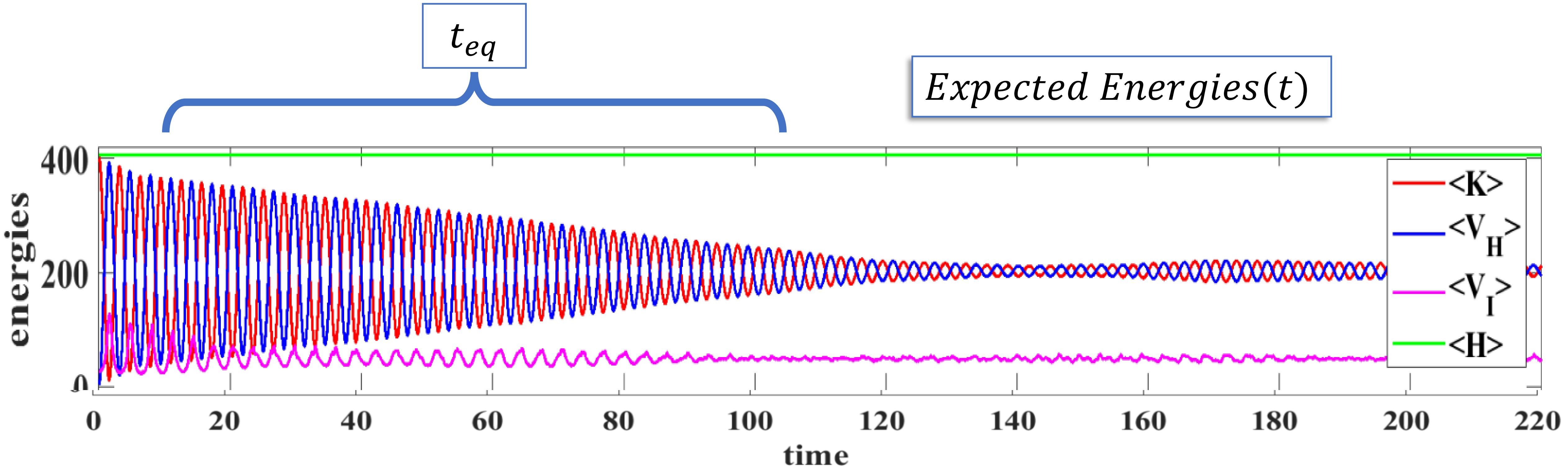


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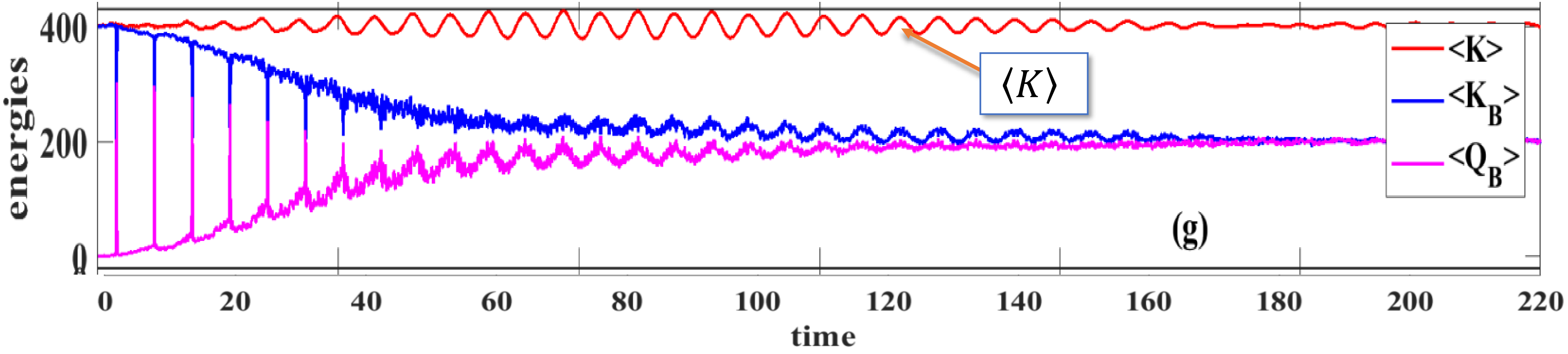
## (a) Closed system Quantum Thermalization

$$\langle K \rangle = \langle K_B \rangle + \langle K_O \rangle$$

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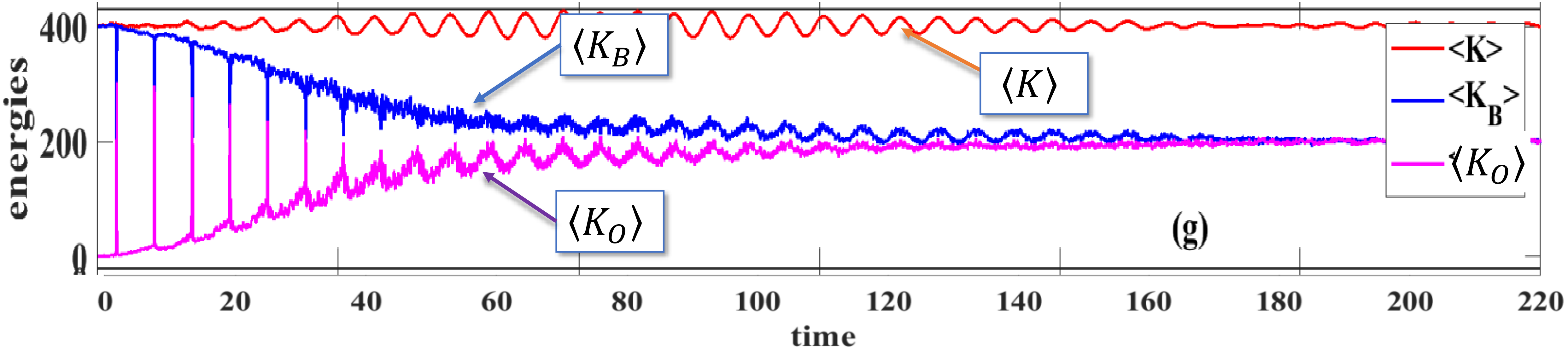


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***Trajectory formulations as heuristic tools?***

$$g^{\psi}(\vec{x},t) = Re\left\{ \frac{\langle \vec{x}|\hat{G}|\psi(t)\rangle}{\langle \vec{x}|\psi(t)\rangle} \right\}$$

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$$\mathbb{E}[G] = \langle \psi|\hat{G}|\psi\rangle = \int |\psi(\vec{x},t)|^2 g^{\psi}(\vec{x},t)dx$$

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- $\mathcal{E}^{\psi}(\vec{x},t) = Re\left\{ \frac{\langle \vec{x}|\hat{\boldsymbol{H}}|\psi(t)\rangle}{\langle \vec{x}|\psi(t)\rangle} \right\} = \frac{\vec{p}^{\,\psi}(\vec{x},t)^2}{2m} + V(\vec{x},t) + Q(\vec{x},t)$ Bohm Energy

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- $\mathcal{E}^{\psi}(\vec{x},t) = Re\left\{ \frac{\langle \vec{x}|\hat{\boldsymbol{H}}|\psi(t)\rangle}{\langle \vec{x}|\psi(t)\rangle} \right\} = \frac{p^{\psi}(\vec{x},t)^2}{2m} + V(\vec{x},t) + Q(\vec{x},t)$ Bohm Energy
- $\ell_z^{\psi}(\vec{x},t) = Re\left\{ \frac{\langle \vec{x}|\hat{\boldsymbol{L}}_{\boldsymbol{z}}|\psi(t)\rangle}{\langle \vec{x}|\psi(t)\rangle} \right\} = x_1 p_2(\vec{x},t) - x_2 p_1(\vec{x},t)$ Bohm Angular Momentum

… …

*D. Pandey, R. Sampaio, T. Ala-Nissila, G. Albareda, and X. Oriols, Phys. Rev. A 103, 052219 (2021).*

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## (b) Expected current in THz electronics

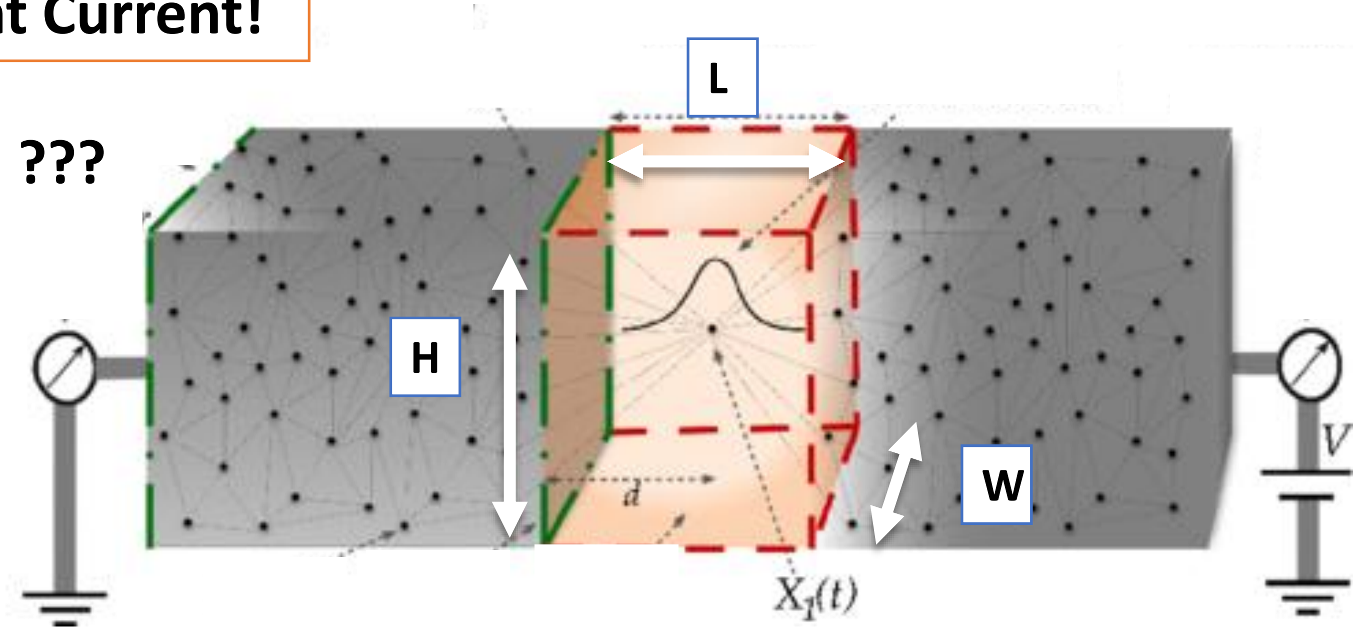
$$I_k^{\xi}(t) = \int_{\Omega} \vec{J}_k^{\xi}(\vec{r},t) \cdot d\vec{s} + \int_{\Omega} \varepsilon(\vec{r}\,,t) \frac{\partial \vec{E}_k^{\xi}(\vec{r},t)}{\partial t} \cdot d\vec{s}$$

