Micro : $H = \sum_{j=1}^{N} (i\nabla_j + \lambda \hat{A}(x_j))^2 + \frac{1}{N} \sum_{j \neq \ell} v(x_j - x_\ell) + \sum |k| a_k^* a_k$ $\quad i\partial_t \psi = H \psi_t$

Macro : $i \frac{d}{dt} \varphi_t = \underbrace{\left((i\nabla + \lambda A)^2 + V * |\varphi_t|^2 \right)}_{h^{mf}} \varphi_t$

$\nabla A = 0$ ✓

$\partial_t A = -E_\perp$ ✓ $\qquad -i[H, A] = E$

$\partial_t E_\perp = -\Delta A - j_\perp$

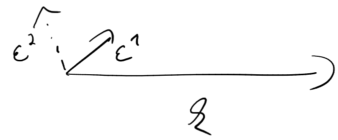
$\to j := \mathfrak{Im} \left(\varphi_t (\nabla - iA) \varphi_t^* \right)$

$$\alpha := \underbrace{\langle \psi, a_1 \psi \rangle}_{\alpha_a} + \underbrace{\lambda^2 \langle \psi, (\hat{A} - A)^2 \psi \rangle}_{\alpha_b} + \underbrace{\lambda^2 \langle \psi, (\hat{E}_\perp - E_\perp)^2 \psi \rangle}_{\alpha_c} \qquad \lambda = \frac{1}{\sqrt{N}}$$

$$\Rightarrow \hat{A} = \sum_{\kappa=1}^{2} \sum_{k} \frac{\vec{\varepsilon}_k^{\,\kappa}}{\sqrt{|k|}} \left(e^{ikx} \cdot a^*(k,\kappa) + e^{-ikx} a(k,\kappa) \right)$$

$\vec{\varepsilon}_k^{\,1/2}$ are unit vectors, orthogonal to $\vec{k}$ and to each other

$$\hat{E}_\perp = \sum_{k=1}^{2} \sum_{k} \vec{\varepsilon}_k^{\,\kappa} \cdot \sqrt{|k|}\, i \left(e^{ikx} a^*(k,\kappa) - e^{-ikx} a(k,\kappa) \right)$$



The time derivative of quantum mech. operators are given by the commutator with H.

Theorem : "Assuming that ∇ is bounded" (replace $\hat{\nabla} = k$ by $\frac{k}{1+|k|}$) and introducing an IR and UV-cutoff we can show that α fulfills a Grönwall inequality :

$$\dot{\alpha} \leq C \cdot \alpha + o(1) \qquad \lim_{N\to\infty} o(1) = 0.$$

Proof : We proof that $\alpha_{a,b,c} \leq C \cdot \alpha + o(1)$ for a, b, c separately.

a) $\left| \frac{d}{dt} \alpha_a \right| = \left| \langle \psi_t \left[(H - H^{mf}), q_1 \right] \psi_t \rangle \right| \qquad H^{mf} = \sum_{j=1}^{N} h_j^{mf}$

$$\leq C \cdot (\mathrm{I} + \mathrm{II} + \mathrm{III}) + \left| \langle \psi_t, \left[(i\nabla_1 + \hat{A}(x_1))^2 - (i\nabla_1 + A(x_1))^2, q_1 \right] \psi_t \rangle \right|$$

$|\langle \psi_{t}, [\sum v(x_1 - x_j) - v * |\varphi|^2, q_1] \psi \rangle|$

old

$$\leq C \left(\alpha_a + \frac{1}{N} \right) + 2 \left| \langle \psi_t, \left(2\nabla_1 \lambda (\hat{A} - A) + (\underbrace{\lambda^2 \hat{A}^2 - \lambda^2 A^2}_{(\lambda\hat{A} - \lambda A)(\lambda\hat{A} + \lambda A)}) \right) q_1 \psi_t \rangle \right|$$

$$\leq C \left(\alpha_a + \frac{1}{N} \right) + C \lambda \underbrace{\| (\hat{A} - A) \psi \|}_{\sqrt{\alpha_b} \leq \sqrt{\alpha}} \cdot \underbrace{\| q_1 \psi \|}_{\sqrt{\alpha_a} \leq \sqrt{\alpha}} \leq \boxed{C \left(\alpha + \frac{1}{N} \right)}$$

under our assumptions ∇_1 and $(\lambda\hat{A} + \lambda A)$ are bounded operators.

