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Winter semester 2025/26
November 20, 2025

Foundations of Quantum Mechanics: Assignment 7

Exercise 26: Essay question (20 points)
Explain the concept of contextuality for the example of a quantum measurement of σ_3.

Exercise 27: Half Angles (30 points)
(a) Show that for unit vectors ϕ, χ in spin space S,

$$2\big|\langle\phi|\chi\rangle\big|^2 = 1 + \sum_{a=1}^{3} \langle\phi|\sigma_a\phi\rangle\langle\chi|\sigma_a\chi\rangle\,.$$

(b) Conclude further that if $\mathbb{C}\phi$ and $\mathbb{C}\chi$ have angle $\theta = \arccos\big|\langle\phi|\chi\rangle\big|$ in S, then $\boldsymbol{\omega}(\phi)$ and $\boldsymbol{\omega}(\chi)$ have angle 2θ in $\mathbb{R}^3$.

Exercise 28: Iterated Stern-Gerlach experiment (20 points)
Consider the following experiment on a single electron. Suppose it has a wave function of the product form $\psi_s(\boldsymbol{x}) = \phi_s\,\chi(\boldsymbol{x})$, and we focus only on the spinor. The initial spinor is $\phi = (1, 0)$.

(a) A Stern–Gerlach experiment in the y-direction (or σ_2-measurement) is carried out, then a Stern–Gerlach experiment in the z-direction (or σ_3-measurement). Both measurements taken together have four possible outcomes: up-up, up-down, down-up, down-down. Find the probabilities of the four outcomes.

(b) As in (a), but now the z-experiment comes first and the y-experiment afterwards.

Exercise 29: Projections (30 points)
We defined a projection to be an operator P such that there is an ONB $\{\phi_n : n \in \mathbb{N}\}$ diagonalizing P, $P\phi_n = \lambda_n\phi_n$, with eigenvalues λ_n that are 0 or 1.

(a) Show that the projections are exactly the self-adjoint operators P with $P^2 = P$.

(b) Suppose that $P : \mathscr{H} \to \mathscr{H}$ is a projection with range $\mathscr{K}$; one says that P is the projection to $\mathscr{K}$. Show that $I - P$ is the projection to the orthogonal complement of $\mathscr{K}$, i.e., to $\mathscr{K}^\perp = \{\phi \in \mathscr{H} : \langle\phi|\psi\rangle = 0\,\forall\psi \in \mathscr{K}\}$.

(c) Suppose that P is the projection to $\mathscr{K}$. Show that the element in $\mathscr{K}$ closest to a given vector $\psi \in \mathscr{H}$ is $P\psi$.

Hand in: By Saturday November 29, 2025, at noon via `urm.math.uni-tuebingen.de`

Reading assignment due Thursday December 4, 2025: T. Maudlin, Three Measurement Problems. *Topoi* **14(1)**: 7–15 (1995). Read pages 7–12 and the first two paragraphs on page 13.