**Particle Current**

**Operator:** $\hat{J}$

**Displacement Current!**

**Operator: ???**

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Quantum RSP theorem $\Longrightarrow$ $L \ll H, W$

$$I_k^{\xi}(t) \approx \frac{q}{L} \frac{d\vec{x}_k^{\xi}(t)}{dt}$$

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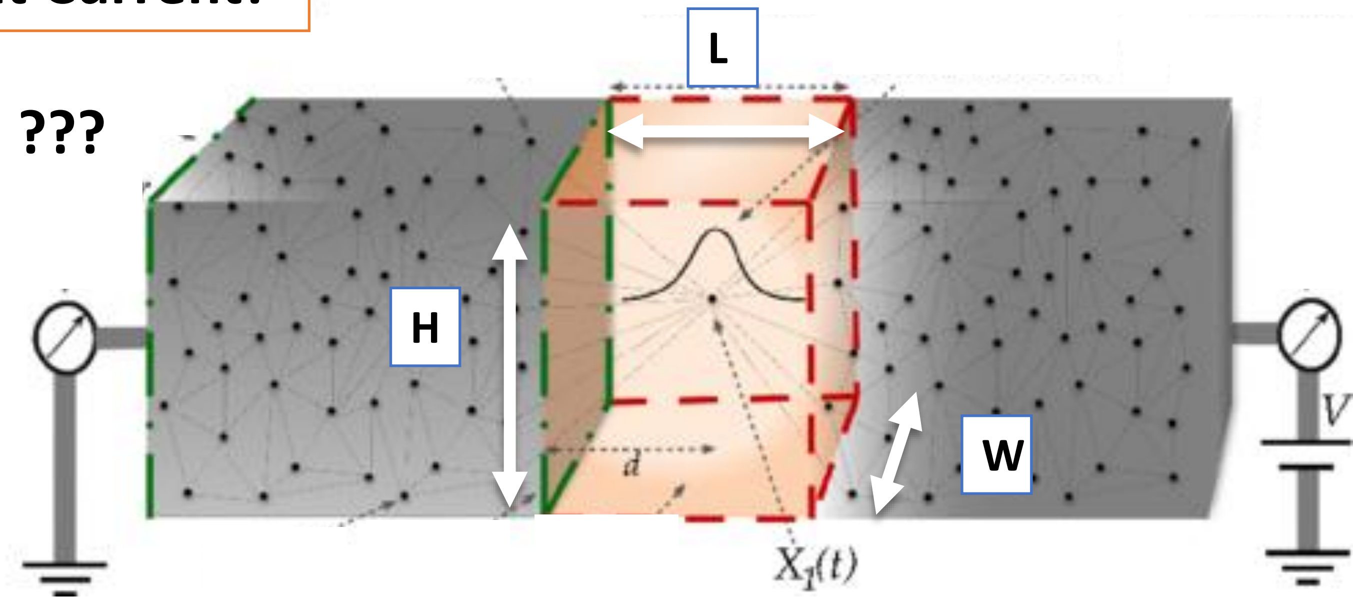


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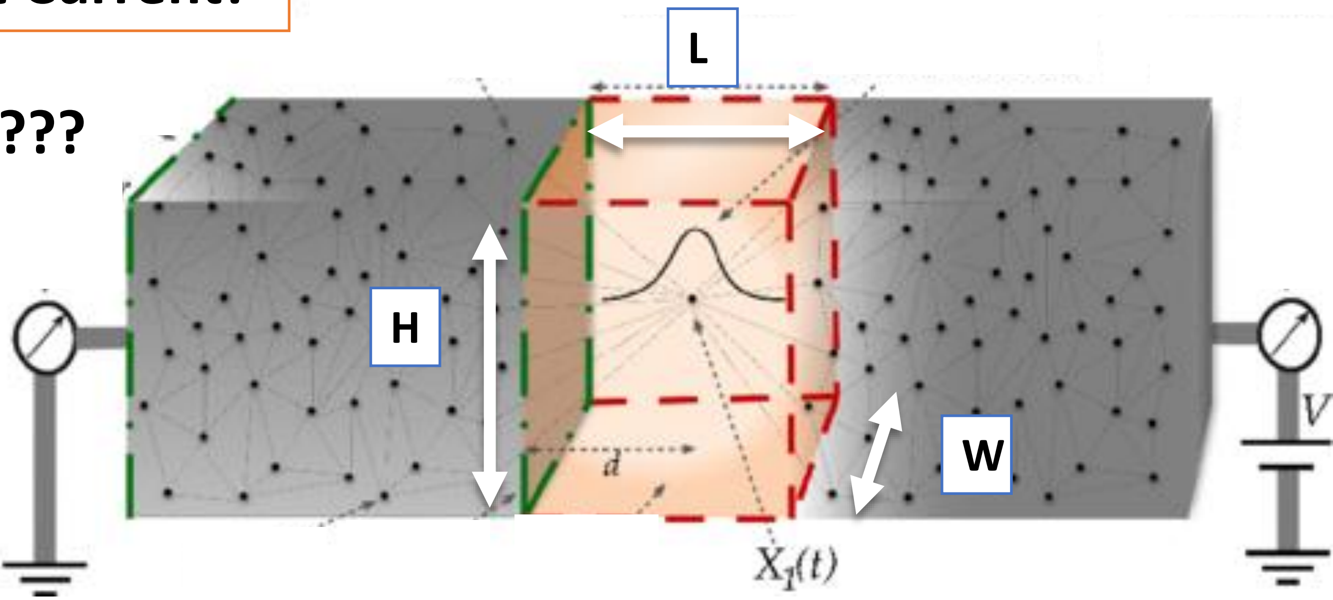
$$\mathbb{E}[I_T](t) = \lim_{|\sigma|\to\infty} \frac{1}{|\sigma|} \sum_{\xi\in\sigma} \sum_{k=1}^{N} I_k^{\xi}(t)$$

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## (c) The dwell-time in nanoscale transistors

**Consensus** dwell time

$$\tau_D := \int_0^{\infty} dt \int_{\Gamma} |\psi(\vec{r}, t)|^2 dr$$

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$$\lim_{|\sigma|\to\infty} \frac{\sum_{\xi\in\sigma} \tau^\xi}{|\sigma|} = \tau_D$$

$$\sigma \equiv \boldsymbol{All}\ sampled\ trajs$$

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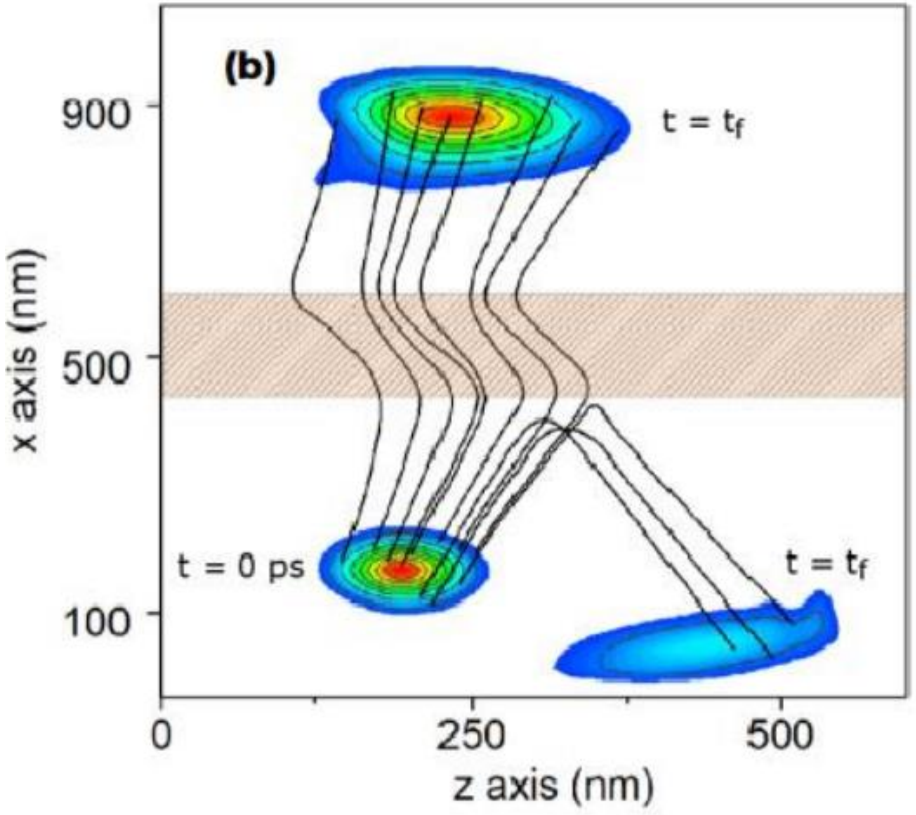
$\sigma^* \equiv \textbf{Only } \textit{those } \textbf{entering } \textit{the barrier}$

$$\tau_{DB} := \lim_{|\sigma^*| \to \infty} \frac{\sum_{\xi \in \sigma^*} \tau^{\xi}}{|\sigma^*|}$$

