Prof. Dr. Roderich Tumulka
Eberhard Karls University Tübingen

Winter semester 2025/26
January 15, 2026

Foundations of Quantum Mechanics: Assignment 12

Exercise 46: Essay question. (25 points)
Why does GRW theory make approximately the same predictions as the quantum formalism?

Exercise 47: Positive operators (25 points)
An operator $S:\mathbb{C}^d\to\mathbb{C}^d$ is positive (= positive semi-definite) iff $\langle\psi|S\psi\rangle\geq 0$ for all ψ. Are the following statements about operators on $\mathbb{C}^d$ true or false? Justify your answers.

a) $R^\dagger R$ is always a positive operator.

b) If E is a positive operator, then so is $R^\dagger E R$.

c) The positive operators form a subspace of the space of self-adjoint operators.

d) The sum of two projections is positive only if they commute.

e) e^{At} is a positive operator for every self-adjoint A and $t\in\mathbb{R}$.

Exercise 48: Sum of projections (25 points)
Let $\mathscr{H}$ be a Hilbert space of finite dimension, let P_1 and P_2 be projections in $\mathscr{H}$, $P_i=P_i^\dagger$ and $P_i^2=P_i$, and let $\mathscr{H}_i$ be the range of P_i. Show that if $Q:=P_1+P_2$ is also a projection ($Q=Q^\dagger$ and $Q^2=Q$), then **(a)** $\mathscr{H}_1\perp\mathscr{H}_2$, and **(b)** the range $\mathscr{K}$ of Q is the span of $\mathscr{H}_1\cup\mathscr{H}_2$.

Exercise 49: POVMs (25 points)
(a) Suppose E_1 and E_2 are POVMs on $\mathscr{Z}_1$ and $\mathscr{Z}_2$, respectively, both acting on $\mathscr{H}$; let $q_1,q_2\in[0,1]$ with $q_1+q_2=1$. Show that $E(B):=q_1E_1(B\cap\mathscr{Z}_1)+q_2E_2(B\cap\mathscr{Z}_2)$ defines a POVM on $\mathscr{Z}_1\cup\mathscr{Z}_2$.

(b) Suppose experiment $\mathscr{E}_1$ has distribution of outcomes $\langle\psi|E_1(\cdot)|\psi\rangle$, and $\mathscr{E}_2$ has distribution of outcomes $\langle\psi|E_2(\cdot)|\psi\rangle$. Describe an experiment with distribution of outcomes $\langle\psi|E(\cdot)|\psi\rangle$.

(c) Give an example of a POVM for which the E_z do not pairwise commute. *Suggestion*: Choose $E_1(z)$ that does not commute with $E_2(z')$ for $\mathscr{Z}_1\cap\mathscr{Z}_2=\emptyset$.

Hand in: by Saturday January 24, 2026, at noon via `urm.math.uni-tuebingen.de`

Reading assignment due Thursday January 22, 2026:
J. Bell: Six possible worlds of quantum mechanics. Talk given at the Symposium "Possible Worlds in Arts and Sciences" (1986), reprinted in *Speakable and Unspeakable in Quantum Mechanics* (1987), pages 181–195.