Prof. Dr. Roderich Tumulka
Eberhard Karls University Tübingen

Winter semester 2025/26
January 22, 2026

Foundations of Quantum Mechanics: Assignment 13

Exercise 50: Essay question. (25 points)
Describe Einstein's boxes argument.

Exercise 51: Can't distinguish non-orthogonal state vectors with POVMs (25 points)
In Exercise 32(b) in Assignment 8, it was shown that Bob, when allowed to use a quantum measurement of *any self-adjoint operator* on a given particle, is unable to decide with certainty whether the quantum state was $(1,0)$ or $\frac{1}{\sqrt{2}}(1,1)$. What if Bob is allowed to use *any experiment whatsoever*? Use the main theorem about POVMs.

Exercise 52: Main theorem about POVMs (25 points)
The proof of the main theorem from Bohmian mechanics assumes that at the initial time t_i of the experiment, the joint wave function factorizes, $\Psi_{t_i} = \psi \otimes \phi$. What if factorization is not exactly satisfied, but only approximately? Then the probability distribution of the outcome Z is still approximately given by $\langle\psi|E(\cdot)|\psi\rangle$. To make this statement precise, suppose that

$$\Psi_{t_i} = c\psi \otimes \phi + \Delta\Psi\,, \tag{1}$$

where $\|\Delta\Psi\| \ll 1$ (you can use $\|\Delta\Psi\| < 1/2$), $\|\psi\| = \|\phi\| = 1$, and $c = \sqrt{1 - \|\Delta\Psi\|^2}$ (which is close to 1). Use the Cauchy-Schwarz inequality,

$$\big|\langle f|g\rangle\big| \leq \|f\|\,\|g\|\,, \tag{2}$$

to show that, for any $B \subseteq \mathscr{Z}$,

$$\Big|\mathbb{P}(Z \in B) - \langle\psi|E(B)|\psi\rangle\Big| < 3\|\Delta\Psi\|\,. \tag{3}$$

Exercise 53: Statistical density matrix (25 points)
Show that a probability distribution μ over $\mathbb{S}(\mathscr{H})$ has density matrix $\rho_\mu = |\psi\rangle\langle\psi|$ if and only if μ is concentrated on the 1d subspace $\mathbb{C}\psi$ (i.e., $\Psi = e^{i\Theta}\psi$ with a random global phase factor).

Hand in: by Saturday January 31, 2026, at noon via `urm.math.uni-tuebingen.de`

Reading assignment due Thursday January 29, 2026:
J. Bell: Against 'measurement.' *Physics World*, August 1990, pages 33–40.