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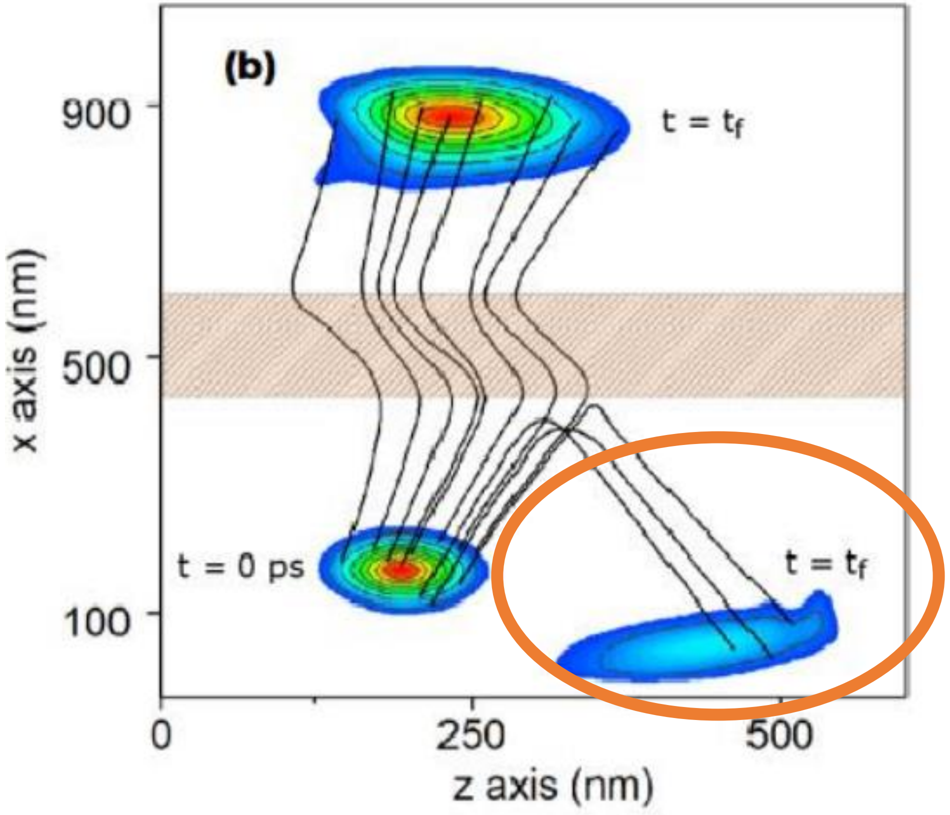



*D. Pandey, M.Villani, E. Colomes, Z. Zhan and X.Oriols, Sci. Technol. 34, 034002 (2019)*

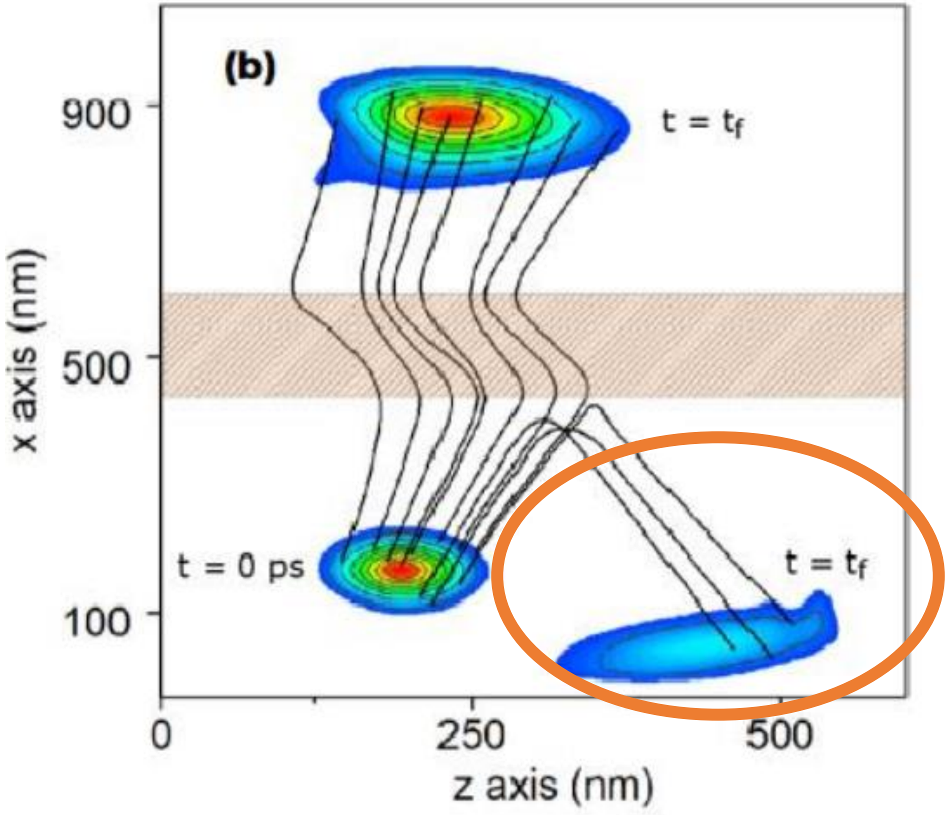
## (c) The dwell-time in nanoscale transistors

$\sigma^* \equiv$ ***Only*** *those* ***entering*** *the barrier*

$$\tau_{DB} := \lim_{|\sigma^*| \to \infty} \frac{\sum_{\xi \in \sigma^*} \tau^{\xi}}{|\sigma^*|}$$

**More informative Cut-off Frequency?**

$$f_B = \frac{1}{\tau_{DB}}$$

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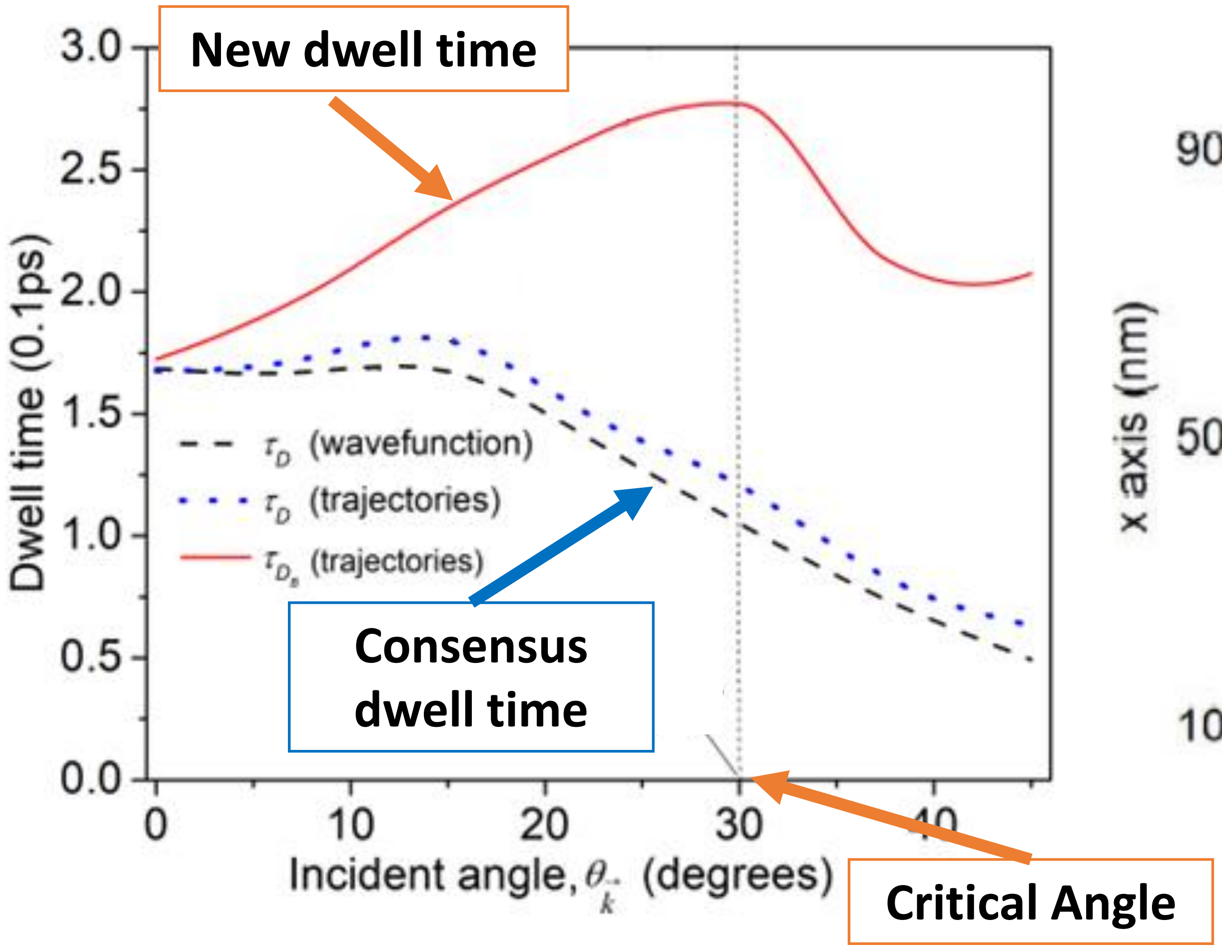
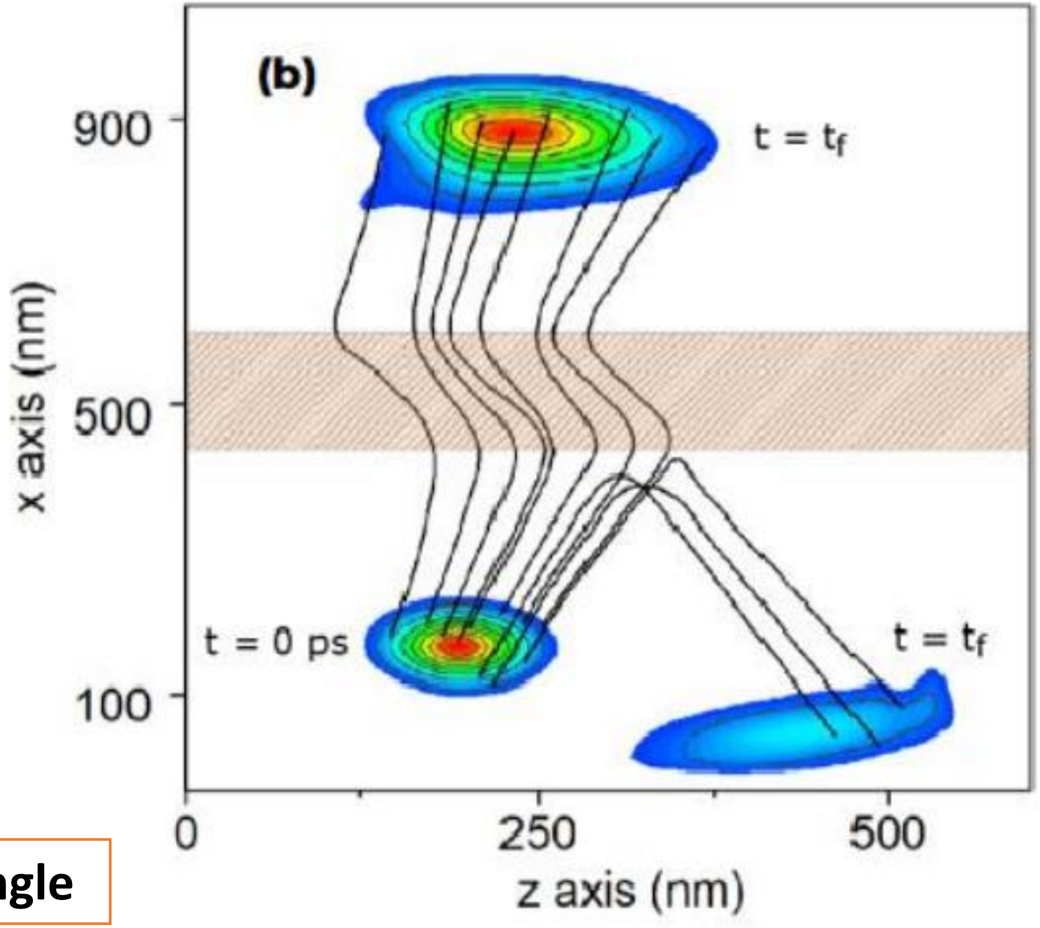
# II – Unprecedented Characterization

