Foundations of Quantum Mechanics

Written homework due Wednesday December 13, 2017

Exercise 27: Essay question. Why does GRW theory make approximately the same predictions as the quantum formalism?

Exercise 28: Uncertainty relation
Compute both sides of the generalized uncertainty relation

$$\sigma_A \, \sigma_B \geq \frac{1}{2} \Big| \langle \psi | [A, B] | \psi \rangle \Big| \tag{1}$$

for $A = \sigma_1$, $B = \sigma_2$, and $\psi = |z\text{-down}\rangle$.
Hint: In order to obtain the standard deviations σ_A and σ_B, compute first the probability distribution for A and B according to Born's rule.

Exercise 29: Poisson process
For the Poisson process with rate $\lambda > 0$, determine for any fixed $t_0 > 0$ the distribution of $X_{t_0} = \#\{k : T_k < t_0\}$, the number of events up to time t_0. Follow two reasonings:

(a) Heuristically, assume that an event occurs in every infinitesimal time interval $[t, t + dt]$ independently of disjoint intervals with probability $\lambda \, dt$.
Hint: Divide $[0, t_0]$ in $n \gg 1$ subintervals of length $dt = t_0/n$.

(b) Rigorously, assume that the random variables $T_1, T_2, \ldots$ are defined to be $T_k = W_1 + \ldots + W_k$ with all waiting times W_k independent and exponentially distributed with parameter λ, i.e., with density $\rho(w) = 1_{w>0} \, \lambda \, e^{-\lambda w}$.
Hint:

$$\mathbb{P}\big(X_{t_0} \geq 2\big) = \mathbb{P}\Big(W_1 + W_2 < t_0\Big) = \int_0^{t_0} dw_1 \int_0^{t_0 - w_1} dw_2 \, \rho(w_1) \, \rho(w_2) \quad \text{and}$$

$$\mathbb{P}(X_{t_0} = k) \; = \; \mathbb{P}\big(X_{t_0} \geq k\big) - \mathbb{P}\big(X_{t_0} \geq k+1\big) \, .$$

Exercise 30: Spin singlet state
Verify through direct computation that in the spin space $\mathbb{C}^4 = \mathbb{C}^2 \otimes \mathbb{C}^2$ of two spin-$\frac{1}{2}$ particles,

$$\begin{aligned} & |x\text{-up}\rangle |x\text{-down}\rangle - |x\text{-down}\rangle |x\text{-up}\rangle \\ = \; & |y\text{-up}\rangle |y\text{-down}\rangle - |y\text{-down}\rangle |y\text{-up}\rangle \\ = \; & |z\text{-up}\rangle |z\text{-down}\rangle - |z\text{-down}\rangle |z\text{-up}\rangle \end{aligned} \tag{2}$$

up to phase factors.

Reading assignment due Friday December 15, 2017: J. Bell: Six possible worlds of quantum mechanics. *Speakable and Unspeakable in Quantum Mechanics,* pages 181–195.