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Assignment 3
Wintersemester 2017/18

Foundations of Quantum Mechanics

Written homework due Wednesday November 15, 2017

Exercise 9: Essay question. How does Bohmian mechanics explain the double-slit experiment?

Exercise 10: Quantile rule
For a continuous probability distribution on $\mathbb{R}$ with density $\rho(x)$, the α-*quantile* for $0 < \alpha < 1$ is the point x_α where

$$\int_{-\infty}^{x_\alpha} \rho(x)\,dx = \alpha. \qquad (1)$$

The $\frac{1}{2}$-quantile is also known as the median, the $\frac{1}{4}$-quantile as the first quartile, the $\frac{3}{4}$-quantile as the third quartile, and the $\frac{n}{100}$-quantile as the n-th percentile. Show that in Bohmian mechanics in 1 dimension, if Q_0 is the α-quantile of $\rho = |\psi_0|^2$, then Q_t is the α-quantile of $\rho = |\psi_t|^2$.

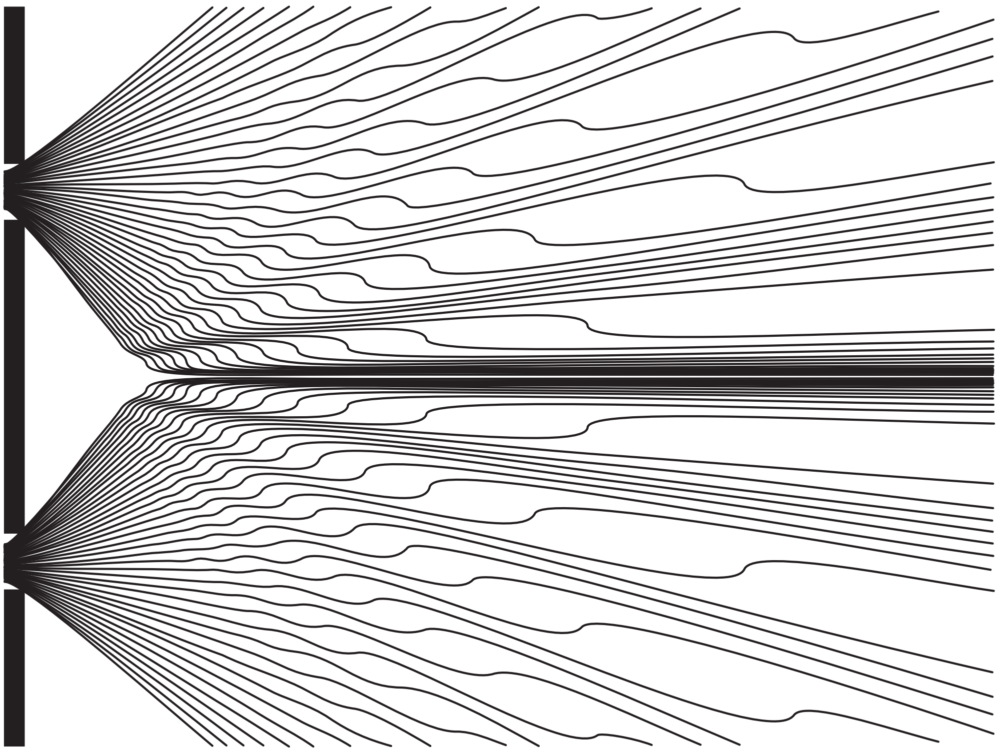

Abbildung 1: Bohmian trajectories for the double-slit experiment

Exercise 11: Optional team-work problem
Write a computer program (in a coding language of your choice) that generates Figure 1. Instructions: To simplify the calculation, it is assumed that the x-velocity is constant, that the x-axis is thus really the time axis, and that the problem can be regarded as Bohmian motion in 1 dimension (the y-axis). It is assumed that the wave packet coming out of each slit is a Gaussian wave packet as in Exercise 4 with mean velocity $\boldsymbol{k} = \boldsymbol{0}$ and equal width σ. The trajectories are computed by computing the quantiles of $|\psi_t|^2$ at every t.

Exercise 12: Entanglement
In a world with N particles, consider the subsystem formed by particles $1, \ldots, M$ (with $M < N$) and call it the x-system; let the y-system consist of all other particles. Let us write $x = (\boldsymbol{q}_1, \ldots, \boldsymbol{q}_M)$ and $y = (\boldsymbol{q}_{M+1}, \ldots, \boldsymbol{q}_N)$ for their respective configuration variables, and $q = (x, y)$ for the full configuration variable. One says that a wave function ψ is *disentangled* iff it is a product of a function of x and a function of y,

$$\psi(x, y) = \psi_1(x)\, \psi_2(y)\,, \tag{2}$$

otherwise ψ is called *entangled*. (Time plays no role in this consideration; all wave functions are considered at some fixed time.) For $N = 2$ particles, $M = 1$ in each system, give an example of an entangled and a disentangled wave function.

Exercise 13: Interaction and entanglement
Consider again the x-system and y-system of Exercise 12. If

$$V(x, y) = V_1(x) + V_2(y) \tag{3}$$

then one says that the two systems *do not interact*. In this exercise, we investigate the consequences of this condition.

(a) Show that in Newtonian mechanics, as given by Eq. (4.1) with the potential (3), the force acting on any particle belonging to the x-system is independent of the configuration of the y-system. Conclude further that the Newtonian trajectory $Q(t) = (X(t), Y(t))$ is such that $X(t)$ obeys the M-particle version of (4.1) with potential V_1, and $Y(t)$ the $(N - M)$-particle version with potential V_2.

(b) Show that if Eq. (3) holds and the wave function is disentangled initially,

$$\psi_0(x, y) = \psi_{1,0}(x)\, \psi_{2,0}(y) \tag{4}$$

then it is disentangled at all times t,

$$\psi_t(x, y) = \psi_{1,t}(x)\, \psi_{2,t}(y)\,, \tag{5}$$

where each factor $\psi_{i,t}$ evolves according to the (M-particle, respectively $(N - M)$-particle) Schrödinger equation with potential V_i.

(c) Show that, in the situation of (b), the Bohmian velocity of any particle belonging to the x-system is independent of the configuration of the y-system. Conclude further that the Bohmian trajectory $Q(t) = (X(t), Y(t))$ is such that $X(t)$ obeys the M-particle version of Bohm's equation of motion (6.1) with wave function ψ_1, and $Y(t)$ the $(N - M)$-particle version with wave function ψ_2.

No reading assignment this week.