## (c) The dwell-time in nanoscale transistors




*D. Pandey, M.Villani, E. Colomes, Z. Zhan and X.Oriols, Sci. Technol. 34, 034002 (2019)*

# III – A Recipe for non-Markovian Open Quantum Systems

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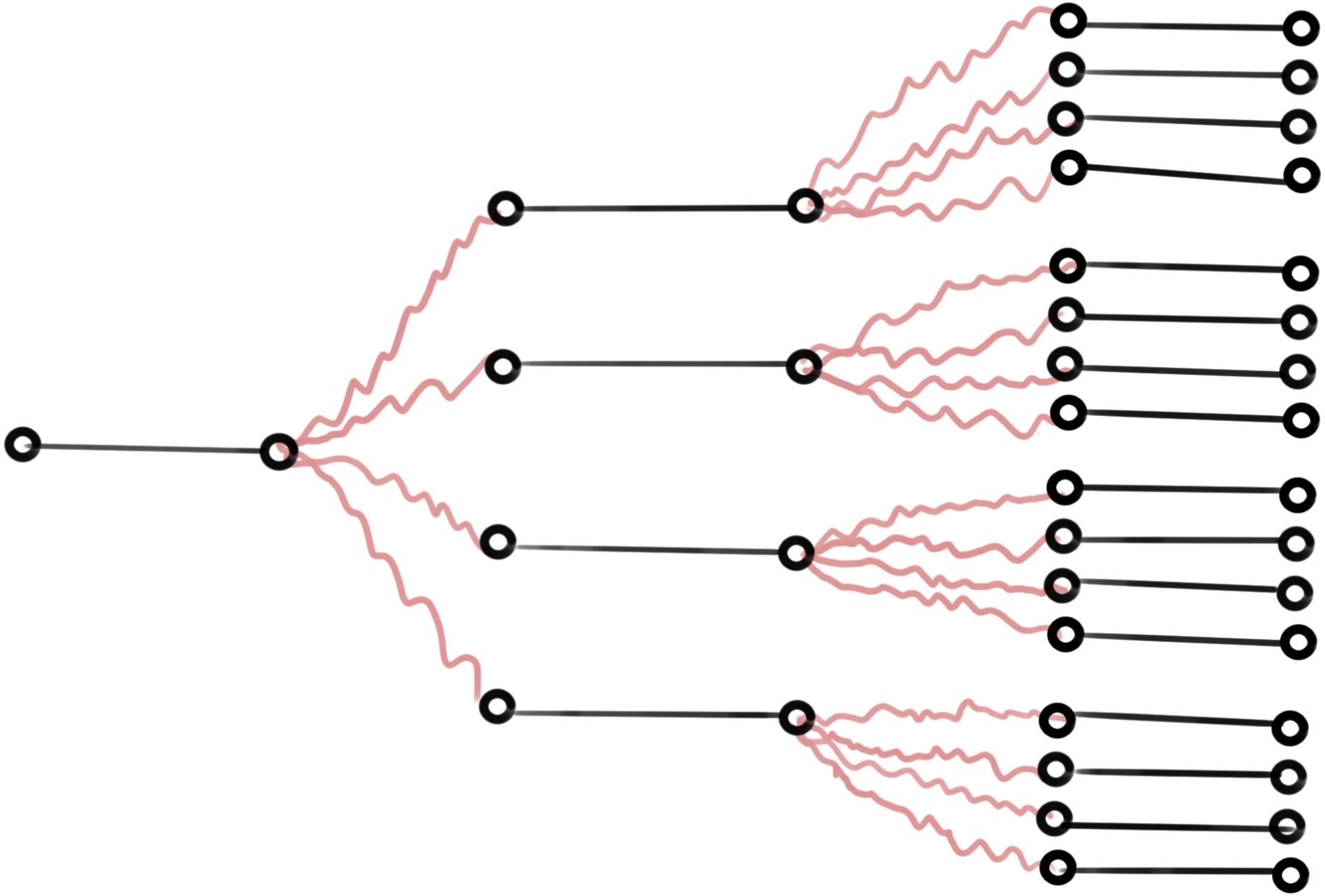
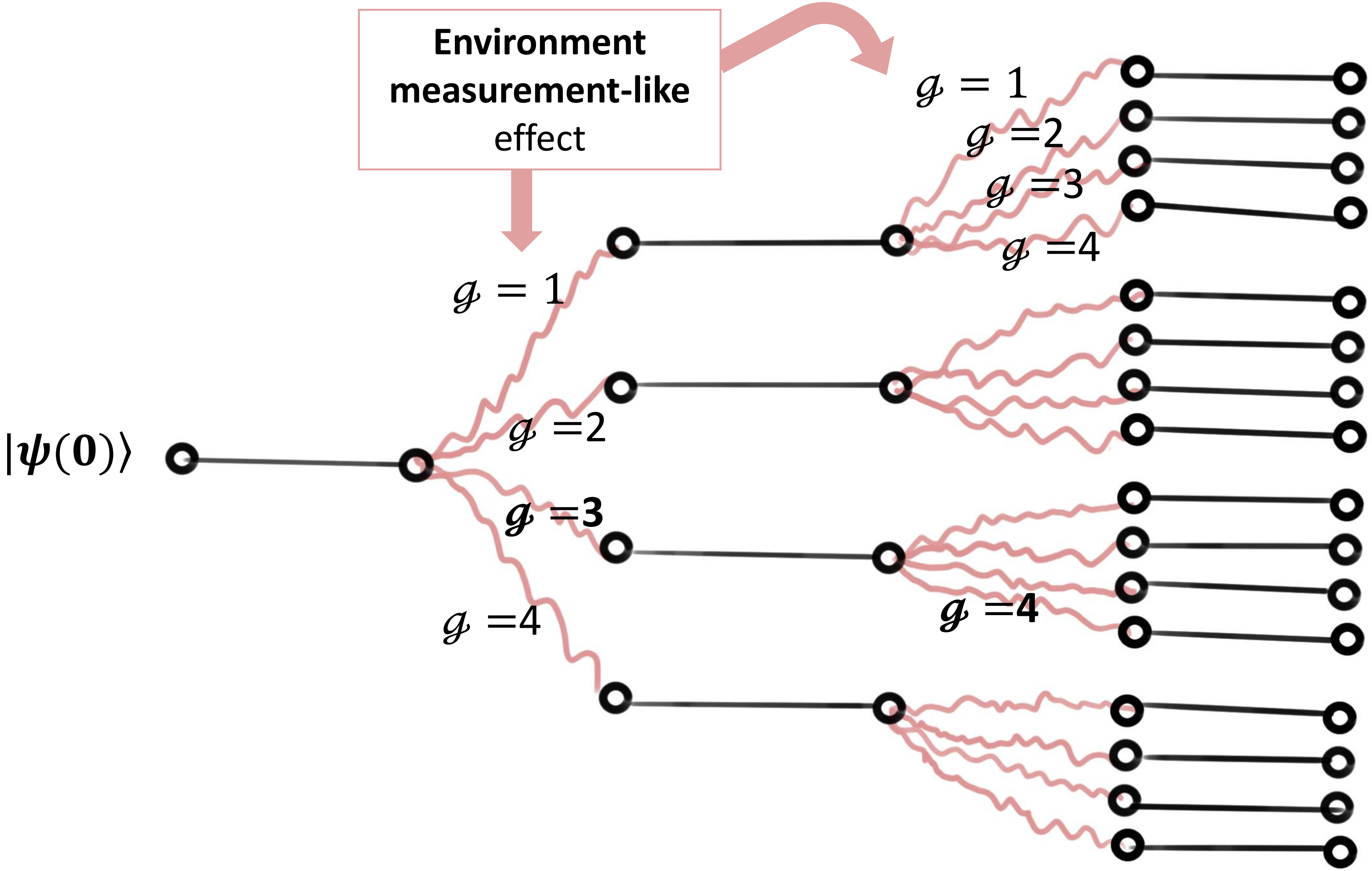
$|\psi(0)\rangle$



