Exercise Sheet 3: Topology II

1. Let $f : X \to Y$ be a map between topological spaces and assume that it is continuous at $x \in X$. Prove that it is also sequentially continuous at x.

2. Show that a map $f : X \to Y$ between metric spaces X, Y is continuous at $x \in X$, if and only if

$$\forall \varepsilon > 0\ \exists \delta > 0 : \quad f(B_\delta(a)) \subset B_\varepsilon(f(a))\,.$$

3. Let $K \subset X$ be a compact subset of a topological space. Show that any sequence in K has a cluster point in K.

4. Show that any compact subset of a Hausdorff space is closed.

5. Find an example of

(i) a sequence of maps that converges pointwise but not uniformly.

(ii) a connected but not path connected topological space.