$\lambda \hat{A}$ can be bounded : Due to the IR. cutoff the number of Photons is bounded by the total energy $\sim N$.

b) $\left| \frac{d}{dt} \langle \psi_t, (\hat{A} - A_t)^2 \psi_t \rangle \right| \leq \left| \langle \psi_t, 2(-\dot{A})(\hat{A} - A) \psi_t \rangle - i \langle \psi_t, [H, (\hat{A} - A)^2] \psi_t \rangle \right|$

$$[A, B^2] = AB^2 - B^2 A = [A, B]B + B[A, B]$$

$$\leq 2 \left| \langle \psi_t, \left[-\dot{A} (\hat{A} - A) - i [H, \hat{A}] (\hat{A} - A) \right] \psi_t \rangle \right.$$

$$\leq 2 \left| \langle \psi_t, \left(E_\perp - i\left[\sum |k| a_k^* a_k, \hat{A}\right] \right) (\hat{A} - A)\psi_t \rangle \right. = \circledast$$

$$\left[\cancel{\sum} |k| a_k^* a_k, \sum \frac{\varepsilon}{\sqrt{|k|}} \left(e^{ikx} a^* + e^{-ikx} a\right)\right] = \sum \overbrace{|k|}^{\sqrt{|k|}} \frac{\varepsilon}{\sqrt{|k|}} \left(a^* e^{ikx} - a e^{-ikx}\right)$$

$$-i\, E_\perp$$

$$\circledast = 2 \left| \langle \psi_t, (E_\perp - \hat{E}_\perp) (\hat{A} - A)\psi_t \rangle \right| \leq 2 \|(E_\perp - \hat{E}_\perp)\psi\| \cdot \|(\hat{A} - A)\psi\| \leq \frac{2\sqrt{\alpha_b}\sqrt{\alpha_c}}{\lambda^2} \leq \frac{2\alpha}{\lambda^2}$$

$$\Rightarrow \quad \frac{d}{dt} \alpha_b \leq 2\alpha \quad \checkmark$$

c) $$\dot{\alpha}_c = \lambda^2 \langle \psi_t, 2(-\dot{E}_\perp) \cdot (\hat{E} - E_\perp)\psi_t \rangle - i \langle \psi_t [H, (\hat{E} - E)^2] \psi_t \rangle$$

$$|\dot{\alpha}_c| \leq \frac{2}{N} \left| \langle \psi_t \left(\boxed{-\dot{E}_\perp - i[H, (\hat{E} - E)]} \right) (\hat{E} - E)\psi_t \rangle \right. \quad + \text{ conjugat}$$

$$i[H, \hat{E}] = i\left[\sum |k| a^*(k) a(k), \hat{E}\right] + i\left[\sum_j [(i\nabla_j + \hat{A}(x_j)]^2, \hat{E}(x_1)\right]$$

$$\boxed{i[H, E] = i\left[\sum_j (i\nabla_j + A(x_j))^2, E_\perp\right]} = 0 \qquad \text{since} \quad \partial_t \nabla A = 0 \quad \nabla \partial_t A = \nabla E_\perp$$

$$\dot{E}_\perp = -\Delta A - j_\perp$$

$$i\left[\sum k\, a^*(k) a(k), \hat{E}\right] = i\left[\cancel{\sum} |k| a^*(k) a(k), \sum \sqrt{|k|}\, \varepsilon \cdot i \left(e^{ikx} a^*(k) - e^{-ikx} a(k)\right)\right]$$

$$= i \sum \frac{k^2}{\sqrt{|k|}} \varepsilon \cdot i \left(e^{ikx} a^* + e^{-ikx} a(k)\right) = -\Delta \hat{A}$$

Assum the Δ is bounded the green parts almost cancel: $C \cdot \|(\hat{A} - A)\psi\| \cdot \|(\hat{E} - E)\psi\| \leq C \cdot \alpha.$