**Pure** Stochastic quantum **State Trajectory**

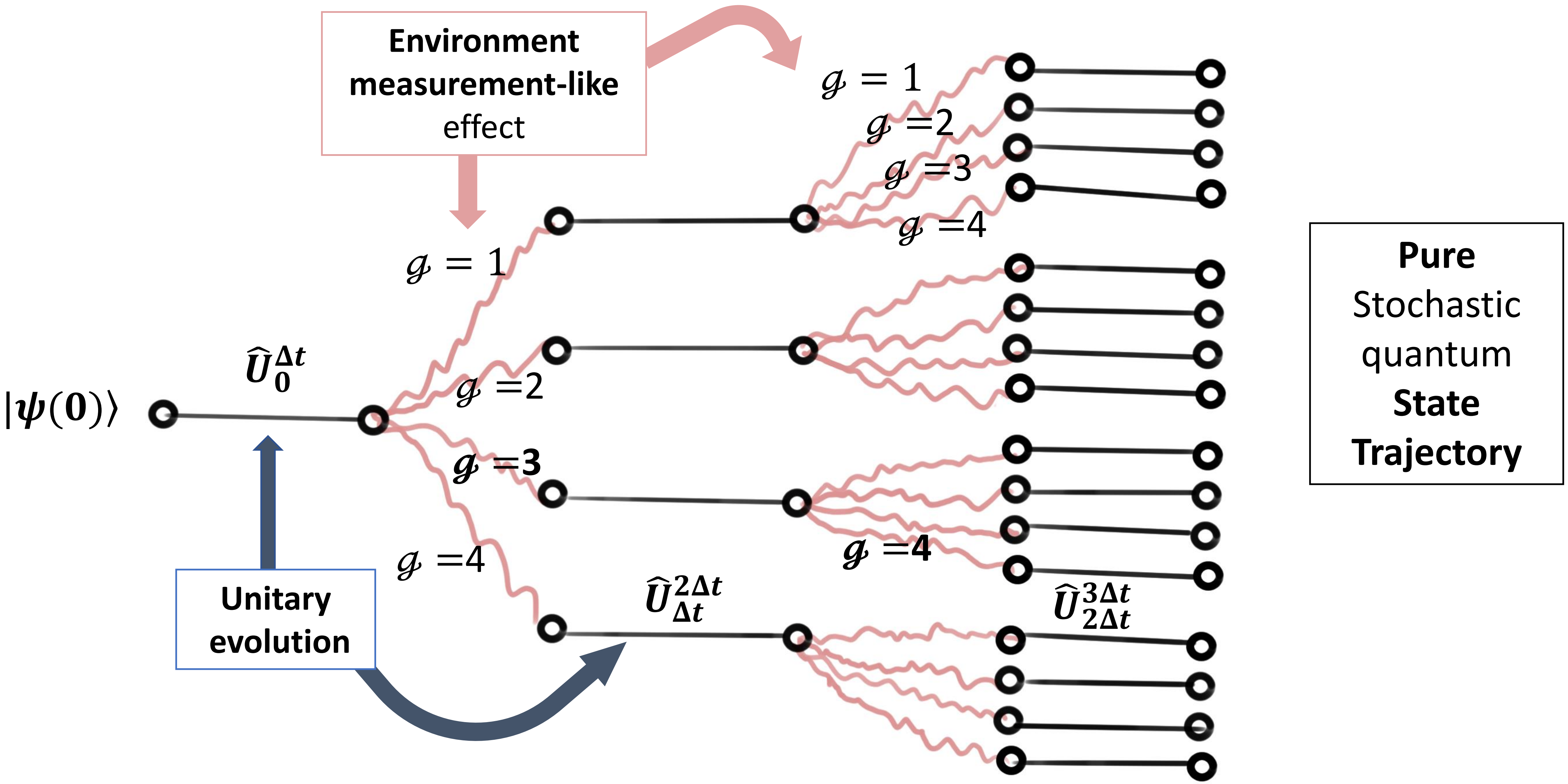
# III – A Recipe for non-Markovian Open Quantum Systems





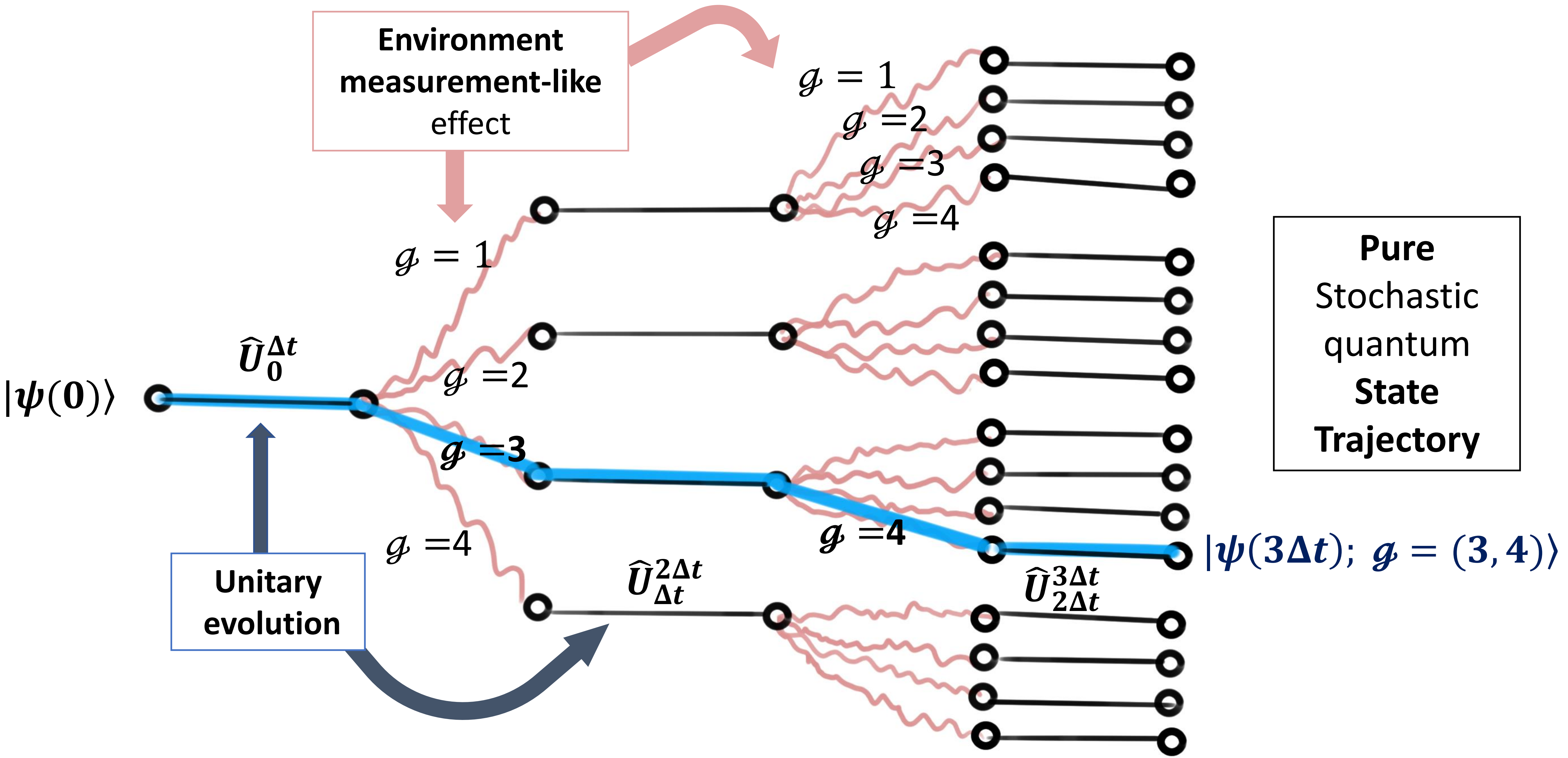

**Pure** Stochastic quantum **State Trajectory**

# III – A Recipe for non-Markovian Open Quantum Systems



# III – A Recipe for non-Markovian Open Quantum Systems





# III – A Recipe for non-Markovian Open Quantum Systems



## (a) Markovian open quantum systems

**Markovian Master Equation** for **reduced density matrix**

**Markovian Stochastic Schrödinger Equation** unravelling some $\hat{G}$

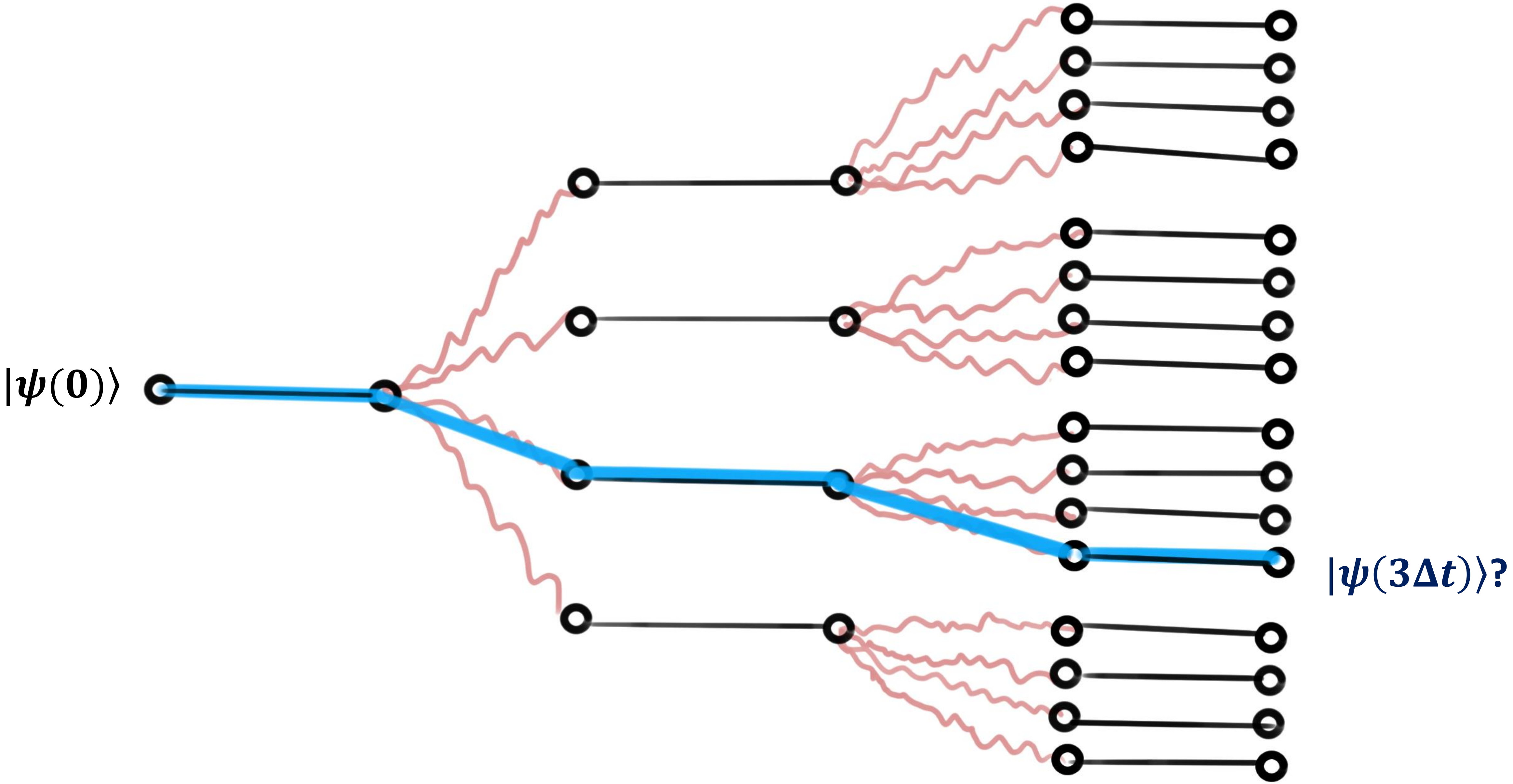
$$\hat{\rho}_S(t) := tr_E[\hat{\rho}_{SE}] = \mathbb{E}_\xi \left[ \, |\psi(t); g^\xi(t)\rangle\langle\psi(t); g^\xi(t)| \, \right]$$

