

## FOUNDATIONS OF QUANTUM MECHANICS: ASSIGNMENT 9

### Exercise 33: Essay question. (20 points)

Describe the measurement problem of quantum mechanics. (Use formulas where appropriate.)

### Exercise 34: Exponential distribution (25 points)

Find the mean and variance of the exponential distribution with parameter  $\lambda$ ; it has density  $\rho(t) = 1_{t>0}\lambda e^{-\lambda t}$ .

### Exercise 35: GRW theory (25 points)

Consider the GRW theory with the constant  $\sigma$  much smaller than the value  $10^{-7}$  m suggested by GRW; say,  $\sigma = 10^{-12}$  m. Explain why Heisenberg's uncertainty relation implies that a free electron, after being hit by a GRW collapse, could move very fast. Use the uncertainty relation to compute the order of magnitude of how fast it can be (assuming it was more or less at rest before the collapse); the mass of an electron is about  $10^{-30}$  kg and  $\hbar \approx 10^{-34}$  kg m<sup>2</sup> s<sup>-1</sup>.

### Exercise 36: Poisson process (30 points)

For the Poisson process with rate  $\lambda > 0$ , determine for any fixed  $t_0 > 0$  the distribution of  $X_{t_0} = \#\{k \in \mathbb{N} : T_k \leq t_0\}$ , the number of events up to time  $t_0$ . Follow two reasonings:

(a) Heuristically, assume that an event occurs in every infinitesimal time interval  $[t, t + dt]$  independently of disjoint intervals with probability  $\lambda dt$ .

*Hint:* Divide  $[0, t_0]$  in  $n \gg 1$  subintervals of length  $dt = t_0/n$ .

(b) Rigorously, assume that the random variables  $T_1, T_2, \dots$  are defined to be  $T_k = W_1 + \dots + W_k$  with all waiting times  $W_k$  independent and exponentially distributed with parameter  $\lambda$ , i.e., with density  $\rho(w) = 1_{w>0}\lambda e^{-\lambda w}$ .

*Hint:*

$$\mathbb{P}(X_{t_0} \geq 2) = \mathbb{P}(W_1 + W_2 < t_0) = \int_0^{t_0} dw_1 \int_0^{t_0-w_1} dw_2 \rho(w_1) \rho(w_2) \quad \text{and}$$

$$\mathbb{P}(X_{t_0} = k) = \mathbb{P}(X_{t_0} \geq k) - \mathbb{P}(X_{t_0} \geq k + 1).$$

**Hand in:** by Saturday December 13, 2025, at noon via [urm.math.uni-tuebingen.de](http://urm.math.uni-tuebingen.de)

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**Reading assignment** due Thursday December 18, 2025:

Sections 1–3 and 5 of J. Bell: Are There Quantum Jumps? In *Schrödinger. Centenary Celebration of a Polymath*. Cambridge University Press (1987). Reprinted as chapter 22 in J. Bell, *Speakable and Unspeakable in Quantum Mechanics*, Cambridge University Press (1987).