

FOUNDATIONS OF QUANTUM MECHANICS: ASSIGNMENT 1

Exercise 1: Essay question. What got you interested in the foundations of quantum mechanics?

Exercise 2: Plane waves.

Show that for every constant vector $\mathbf{k} \in \mathbb{R}^3$ (the *wave vector*), there is a unique constant $\omega \in \mathbb{R}$ so that

$$\psi(t, \mathbf{x}) = e^{i\mathbf{k} \cdot \mathbf{x}} e^{-i\omega t} \quad (1)$$

satisfies the free (i.e., $V = 0$) Schrödinger equation (1.1) for $N = 1$. Specify ω in terms of $\mathbf{k}$. Remark: Since for every t , $\|\psi_t\| = \infty$, this function (called a plane wave) is not square-integrable (not normalizable) and thus not physically possible as a wave function; but it is a useful toy example.

Exercise 3: Time reversal invariance.

Show that if $\psi(t, q)$ is a solution of the Schrödinger equation (1.1) with real-valued potential V , then so is $\psi^*(-t, q)$.

Exercise 4: Gaussian wave packet.

(a) Show that the function

$$\psi(\mathbf{x}, t) = (2\pi\lambda_t^2\sigma^2)^{-3/4} e^{i\mathbf{k} \cdot (\mathbf{x} - \hbar\mathbf{k}t/2m)} e^{-\frac{(\mathbf{x} - \hbar\mathbf{k}t/m)^2}{4\lambda_t^2\sigma^2}} \quad (2)$$

with

$$\lambda_t = 1 + \frac{i\hbar t}{2m\sigma^2} \quad (3)$$

and arbitrary constants $\mathbf{k} \in \mathbb{R}^3$, $\sigma > 0$, is a solution of the free Schrödinger equation of a single particle in 3 dimensions,

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi. \quad (4)$$

The main difficulty here is to organize the calculation so as to make it manageable.

(b) Show that the probability density $\rho_t(\mathbf{x})$ is, at every t , Gaussian and specify its mean and standard deviation.

Hand in: Via `urm.math.uni-tuebingen.de` by Saturday October 18, 2025, at noon.

Reading assignment due Thursday October 23, 2025: R. Feynman, *Feynman Lectures on Physics* vol. 3, chapter 1.