*K. Jacobs and D. A. Steck, Contemp. Phys. 47, 279 (2006)*

*L. Li, M. J. Hall, and H. M. Wiseman, Phys. Rep. 759, 1 (2018)*

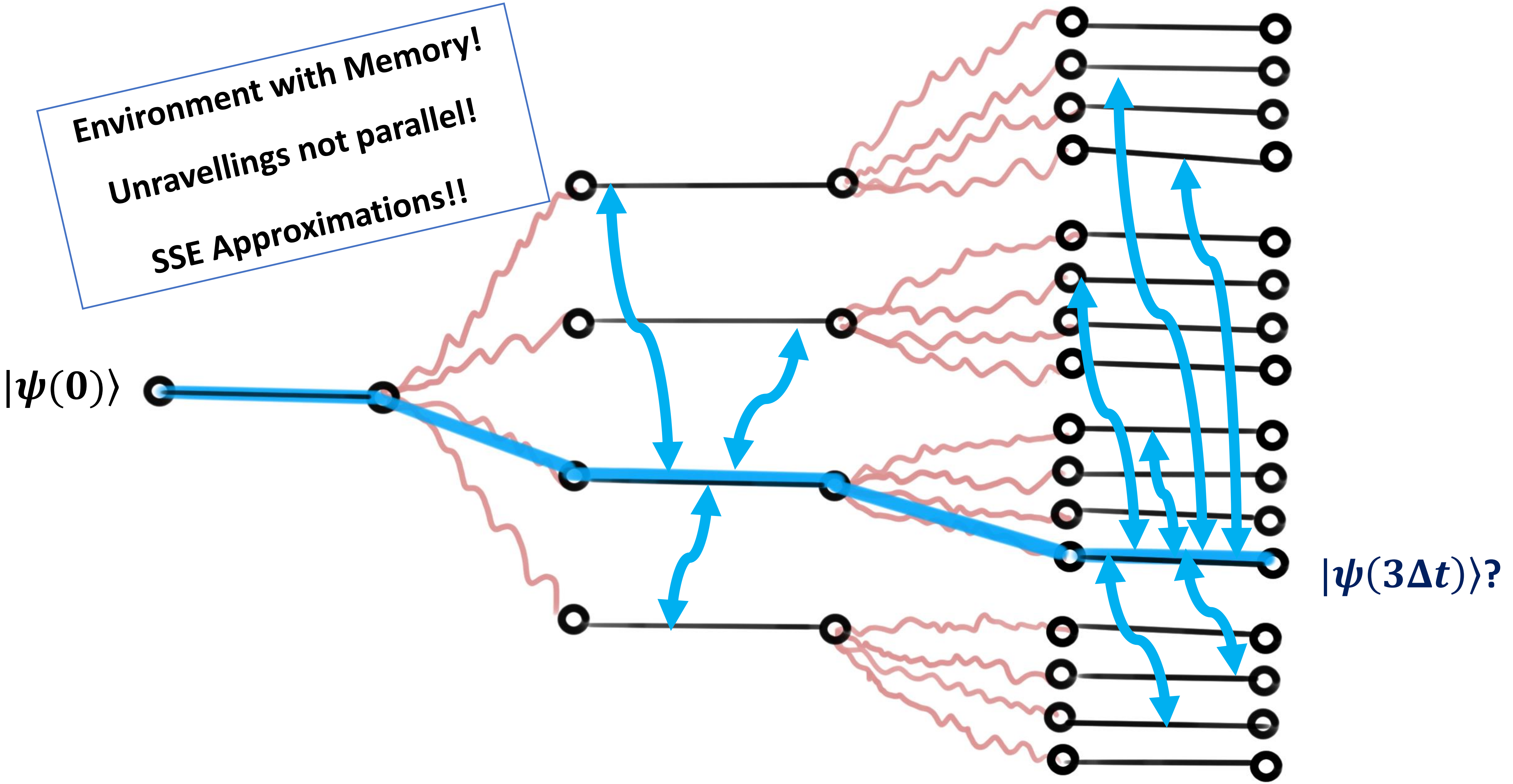
# III – A Recipe for non-Markovian Open Quantum Systems





# III – A Recipe for non-Markovian Open Quantum Systems





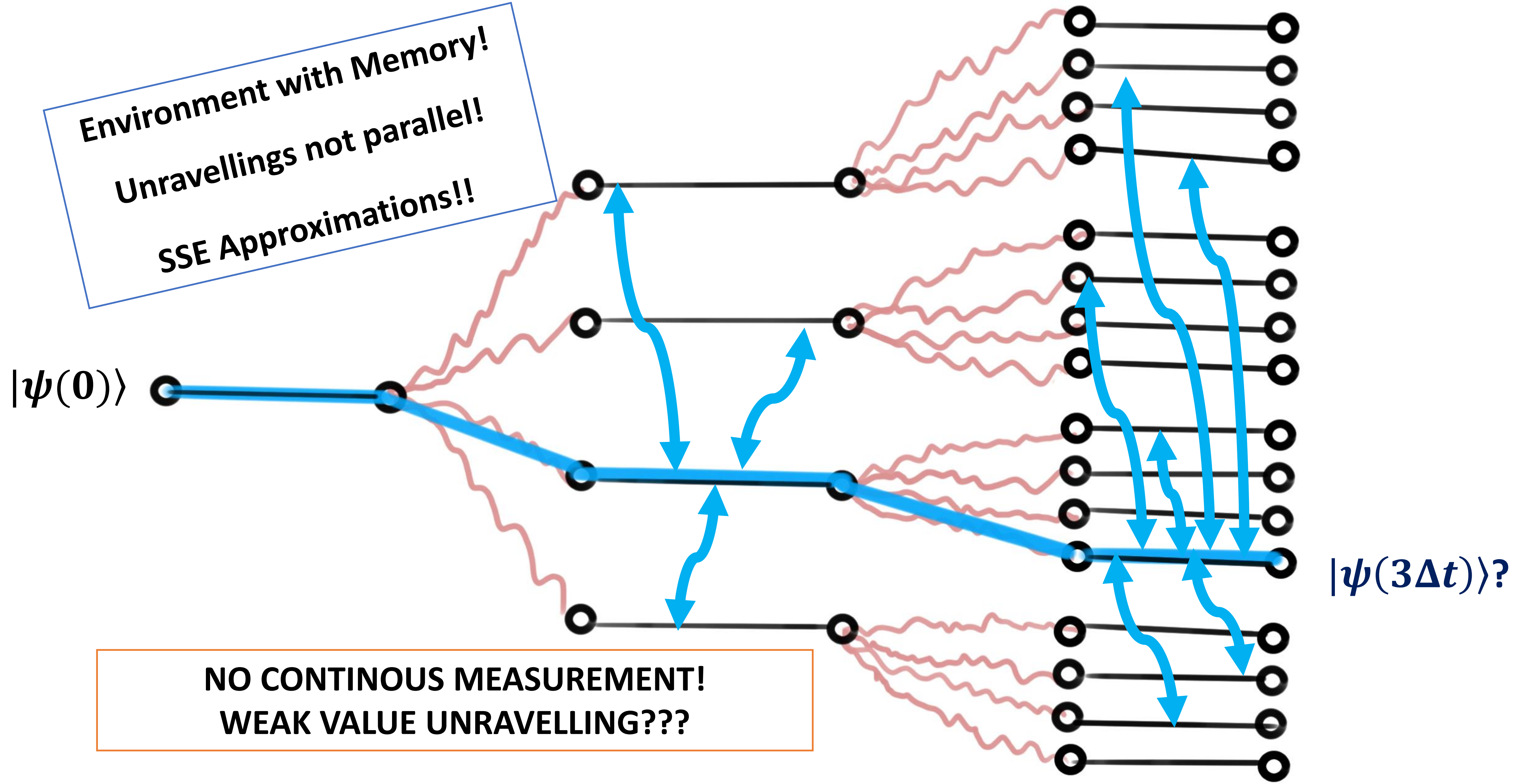
# III – A Recipe for non-Markovian Open Quantum Systems



Environment with Memory!

Unravellings not parallel!

SSE Approximations!!




NO CONTINOUS MEASUREMENT!
WEAK VALUE UNRAVELLING???

# III – A Recipe for non-Markovian Open Quantum Systems



## (b) SSEs Unravelling Weak Values

$$g^{\xi}(t) \rightarrow \overrightarrow{x_E}^{\xi}(\mathrm{t})$$

$$\left\{ i\hbar \frac{\partial \psi^{\xi}(\overrightarrow{x_S}, t)}{\partial t} = \left[ -\sum_{j \in S} \frac{\hbar^2}{2m_j} \frac{\partial^2}{\partial x_j^2} + U\left( \overrightarrow{x_S}, \overrightarrow{x_E}^{\xi}(t), t \right) + \mathcal{P}\left( \overrightarrow{x_S}, \overrightarrow{x_E}^{\xi}(t), t \right) \right] \psi^{\xi}(\overrightarrow{x_S}, t) \right.$$

*H. M. Wiseman and J. M. Gambetta, Phys. Rev. A 68, 062104 (2003) & Phys. Rev. Lett. 101, 140401 (2008)*

*X. Oianguren-Asua, CF. Destefani , M. Villani, DK. Ferry, X. Oriols. Chapter of: Physics and the Nature of Reality (2023)*

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$$\mathcal{P}(\overrightarrow{x_S}, \overrightarrow{x_E}, t) := \sum_{k\in E} \left[ -\frac{1}{2} m_k v_k(\vec{x}, t) + Q_k(\vec{x}, t) - i\frac{\hbar}{2} \frac{\partial v_k(\vec{x}, t)}{\partial x_k} \right]$$

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**In non-Markovian,** only approximately knowable!