The terms left are " $i\left[\sum_{j=1}^N (i\nabla_j + \hat{A}(x_j))^2, \hat{E}(x_1)\right]$ and $-j_\perp$ "

$$i\left[\sum_{j=1}^N (i\nabla_j + \hat{A}(x_j))^2, \hat{E}(x_1)\right] = i \sum_{j=1}^N \left[\boxed{(i\nabla_j + \hat{A}(x_j)) \cdot [i\nabla_j + \hat{A}(x_j), \hat{E}(x_1)]} + [\ldots] \cdot (i\nabla_j + \hat{A}(x_j)) \right]$$

$$[i\nabla_j + \hat{A}(x_j), \hat{E}_\perp] = [\hat{A}(x_j), \hat{E}_\perp(x_1)] \qquad \text{because} \quad \nabla e^{ikx} \parallel k \qquad \vec{\varepsilon} \perp k$$

$$= \sum_{K=1}^{2} \sum_k i \left[e^{ikx_j} a^*(k,K) + e^{-ikx_j} a(k,K), e^{ikx_1} a^*(k,K) - e^{-ikx_1} a(k,K)\right] =$$

$$= \sum_K \sum_k i \left([a^*(k,K), -a] e^{ik(x_j - x_1)} + [a, a^*] e^{-ik(x_j - x_1)} \right) = \underline{\sum_K} \underbrace{\sum_k \left(e^{ik(x_j - x_1)} + e^{-ik(x_j - x_1)}\right)}_{\delta(x_j - x_1)}$$

$$\frac{2}{N} \langle \psi_t, i \sum_{j=1}^N (i\nabla_j + \hat{A}(x_j)) \delta(x_j - x_1) \left(\hat{E}(x_1) - E(x_1)\right) \psi_t \rangle = \frac{2}{N} \langle \psi, [(i\nabla_1 + A(x_1))^2, (\hat{E} - E)^2] \psi \rangle (\leq CN|$$

$$+ 2\frac{N}{N-1} \langle \psi_t, i(\nabla_2 + A(x_2)) \ldots \rangle + 2\frac{N}{N-1} \langle \psi, i(A_2 - \hat{A}_2) \delta(x_1 - x_2) (\hat{E}_\perp(x_1) - E_\perp(x_1)) \psi \rangle$$

$$\leq C_1' \cdot \sqrt{\alpha_b}\sqrt{\alpha_c} \leq C\alpha$$

$$\langle \Psi_t, \left(i\nabla_2 + A(x_2)\right) \delta(x_1 - x_2) \left(\hat{E}(x_1) - E(x_1)\right) \Psi_t \rangle =$$

$$\langle \Psi_t, (p_2 + q_2)\, i\,(i\nabla_2 + A(x_2))\ \ \delta(x_1 - x_2)\ (p_2 + q_2)\ (\hat{E}(x_1) - E(x_1))\ \Psi_t \rangle =$$

$$\mathrm{I}: \langle \Psi_t\, |\varphi(x_2)\rangle \langle \varphi(x_2)|\, i\,(i\nabla + A)(x_2)\ \ \delta(x_1 - x_2)\ |\varphi(x_2)\rangle \langle \varphi(x_2)|\, \left(\hat{E}(x_1) - E(x_1)\right) \Psi_t \rangle$$

From the expression $\left(\hat{E} - E\right) \cdot \left[\{(i\nabla + A)^2, \hat{E}\}\right]$ we get a conjugate expression

This term $\langle \varphi(x_2),\, i\left(i\nabla_2 + A(x_2)\right) \delta(x_1 - x_2)\, \varphi(x_2) \rangle$ into the imaginary part

$$\parallel$$

$$2\,\mathrm{Im} \underbrace{\int \varphi(x_2)^* \left(i\nabla_2 + A(x_2)\right) \delta(x_1 - x_2)\ \varphi(x_2)\ dx^2}_{\varphi(x_1)^* \left(i\nabla_1 + A(x_1)\right) \varphi(x_1)\ =\ j(x_1)}$$

The part j parallel to $E_\perp$ drops $\rightarrow$ $j_\perp$ is left.

$\boxed{\mathrm{II}}$, $\boxed{\mathrm{III}}$ and $\boxed{\mathrm{IV}}$ all have at least one q_2 somewhere, acting either on the "left" or the "right" Ψ_t. We do Cauchy Schwarz, moving $\hat{E} - E$ to the other Ψ_t. This can be done since $\hat{E}(x_1) - E(x_1)$ commutes with q_1.

We assumed all other operators to be bounded.

$$\leq \|q_1 \Psi_t\| \cdot C_1' \cdot \lambda \|(\hat{E} - E)\Psi\| \leq C\alpha$$