

FOUNDATIONS OF QUANTUM MECHANICS: ASSIGNMENT 11

Exercise 42: Essay question. (25 points)

Explain why Schrödinger's theory Sm has a many-worlds character.

Exercise 43: Boundary conditions (25 points)

On the half axis $(-\infty, 0]$, consider the Schrödinger equation $i\hbar\partial\psi/\partial t = -(\hbar^2/2m)\partial^2\psi/\partial x^2$ with boundary condition

$$\alpha\frac{\partial\psi}{\partial x}(x=0) + \beta\psi(x=0) = 0 \quad (1)$$

with constants $\alpha, \beta \in \mathbb{C}$. For $\alpha = 0, \beta = 1$ this is called a *Dirichlet boundary condition*, for $\alpha = 1$ and $\beta = 0$ a *Neumann boundary condition*. [This is Carl Neumann (1832–1925; in Tübingen 1865–1868), not John von Neumann (1903–1957).] For general $(\alpha, \beta) \neq (0, 0)$ it is called a *Robin boundary condition*. Which choices of (α, β) imply that $j(x=0) = 0$? (They are reflecting boundary conditions and lead to a unitary time evolution.) Which imply that $j(x=0) > 0$ whenever $\psi(x=0) \neq 0$? (They are absorbing boundary conditions and lead to loss of probability.)

Exercise 44: Distinguish ensembles (20 points)

A source generates

(a) either 10,000 particle pairs in the spin singlet state

$$\frac{1}{\sqrt{2}}(|z\text{-up}\rangle|z\text{-down}\rangle - |z\text{-down}\rangle|z\text{-up}\rangle) = -\frac{1}{\sqrt{2}}(|x\text{-up}\rangle|x\text{-down}\rangle - |x\text{-down}\rangle|x\text{-up}\rangle)$$

(b) or, in a random order, 5,000 pairs in $|z\text{-up}\rangle|z\text{-down}\rangle$ and 5,000 in $|z\text{-down}\rangle|z\text{-up}\rangle$

(c) or, in a random order, 5,000 pairs in $|x\text{-up}\rangle|x\text{-down}\rangle$ and 5,000 in $|x\text{-down}\rangle|x\text{-up}\rangle$.

Alice and Bob are far from each other, and each receives one particle of every pair. By carrying out (local) Stern-Gerlach experiments on their particles and comparing their results afterwards, how can they decide whether the source was of type (a), (b), or (c)?

Exercise 45: Allcock's paradox¹ (30 points)

Allcock considered a "soft detector," i.e., one for which the particle may fly through the detector volume for a while before being detected. An as effective description, Allcock proposed an imaginary potential. For example, for a single particle in 1d and a detector in the right half axis, he considered the Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} - iv1_{x\geq 0}\psi \quad (2)$$

with $v > 0$ a constant. The time evolution then is not unitary.

(a) Derive from (2) the continuity equation

$$\frac{\partial\rho}{\partial t} = -\text{div } j - \frac{2v}{\hbar}1_{x\geq 0}\rho \quad (3)$$

for $\rho = |\psi|^2$ and $j = \frac{\hbar}{m}\text{Im}[\psi^*\partial\psi/\partial x]$. Eq. (3) is the evolution of the probability density of a particle that moves along Bohmian trajectories and disappears spontaneously (stochastically) at

¹G.R. Allcock: The time of arrival in quantum mechanics II. The individual measurement. *Annals of Physics* **53**: 286–310 (1969)

rate $2v/\hbar$ whenever it stays in the region $x \geq 0$. $\|\psi_t\|^2$ is a decreasing function of t and represents the probability that the particle has not been detected (and disappeared from the model) yet.

(b) To obtain an effective description of a hard detector (i.e., one that will detect the particle as soon as it reaches the region $x \geq 0$), Allcock assumed that ψ_0 is concentrated in $\{x < 0\}$ and took the limit $v \rightarrow \infty$, but found that the particle never gets detected ($\|\psi_t\|^2 = \text{const.}$)! That is parallel to the quantum Zeno effect.

Prove the following simplified version: In a 2d Hilbert space \mathbb{C}^2 , let $\psi_0 = (1, 0)$ evolve with the (non-self-adjoint) Hamiltonian

$$H_v = \begin{pmatrix} 0 & 1 \\ 1 & -iv \end{pmatrix}. \quad (4)$$

Then for every $t > 0$, $\psi_t = e^{-iH_v t/\hbar} \psi_0 \rightarrow \psi_0$ as $v \rightarrow \infty$.

Hand in: by Saturday January 17, 2026, at noon via urm.math.uni-tuebingen.de

Reading assignment due Thursday January 15, 2026:

N. Bohr: Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?
Physical Review **48**: 696–702 (1935)