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**In non-Markovian,** only approximately knowable!

But**, even in non-Markovian** **so Almost SSE !**

$$\hat{\rho}_S(t) = \mathbb{E}_{\xi}\left[ \,|\psi^{\xi}(t); g_{\xi}(t)\rangle \langle \psi^{\xi}(t); g_{\xi}(t)| \,\right]$$


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$$\hat{\rho}_S(t) = \mathbb{E}_{\xi}\left[ \, |\psi^{\xi}(t); g_{\xi}(t)\rangle \langle \psi^{\xi}(t);\, g_{\xi}(t)| \, \right]$$

➡ **Guide approximation** by classical intuition

*H. M. Wiseman and J. M. Gambetta, Phys. Rev. A 68, 062104 (2003) & Phys. Rev. Lett. 101, 140401 (2008)*

*X. Oianguren-Asua, CF. Destefani , M. Villani, DK. Ferry, X. Oriols. Chapter of: Physics and the Nature of Reality (2023)*

## (c) Quantum electron transport with Monte Carlo trajectories

$$I^{\xi}_{Total}(t) = \sum_{k=1}^{N} I^{\xi}_{k}(t) \Rightarrow$$

Two-time **correlations** (PSD), **noise** distribution**,** logical **operation frequency** etc.

*D. Marian, N. Zanghì, and X. Oriols, Phys. Rev. Lett. 116, 110404 (2016)*
*D. Pandey, E. Colomés, G. Albareda, and X. Oriols, Entropy 21, 1148 (2019)*

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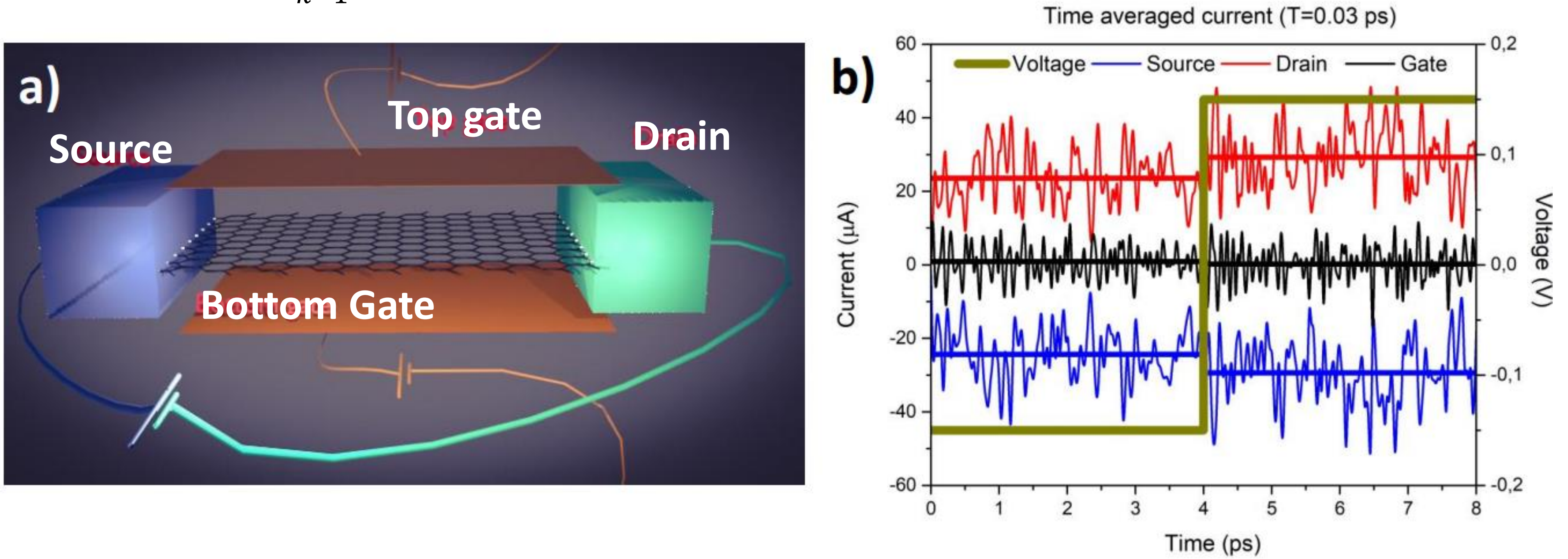


The **BITLLES** simulator.

*D. Marian, N. Zanghì, and X. Oriols, Phys. Rev. Lett. 116, 110404 (2016)*
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# Conclusions

- **Weak Values** (e.g. the $\psi$) **characterize quantum systems**
- They are **experimentally measurable** through **averages**
- **“Any” simulated result is** thus a **prediction** for an **experiment**
- They **promote Bohmian Mechanics to** a **practical tool :**
  - Resolve **pathological scenarios**
  - **Non-Markovian SSE** Toolbox

# Thank you for your attention!

# Questions?

## (d.1.) Quantum Work?

$$W(t_1, t_2) \sim E(t_1), E(t_2), path(t_1, t_2)$$

*D. H. Kobe, J. Phys. A Math. Theor. 40, 5155 (2007)*

*R. Sampaio, S. Suomela, T. Ala-Nissila, J. Anders, and T. Philbin, Phys. Rev. A 97, 012131 (2018)*

# II – Unprecedented Characterization



## (d.1.) Quantum Work?

$$W(t_1, t_2) \sim \; E(t_1), E(t_2), path(t_1, t_2)$$

- **Path *if* measurement** $\neq$ ***if not*** measurement
- **Energy undefined if not measurement**

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- **Energy undefined if not measurement**

⬇

**No uncontextual** quantum **work?**

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*R. Sampaio, S. Suomela, T. Ala-Nissila, J. Anders, and T. Philbin, Phys. Rev. A 97, 012131 (2018)*

## (d.1.) Quantum Work?

**Well…**

$$W(t_1, t_2) \sim E(t_1), E(t_2), path(t_1, t_2)$$

**Energy of** the system in $\vec{x}, t$

$$\mathcal{E}^{\psi}(\vec{x}, t) = Re\left\{ \frac{\langle \vec{x} | \hat{H}(t) | \psi(t) \rangle}{\langle \vec{x} | \psi(t) \rangle} \right\} = \frac{\vec{\wp}^{\psi}(\vec{x}, t)^2}{2m} + V(\vec{x}, t) + Q(\vec{x}, t)$$

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⬇

**Work on** the $\xi$-th trajectory

$$W^{\xi}(t_1,t_2):=\int_{t_1}^{t_2}\frac{d}{dt}\mathcal{E}^{\psi}\big(\vec{x}^{\xi}(t),t\big)dt=\mathcal{E}^{\psi}\big(\vec{x}^{\xi}(t_2),t_2\big)-\mathcal{E}^{\psi}\big(\vec{x}^{\xi}(t_1),t_1\big)$$

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⬇

**Work on** the $\xi$-th trajectory

$$W^{\xi}(t_1, t_2) := \int_{t_1}^{t_2} \frac{d}{dt} \mathcal{E}^{\psi}\big(\vec{x}^{\xi}(t), t\big) dt = \mathcal{E}^{\psi}\big(\vec{x}^{\xi}(t_2), t_2\big) - \mathcal{E}^{\psi}\big(\vec{x}^{\xi}(t_1), t_1\big)$$

⬇

**Work on** the quantum **system**

$$\langle W(t_1, t_2) \rangle := \lim_{|\sigma| \to \infty} \frac{1}{|\sigma|} \sum_{\xi \in \sigma} W^{\xi}(t_1, t_2) = \langle \psi(t_2) | \hat{H}(t_2) | \psi(t_2) \rangle - \langle \psi(t_1) | \hat{H}(t_1) | \psi(t_1) \rangle$$

*D. H. Kobe, J. Phys. A Math. Theor. 40, 5155 (2007)*

*R. Sampaio, S. Suomela, T. Ala-Nissila, J. Anders, and T. Philbin, Phys. Rev. A 97, 012131 (2018)*

# II – Unprecedented Characterization



## (d.2.) Two-time correlations

$$[\hat{G}, \hat{J}] \neq \hat{0} \quad \Rightarrow \quad \langle \hat{G}(t_2)\hat{J}(t_1) \rangle = \langle \psi | \hat{G}(t_2)\hat{J}(t_1) | \psi \rangle \in \mathbb{C}$$

**No well-defined two-time correlation?**

*D. Pandey, R. Sampaio, T. Ala-Nissila, G. Albareda, and X. Oriols, Phys. Rev. A 103, 052219 (2021).*

## (d.2.) Two-time correlations

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**Well…**

$$g^{\psi}(\vec{x},t) := Re\left\{ \frac{\langle \vec{x} | \hat{G} | \psi(t) \rangle}{\langle \vec{x} | \psi(t) \rangle} \right\} \qquad j^{\psi}(\vec{x},t) := Re\left\{ \frac{\langle \vec{x} | \hat{F} | \psi(t) \rangle}{\langle \vec{x} | \psi(t) \rangle} \right\}$$

$$\langle G(t_2) J\,(t_1) \rangle \coloneqq \lim_{|\sigma| \to \infty} \frac{1}{|\sigma|} \sum_{\xi \in \sigma} g^{\psi}\left(\vec{x}^{\,\xi}(t_1), t_1\right) j^{\psi}\left(\vec{x}^{\,\xi}(t_2), t_2\right)$$

*D. Pandey, R. Sampaio, T. Ala-Nissila, G. Albareda, and X. Oriols, Phys. Rev. A 103, 052219 (2021).*

# III – A Recipe for non-Markovian Open Quantum Systems

$$\Psi(\vec{\boldsymbol{x}}_{\boldsymbol{1}}, \dots, \vec{\boldsymbol{x}}_N, \vec{\boldsymbol{y}}_{\boldsymbol{Env}}, \boldsymbol{t})$$

$$\hat{\rho}_{1\dots N,E}(t)$$

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$$\Psi(\vec{\boldsymbol{x}}_{\boldsymbol{1}}, \ldots, \vec{\boldsymbol{x}}_{\boldsymbol{N}}, \vec{\boldsymbol{y}}_{\boldsymbol{Env}}, \boldsymbol{t})$$

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One non-M. open QS

$$\Psi\left(\vec{\boldsymbol{x}}_{\boldsymbol{1}}, \ldots, \vec{\boldsymbol{x}}_{\boldsymbol{N}},;\ \vec{y}^{\xi}_{Env}(t), t\right)$$

$$\rightarrow\ |\psi^{\xi}(t); g_{\xi}(t)\rangle_{1\ldots N}$$

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$\hat{\boldsymbol{\rho}}_{1\dots N}(\boldsymbol{t})$

**OK!**

**But… no** $\mathcal{P}$ **approx.!**

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$\hat{\boldsymbol{\rho}}_{1\ldots N}(\boldsymbol{t})$

**OK!**

**But… no** $\mathcal{P}$ **approx.!**

**N non-M open QS!**

$$\Psi\left(\vec{\boldsymbol{x}}_1, \vec{x}^{\xi}_2(t), \ldots, \vec{x}^{\xi}_N(t),\ \vec{y}^{\xi}_P(t), t\right) \rightarrow \left|\psi^{\xi}_1(t); g_{\xi}(t)\right\rangle_1$$

…

$$\Psi\left(\vec{x}^{\xi}_1(t), \ldots, \vec{x}^{\xi}_{N-1}(t), \vec{\boldsymbol{x}}_N,\ \vec{y}^{\xi}_P(t), t\right) \rightarrow \left|\psi^{\xi}_N(t); g_{\xi}(t)\right\rangle_N$$

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$\hat{\rho}_{1\ldots N,E}(t)$

$$\Psi\left(\vec{\boldsymbol{x}}_1, \ldots, \vec{\boldsymbol{x}}_N, ;\ \vec{y}^{\xi}_{Env}(t), t\right) \rightarrow |\psi^{\xi}(t); g_{\xi}(t)\rangle_{1\ldots N}$$

**One** non-M. **open QS**

$\hat{\boldsymbol{\rho}}_{1\ldots N}(\boldsymbol{t})$

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$$\Psi\left(\vec{\boldsymbol{x}}_1, \vec{x}^{\xi}_2(t), \ldots, \vec{x}^{\xi}_N(t),\ \vec{y}^{\xi}_P(t), t\right) \rightarrow |\psi^{\xi}_1(t); g_{\xi}(t)\rangle_1 \quad \hat{\boldsymbol{\rho}}_1(\boldsymbol{t})$$

…

$$\Psi\left(\vec{x}^{\xi}_1(t), \ldots, \vec{x}^{\xi}_{N-1}(t), \vec{\boldsymbol{x}}_N,\ \vec{y}^{\xi}_P(t), t\right) \rightarrow |\psi^{\xi}_N(t); g_{\xi}(t)\rangle_N \quad \hat{\boldsymbol{\rho}}_N(\boldsymbol{t})$$

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$\hat{\rho}_{1\dots N,E}(t)$

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$\hat{\boldsymbol{\rho}}_{1\dots N}(\boldsymbol{t})$

**OK!**

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…

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$\hat{\boldsymbol{\rho}}_1(\boldsymbol{t})$

…

$\hat{\boldsymbol{\rho}}_N(\boldsymbol{t})$

**Available $\mathcal{P}_k$ apprx.!**

$\hat{\boldsymbol{\rho}}_{1\dots N}(t)$ **?**

$\Psi$ **?**

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$\Psi(\vec{\boldsymbol{x}}_1, \dots, \vec{\boldsymbol{x}}_N, \vec{\boldsymbol{y}}_{Env}, \boldsymbol{t})$
$\hat{\rho}_{1\dots N,E}(t)$

**One** non-M.
**open QS**

$$\Psi\left(\vec{\boldsymbol{x}}_1, \dots, \vec{\boldsymbol{x}}_N, ;\ \vec{y}^{\xi}_{Env}(t), t\right) \rightarrow |\psi^{\xi}(t); g_{\xi}(t)\rangle_{1\dots N}$$

→ $\hat{\boldsymbol{\rho}}_{1\dots N}(\boldsymbol{t})$

**OK!**
**But… no $\mathcal{P}$ approx.!**

**N non-M open QS!**

$$\Psi\left(\vec{\boldsymbol{x}}_1, \vec{x}^{\xi}_2(t), \dots, \vec{x}^{\xi}_N(t),\ \vec{y}^{\xi}_P(t), t\right) \rightarrow |\psi^{\xi}_1(t); g_{\xi}(t)\rangle_1$$

→ $\hat{\boldsymbol{\rho}}_1(\boldsymbol{t})$

…

$$\Psi\left(\vec{x}^{\xi}_1(t), \dots, \vec{x}^{\xi}_{N-1}(t), \vec{\boldsymbol{x}}_N, \vec{y}^{\xi}_P(t), t\right) \rightarrow |\psi^{\xi}_N(t); g_{\xi}(t)\rangle_N$$

→ $\hat{\boldsymbol{\rho}}_N(\boldsymbol{t})$

**Available $\mathcal{P}_k$ apprx.!**

$\hat{\boldsymbol{\rho}}_{1\dots N}(t)$ **?**
$\Psi$ **?**

**No need!**

# III – A Recipe for non-Markovian Open Quantum Systems

$\Psi(\vec{\boldsymbol{x}}_1, \dots, \vec{\boldsymbol{x}}_N, \vec{\boldsymbol{y}}_{Env}, \boldsymbol{t})$

$\hat{\rho}_{1\dots N,E}(t)$

One non-M. open QS

$\Psi\left(\vec{\boldsymbol{x}}_1, \dots, \vec{\boldsymbol{x}}_N, ;\ \vec{y}^{\xi}_{Env}(t), t\right) \to |\psi^{\xi}(t); g_{\xi}(t)\rangle_{1\dots N}$ → $\hat{\boldsymbol{\rho}}_{1\dots N}(\boldsymbol{t})$

**OK!**
**But… no $\mathcal{P}$ approx.!**

**N non-M open QS!**

$\Psi\left(\vec{\boldsymbol{x}}_1, \vec{x}^{\xi}_2(t), \dots, \vec{x}^{\xi}_N(t),\ \vec{y}^{\xi}_P(t), t\right) \to |\psi^{\xi}_1(t); g_{\xi}(t)\rangle_1$ → $\hat{\boldsymbol{\rho}}_1(\boldsymbol{t})$

…

$\Psi\left(\vec{x}^{\xi}_1(t), \dots, \vec{x}^{\xi}_{N-1}(t), \vec{\boldsymbol{x}}_N, \vec{y}^{\xi}_P(t), t\right) \to |\psi^{\xi}_N(t); g_{\xi}(t)\rangle_N$ → $\hat{\boldsymbol{\rho}}_N(\boldsymbol{t})$

**Available $\mathcal{P}_k$ apprx.!** …

$\hat{\boldsymbol{\rho}}_{1\dots N}(t)$ ?
$\Psi$ ?
**No need!**

$$j^{\psi}_k(\vec{x}, t) = Re\left\{\frac{\langle \vec{\boldsymbol{x}}_1, \dots, \vec{\boldsymbol{x}}_N | \widehat{Id}_1 \cdots \hat{J}_k \cdots \widehat{Id}_N | \psi\rangle}{\langle \vec{\boldsymbol{x}}_1, \dots, \vec{\boldsymbol{x}}_N | \psi\rangle}\right\}$$

$$\mathcal{J}^{\psi}_T = \Sigma^N_{j=1}\, j^{\psi}_k$$

# III – A Recipe for non-Markovian Open Quantum Systems

$\Psi(\vec{\boldsymbol{x}}_1, \dots, \vec{\boldsymbol{x}}_N, \vec{\boldsymbol{y}}_{Env}, \boldsymbol{t})$
$\hat{\rho}_{1\dots N,E}(t)$

**One** non-M. **open QS**

$$\Psi\left(\vec{\boldsymbol{x}}_1, \dots, \vec{\boldsymbol{x}}_N, ;\ \vec{y}^{\,\xi}_{Env}(t), t\right) \rightarrow |\psi^{\xi}(t); g_{\xi}(t)\rangle_{1\dots N} \Rightarrow \hat{\boldsymbol{\rho}}_{1\dots N}(\boldsymbol{t})$$

**OK!**
**But… no $\mathcal{P}$ approx.!**

**N non-M open QS!**

$$\Psi\left(\vec{\boldsymbol{x}}_1, \vec{x}^{\,\xi}_2(t), \dots, \vec{x}^{\,\xi}_N(t),\ \vec{y}^{\,\xi}_P(t), t\right) \rightarrow |\psi^{\xi}_1(t); g_{\xi}(t)\rangle_1 \Rightarrow \hat{\boldsymbol{\rho}}_1(\boldsymbol{t})$$

…

$$\Psi\left(\vec{x}^{\,\xi}_1(t), \dots, \vec{x}^{\,\xi}_{N-1}(t), \vec{\boldsymbol{x}}_N,\ \vec{y}^{\,\xi}_P(t), t\right) \rightarrow |\psi^{\xi}_N(t); g_{\xi}(t)\rangle_N \Rightarrow \hat{\boldsymbol{\rho}}_N(\boldsymbol{t})$$

**Available $\mathcal{P}_k$ apprx.!** …

$\hat{\boldsymbol{\rho}}_{1\dots N}(t)$ **?**
$\Psi$ **?**

**No need!**

$$j^{\psi}_k(\vec{x}, t) = Re\left\{\frac{\langle\vec{\boldsymbol{x}}_1, \dots, \vec{\boldsymbol{x}}_N|\widehat{Id}_1 \cdots \hat{J}_k \cdots \widehat{Id}_N|\psi\rangle}{\langle\vec{\boldsymbol{x}}_1, \dots, \vec{\boldsymbol{x}}_N|\psi\rangle}\right\}$$

$$\mathcal{J}^{\psi}_T = \Sigma^N_{j=1}\, j^{\psi}_k$$

$$\mathbb{E}[\mathcal{J}\_T](t) = \lim_{|\sigma|\rightarrow\infty} \frac{1}{|\sigma|} \sum_{\xi\in\sigma} \mathcal{J}^{\psi}_T\left(\vec{x}^{\,\xi}_1(t), \dots, \vec{x}^{\,\xi}_N(t), t\right) = \langle\psi|\hat{\mathcal{J}}|\psi\rangle$$